Astrophysical transport calculations inspired by chemistry

-or-

Computing with tube dynamics

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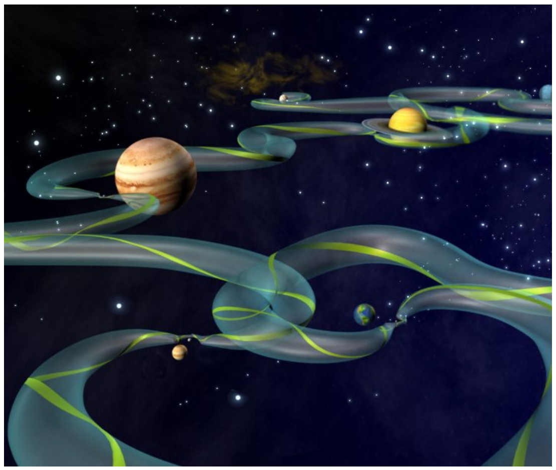
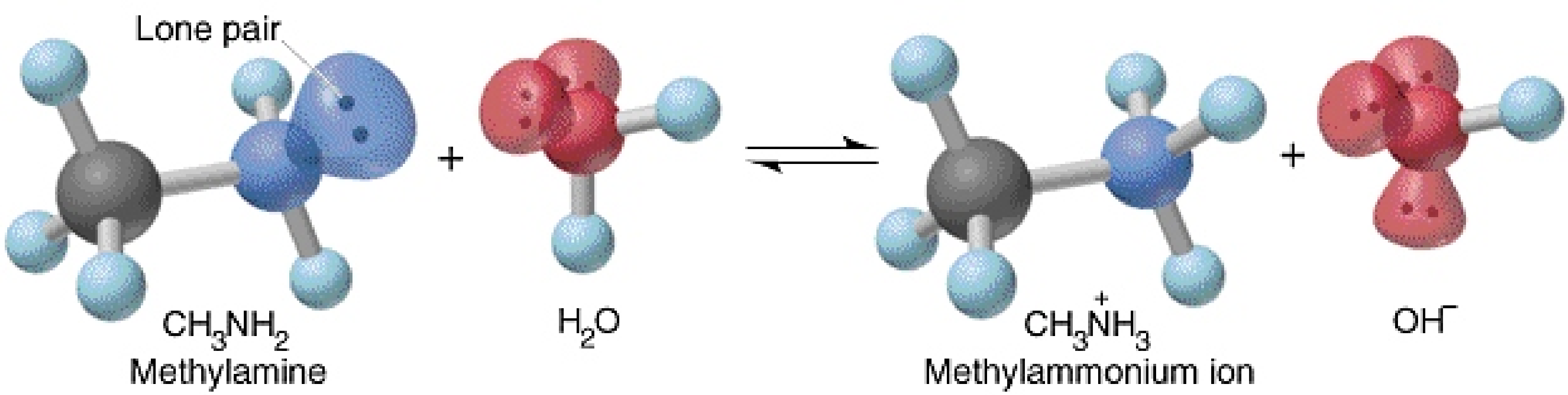
Motivation

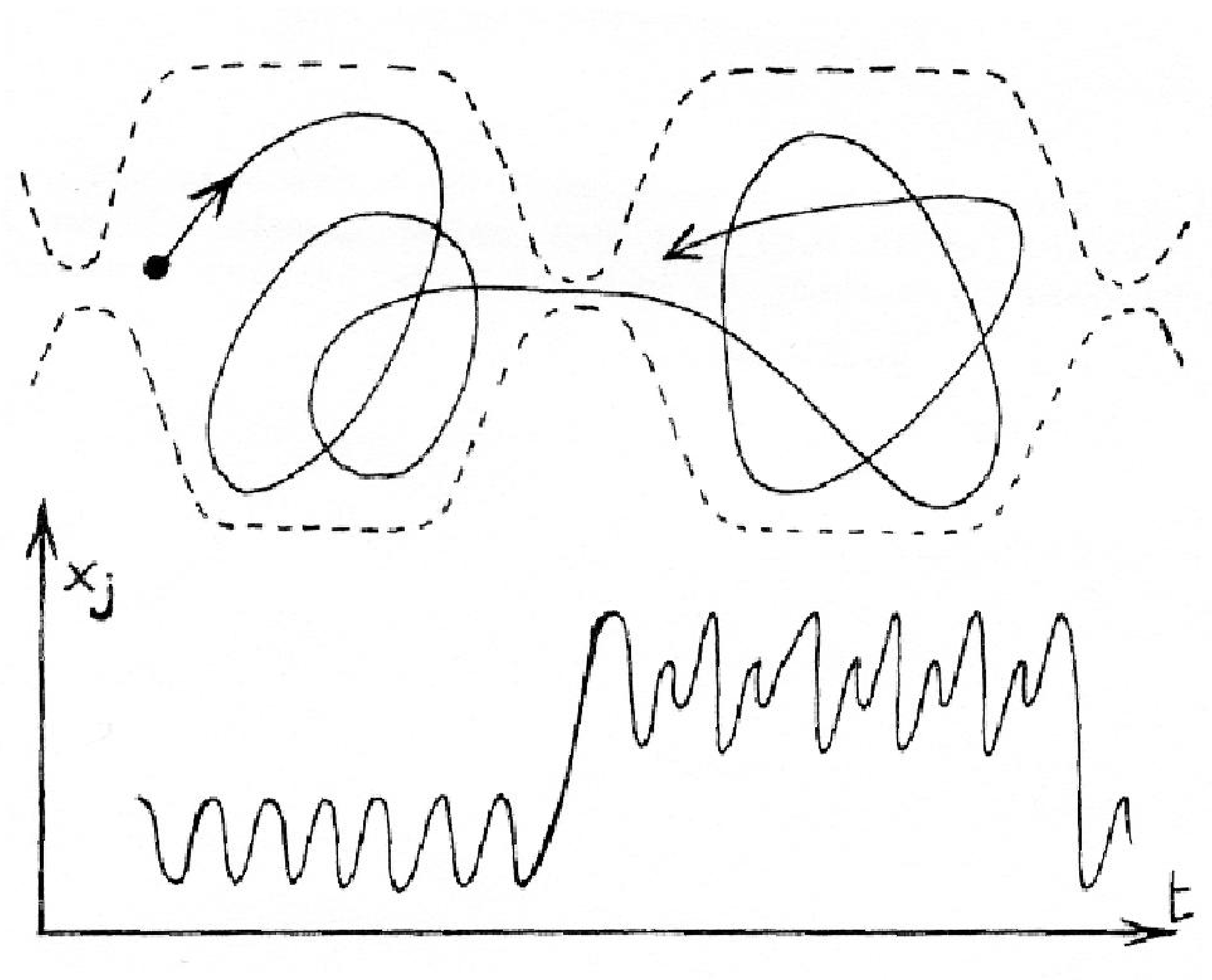
- N-body and fluid systems – phase space transport



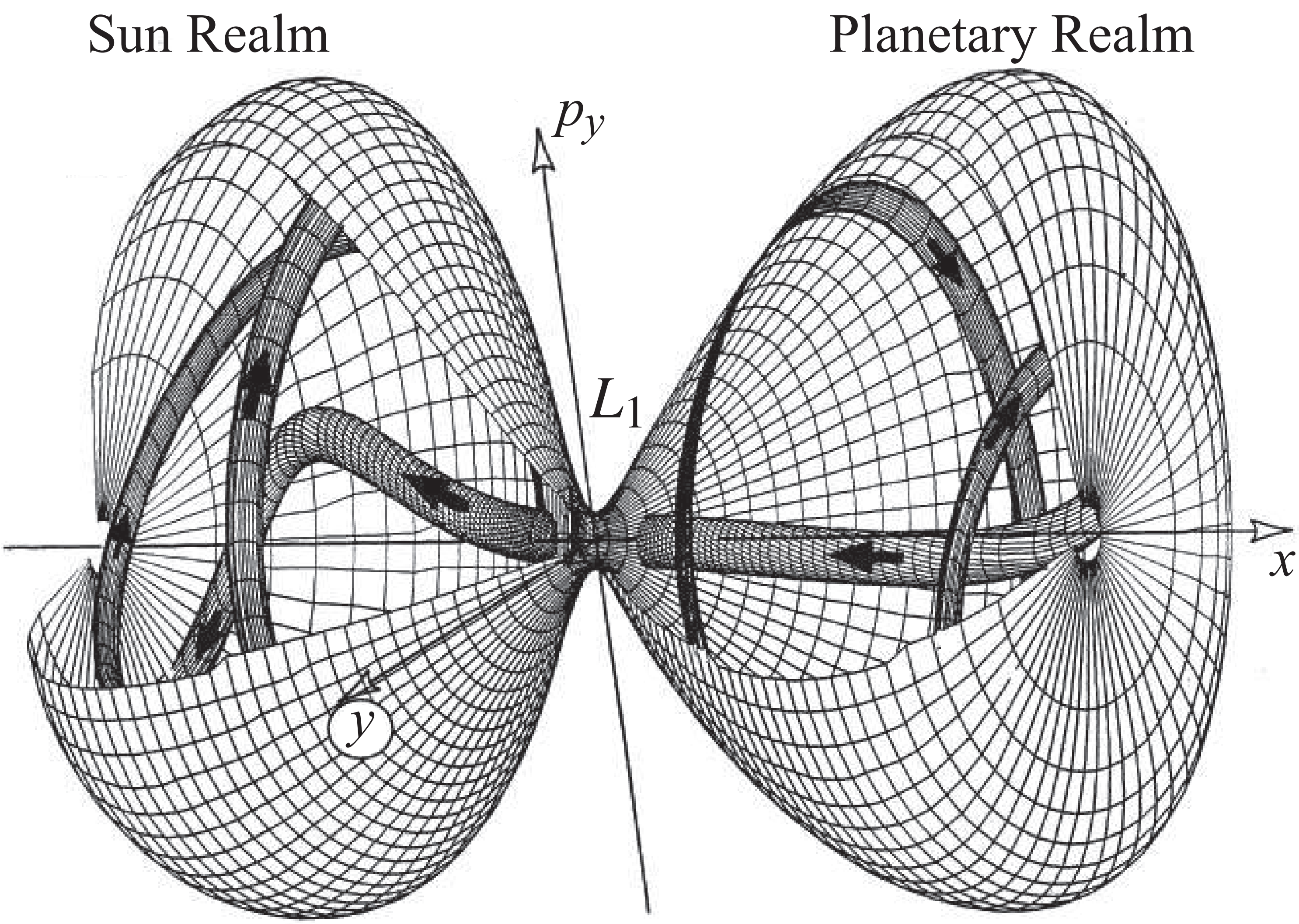


Partition phase space into realms

"reactants" *"products"*



Realms connected by tubes



adapted from Topper [1997]

"Interplanetary Superhighway"

sciencenews.org's version of the tubes

Important ideas

- ***Consider $N \ll$ trillion***
  - Planet and planetary system scale

Important ideas

Consider $N \ll$ *trillion*

- Planet and planetary system scale
- Chaotic transport of small bodies via Hamiltonian flow
  – flow due to point masses or distended bodies
  – low dimensional phase space ($\sim$ 6D)
- Phase space structures mediating transport
  – tube and lobe dynamics

Important ideas

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- Planet and planetary system scale
- Chaotic transport of small bodies via Hamiltonian flow
  – flow due to point masses or distended bodies
  – low dimensional phase space ($\sim$ 6D)
- Phase space structures mediating transport
  – tube and lobe dynamics
- **Approximate statistical models** may be appropriate under certain conditions
  – statistical assumptions in chemistry
  – amounts to phase space volume determinations

Outline of talk

- Some questions about solar system populations
- Crash course in tube dynamics
- Some answers
  – squeezing phase space for all its worth

Some questions

- Transport & evolution of some solar system populations

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  - How do we characterize the motion of
    - Jupiter-family comets
    - scattered Kuiper Belt objects
    - Mars and Earth-encountering asteroids

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  - During encounter:
    - Statistics of temporary capture time
    - Transition probablity between the exterior and interior regions?
    - Probability of comet collision with Jupiter?
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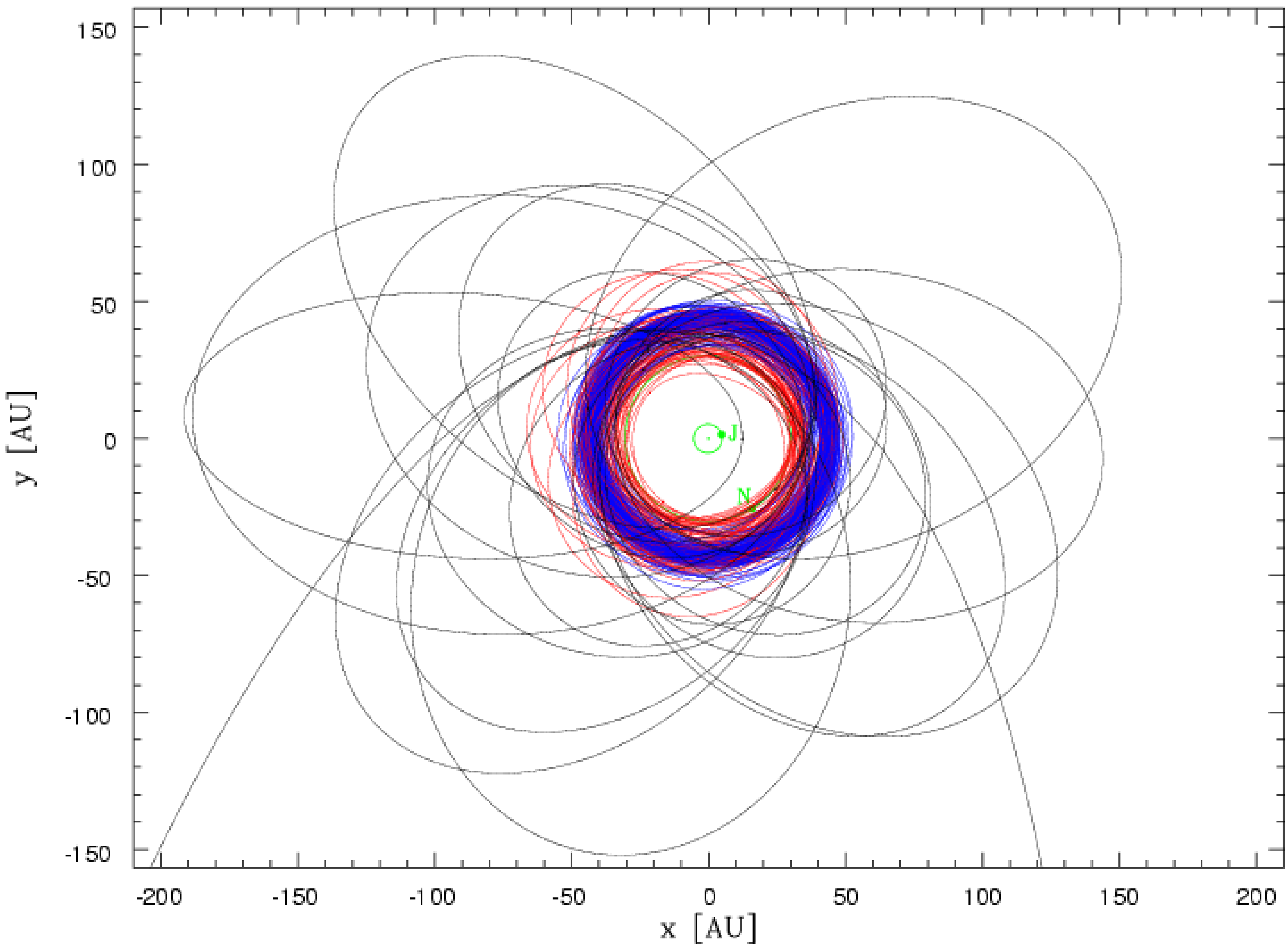
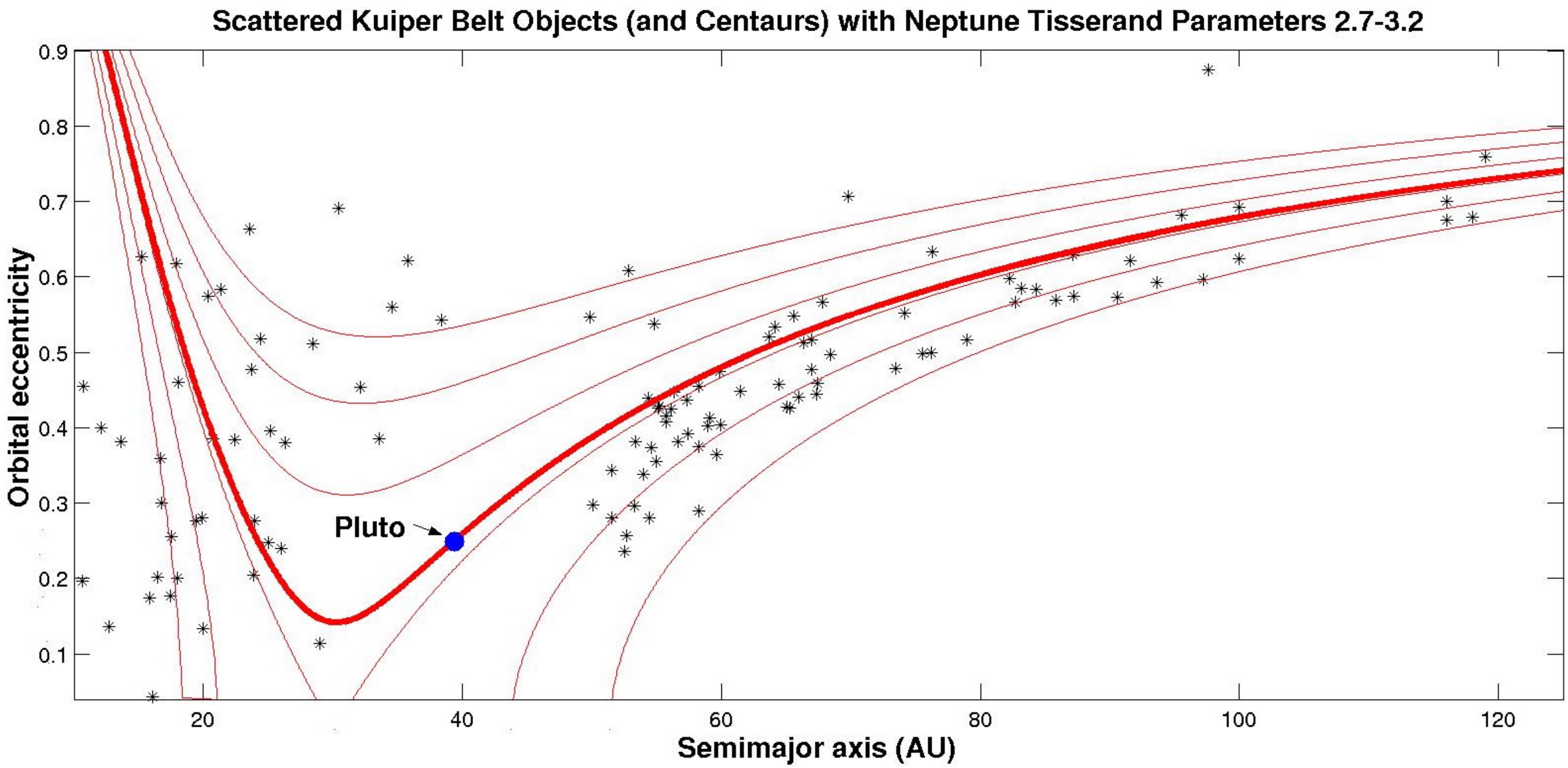
  - Binary asteroids
    - Ejecta escape and re-capture
  - Other situations: planetary ejecta transfer, drag perturbed case

Scattered Kuiper Belt Objects



◦ Seen in inertial space

Scattered Kuiper Belt Objects



- Current SKBO locations in black, with some Tisserand values w.r.t. Neptune in red ($T \approx 3$)

Motion within energy shell

- Recall the circular restricted three-body problem from Jerry Marsden's talk
- Energy shell of energy E is codim-1 surface

$$\mathcal{M}(E) = \{(\mathrm{q}, \mathrm{p}) \mid H(\mathrm{q}, \mathrm{p}) = E\}.$$

- The $\mathcal{M}(E)$ are 5-dimensional surfaces foliating the 6-dimensional phase space.

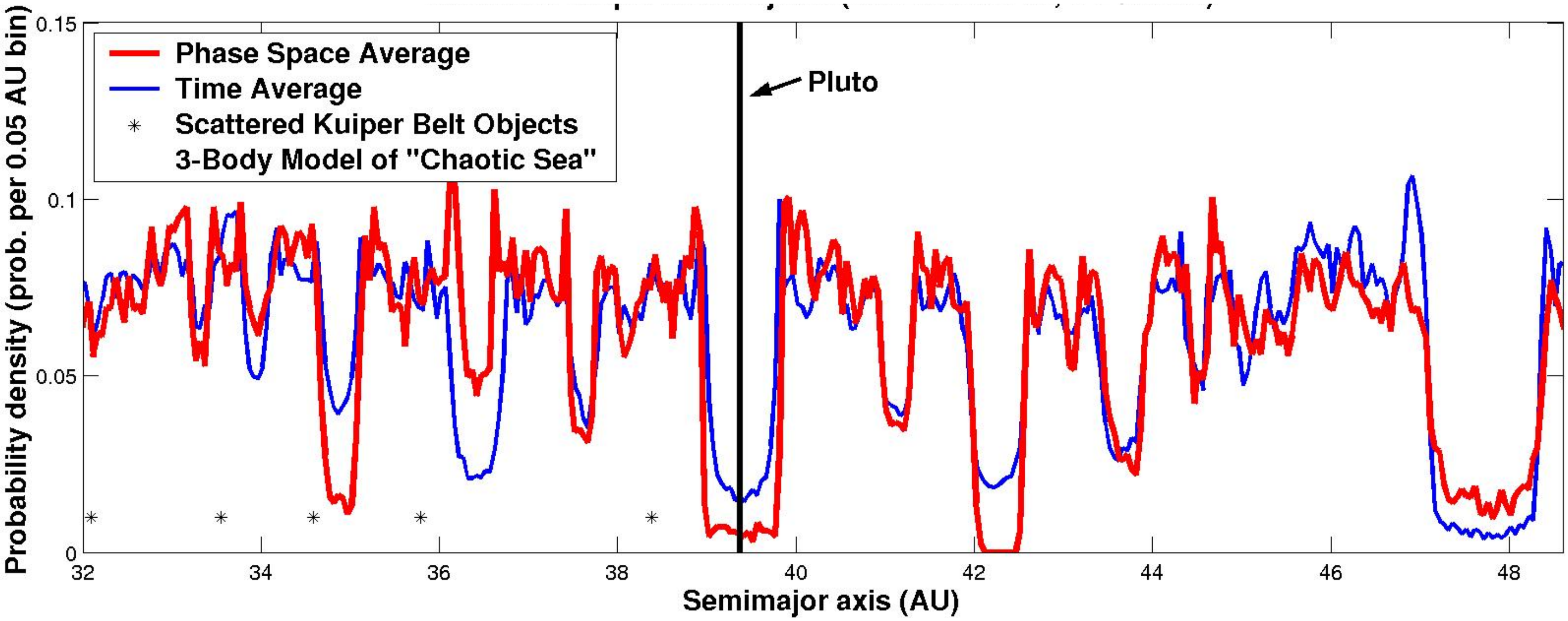
Probability density function

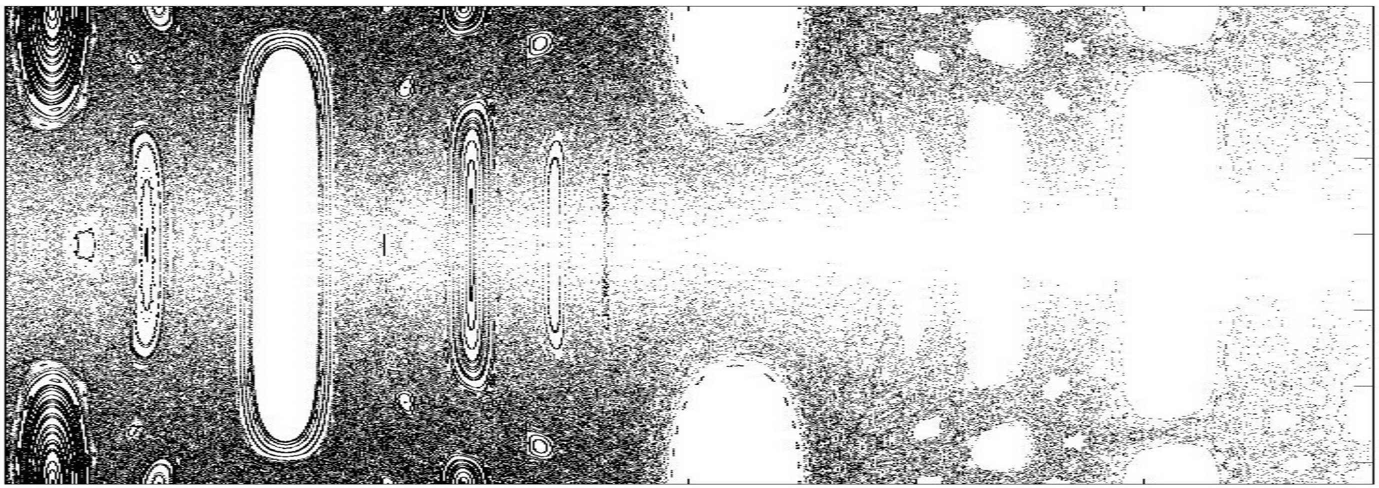
- Recent work suggests there may be regions of the energy shell for which the motion is nearly ergodic, e.g., large connected chaotic sea[1]
- Compute **probability density function** of some function $F(\mathrm{q}, \mathrm{p})$, directly from phase space
  – e.g., semimajor axis probability density function

[1] Jaffe, C., S. D. Ross, M. W. Lo, J. Marsden, D. Farrelly, and T. Uzer [2002] Statistical theory of asteroid escape rates, *Physical Review Letters* 89, 011101, and G. Tancredi [1995] The dynamical memory of Jupiter Family comets, *Astron. Astroph.* 299, 288–292

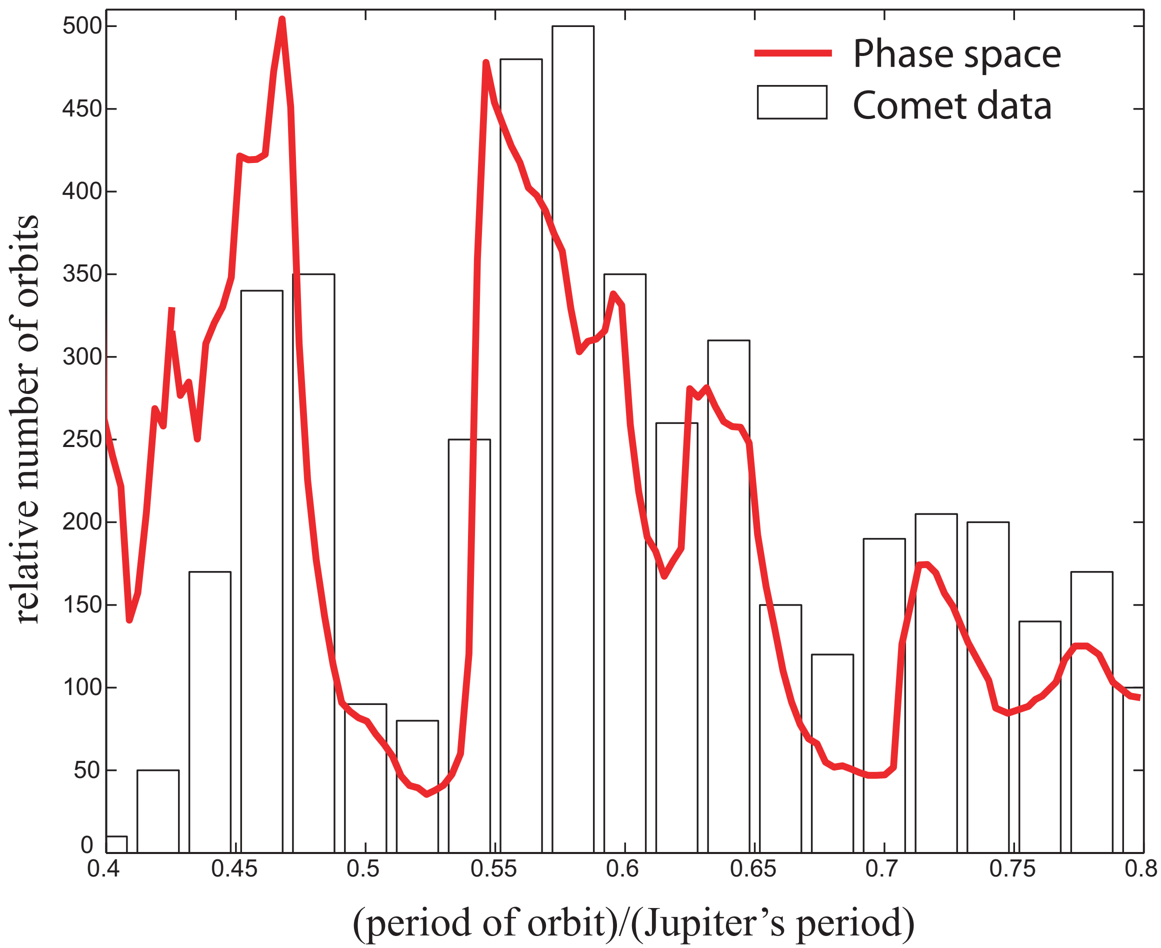
Probability density function

- SKBOs expected in regions of high density.



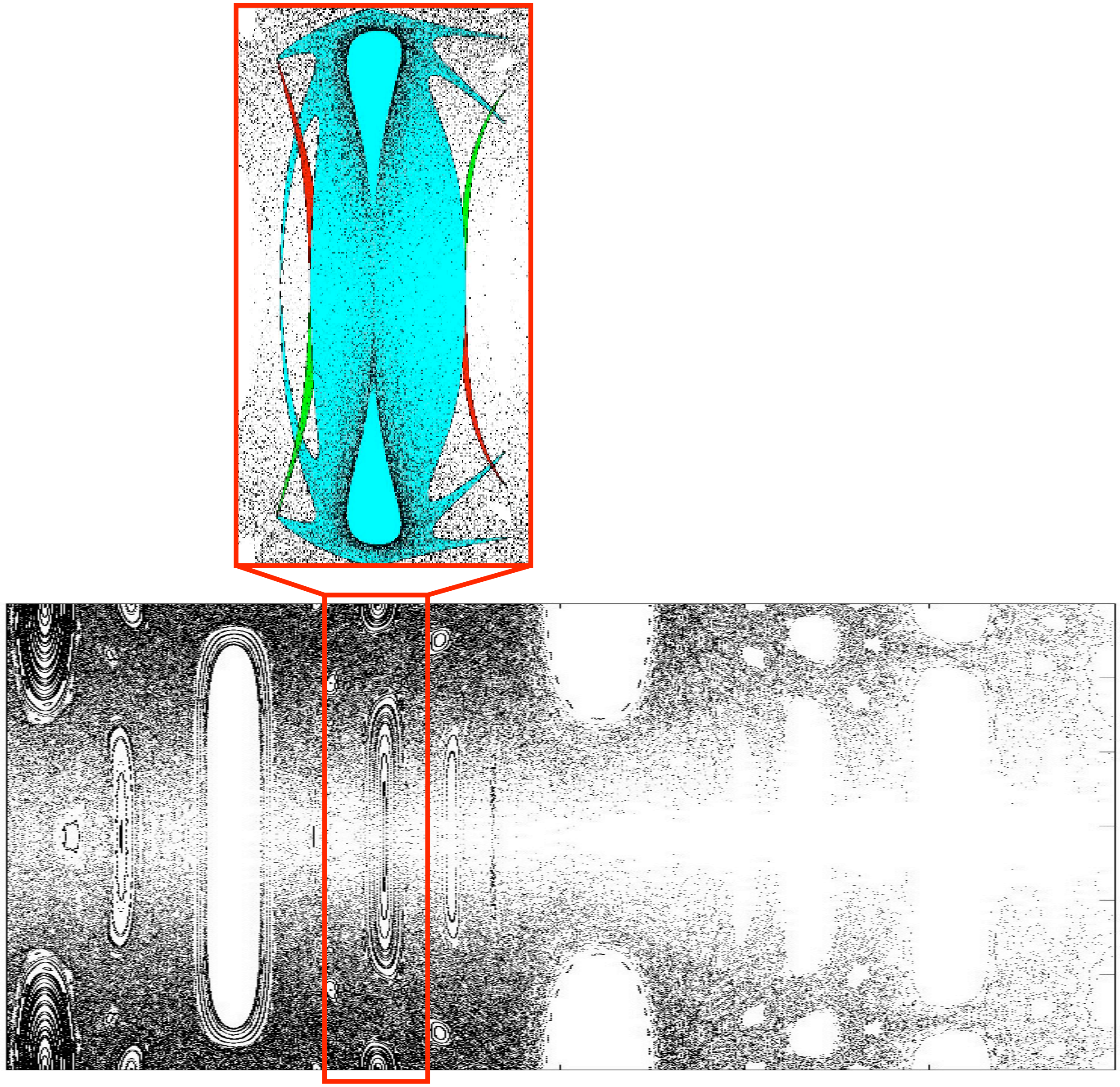


Probability density function



- Similar analysis can be done for
  - Jupiter family comets (above)
  - Earth- and Mars-encountering asteroids
- Summing over energy layers gives full picture

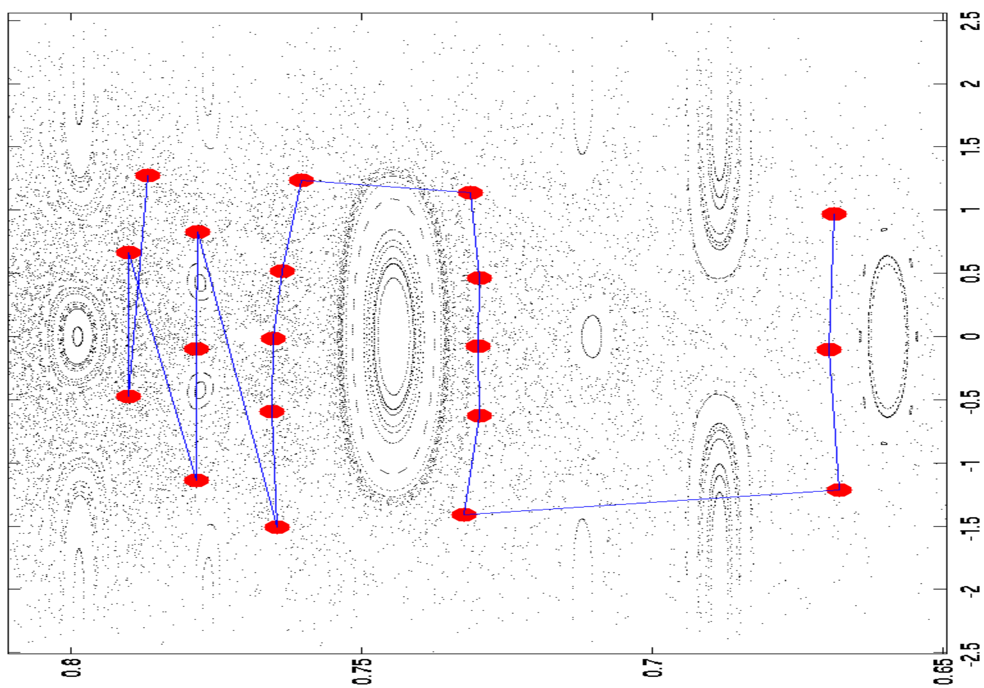
Movement around stable resonances via lobes



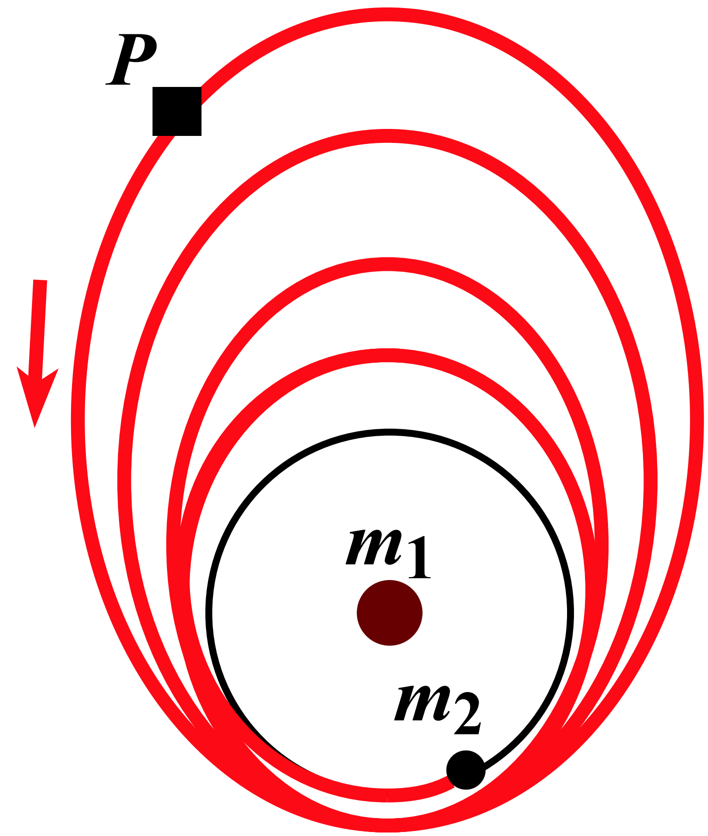
see Ross, Koon, Lo, Marsden [2003], Meiss [1992] and Schroer and Ott [1997]

Movement around stable resonances via lobes

- Scattering via successive close approaches
  – by moving around resonances

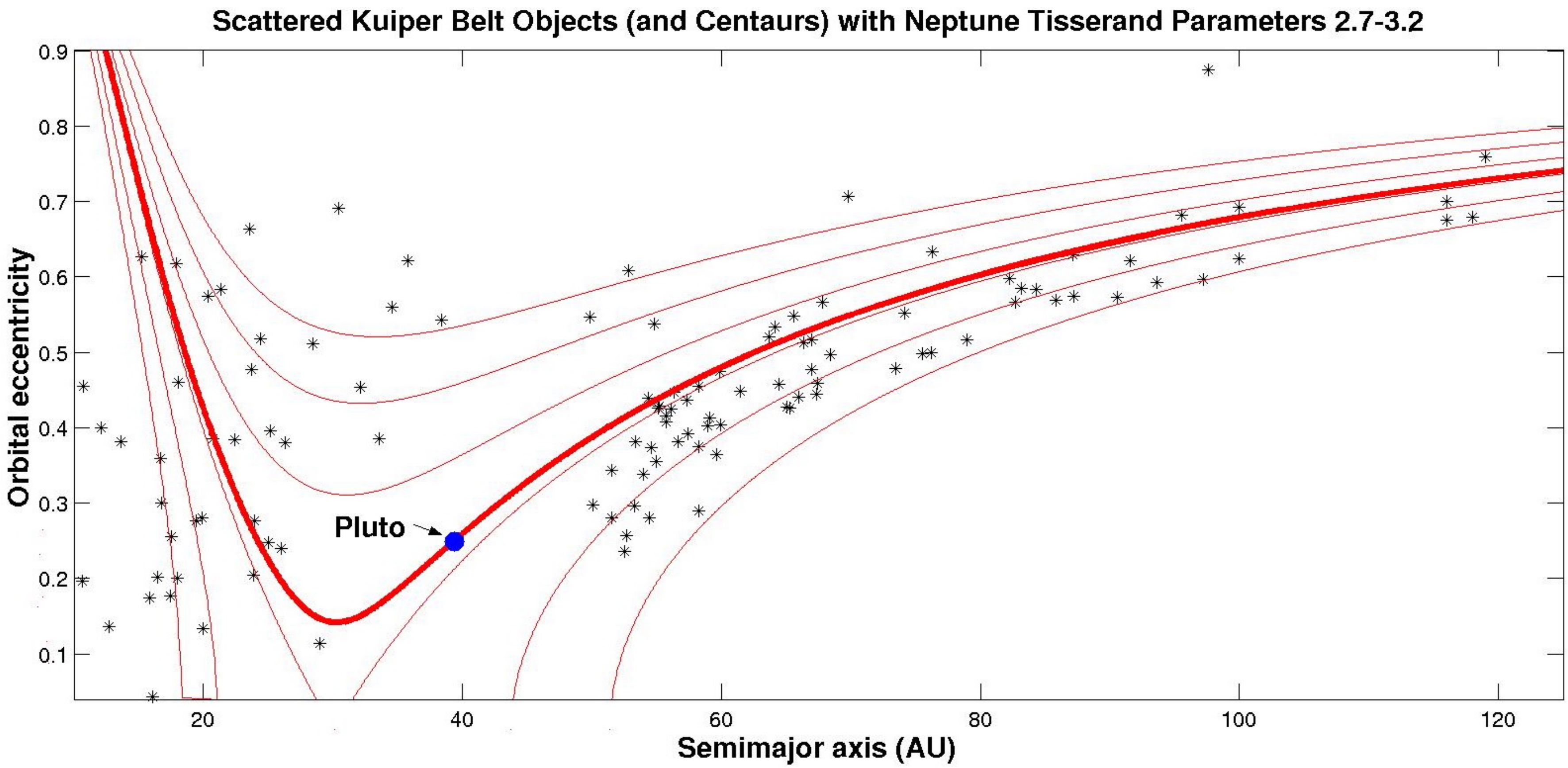


Movement around resonances



Scattering

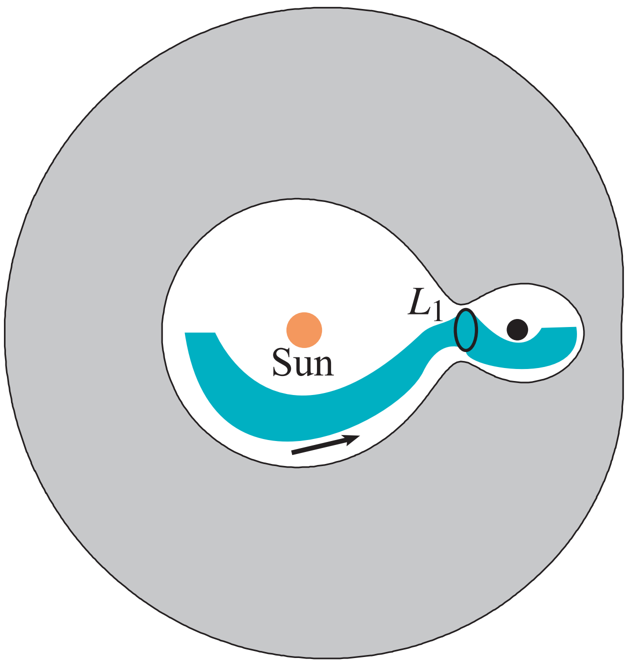
Movement around stable resonances via lobes



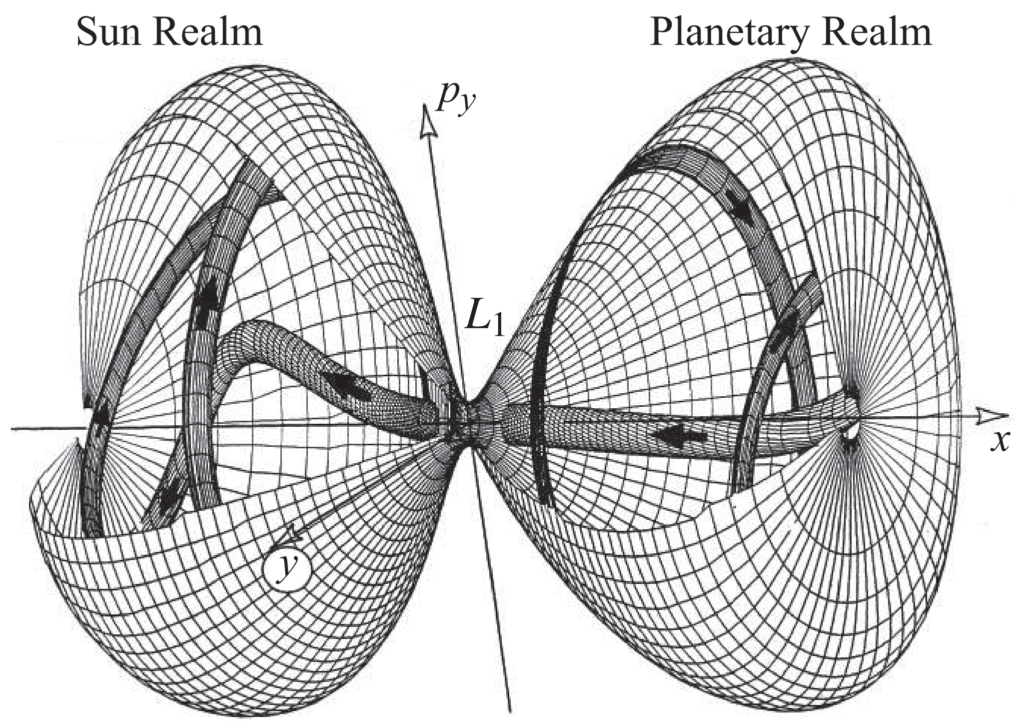
- Scattering looks like lateral movement along Tisserand contour

Realms and tubes

- Planetary and sun realms connected by tubes[2]



Position Space



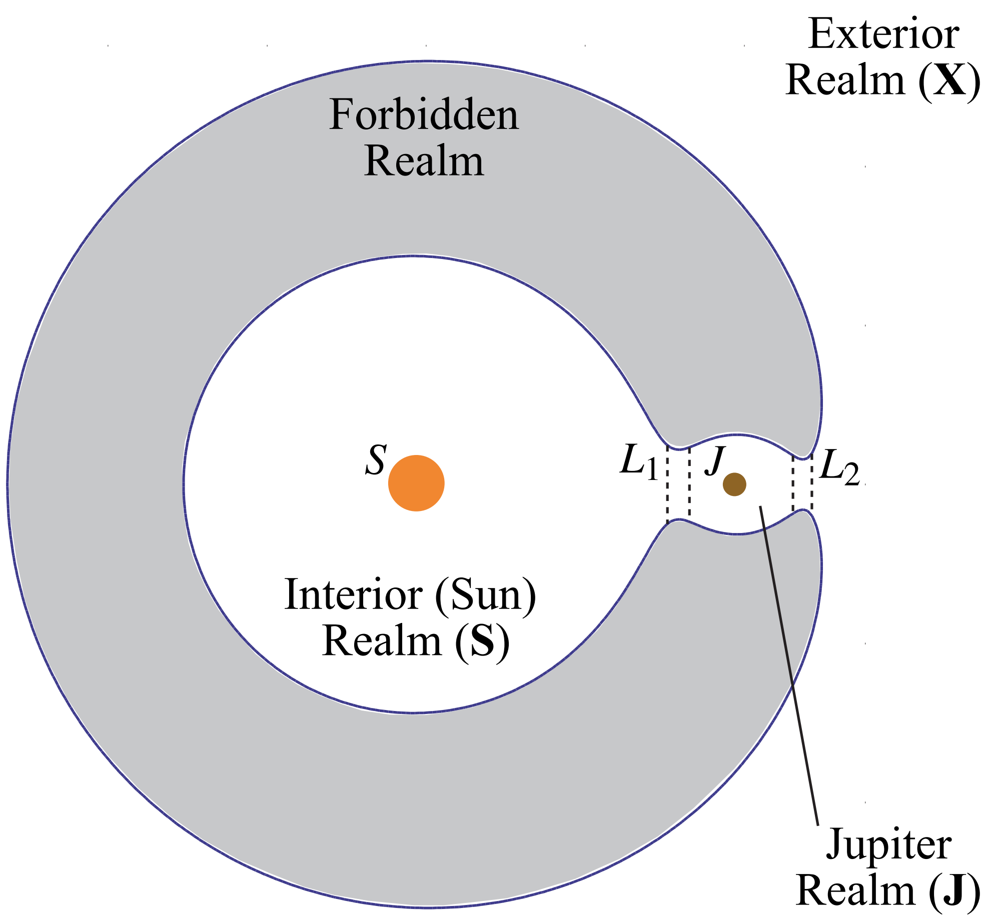
Phase Space (Position + Velocity)

[2]Ross, S. D. [2004] *Cylindrical manifolds and tube dynamics in the restricted three-body problem*, Ph.D. thesis

Restricted 3-body prob.

Planar circular case

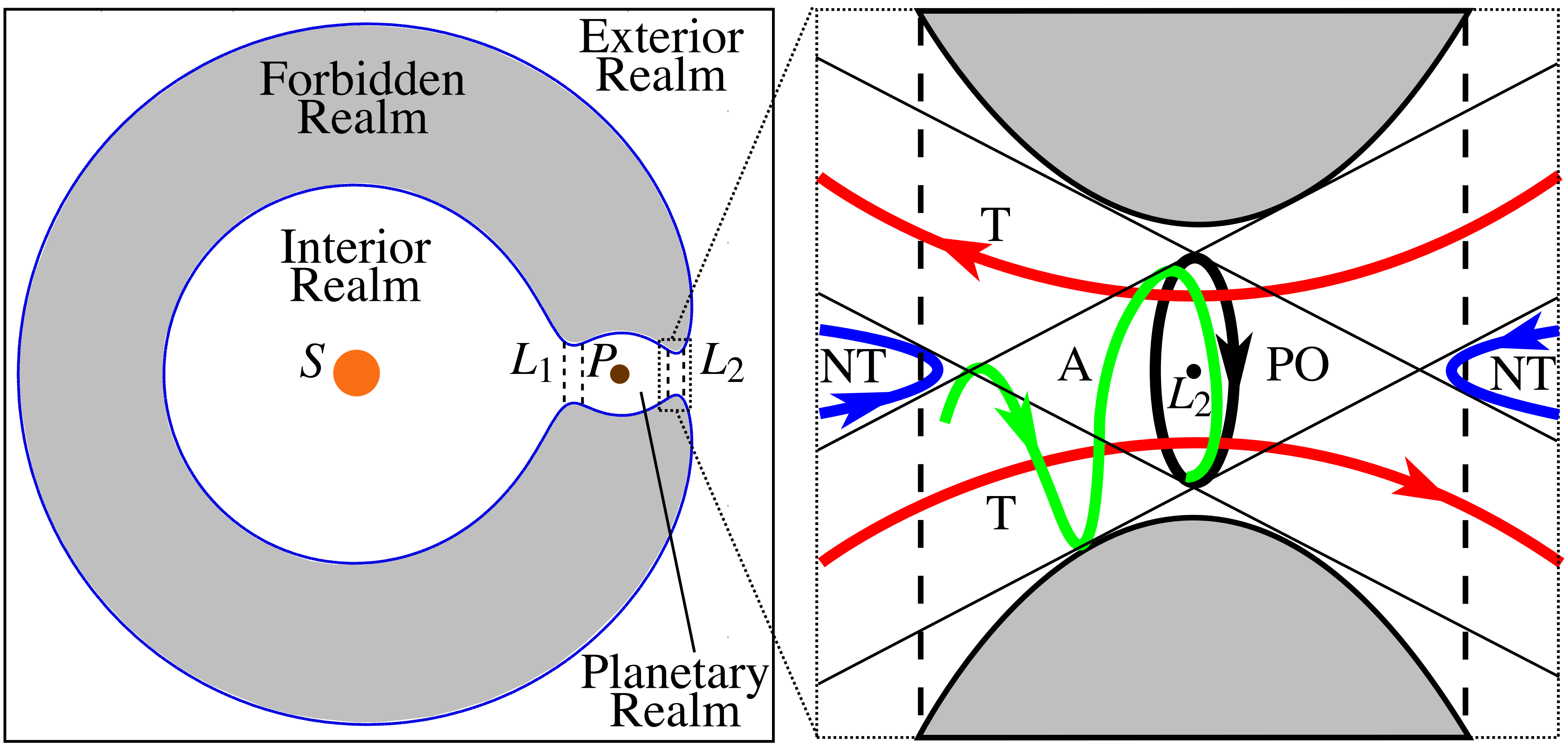
- Partition the energy surface: **S, J, X regions**



Position Space Projection

Equilibrium region

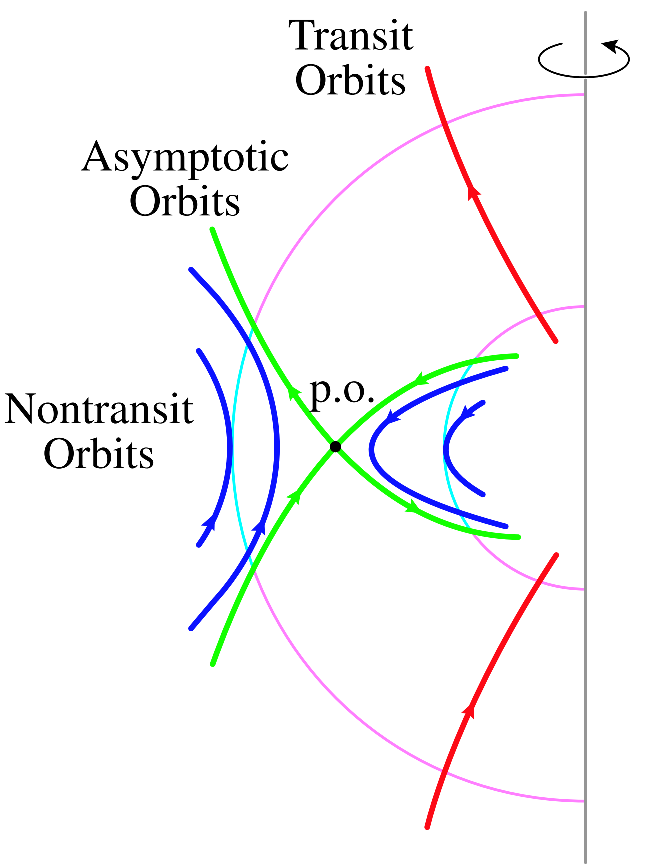
- ***Look at motion near the potential barrier, i.e. the equilibrium region***



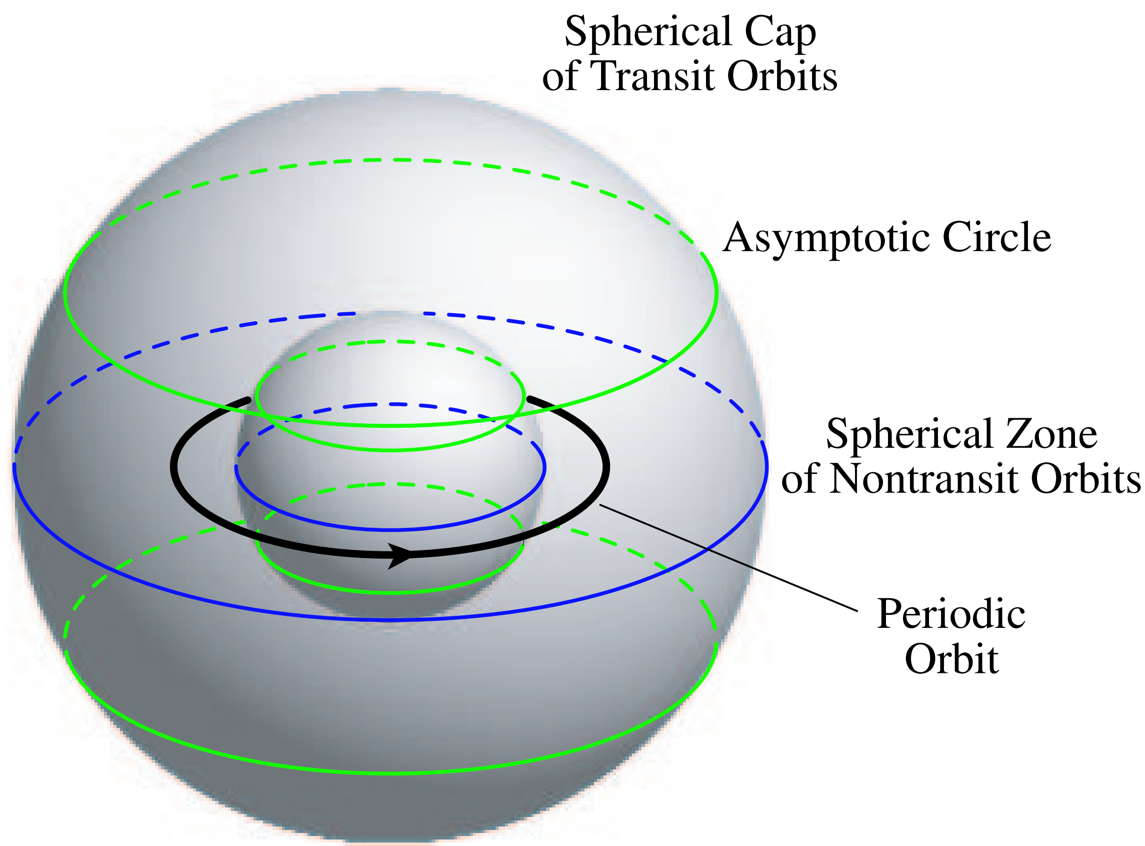
Position Space Projection

Local Dynamics

- For fixed energy, the equilibrium region $\simeq S^2 \times \mathbb{R}$.
  - Stable/unstable manifolds of periodic orbit define mappings between bounding spheres on either side of the barrier



Cross-section of Equilibrium Region



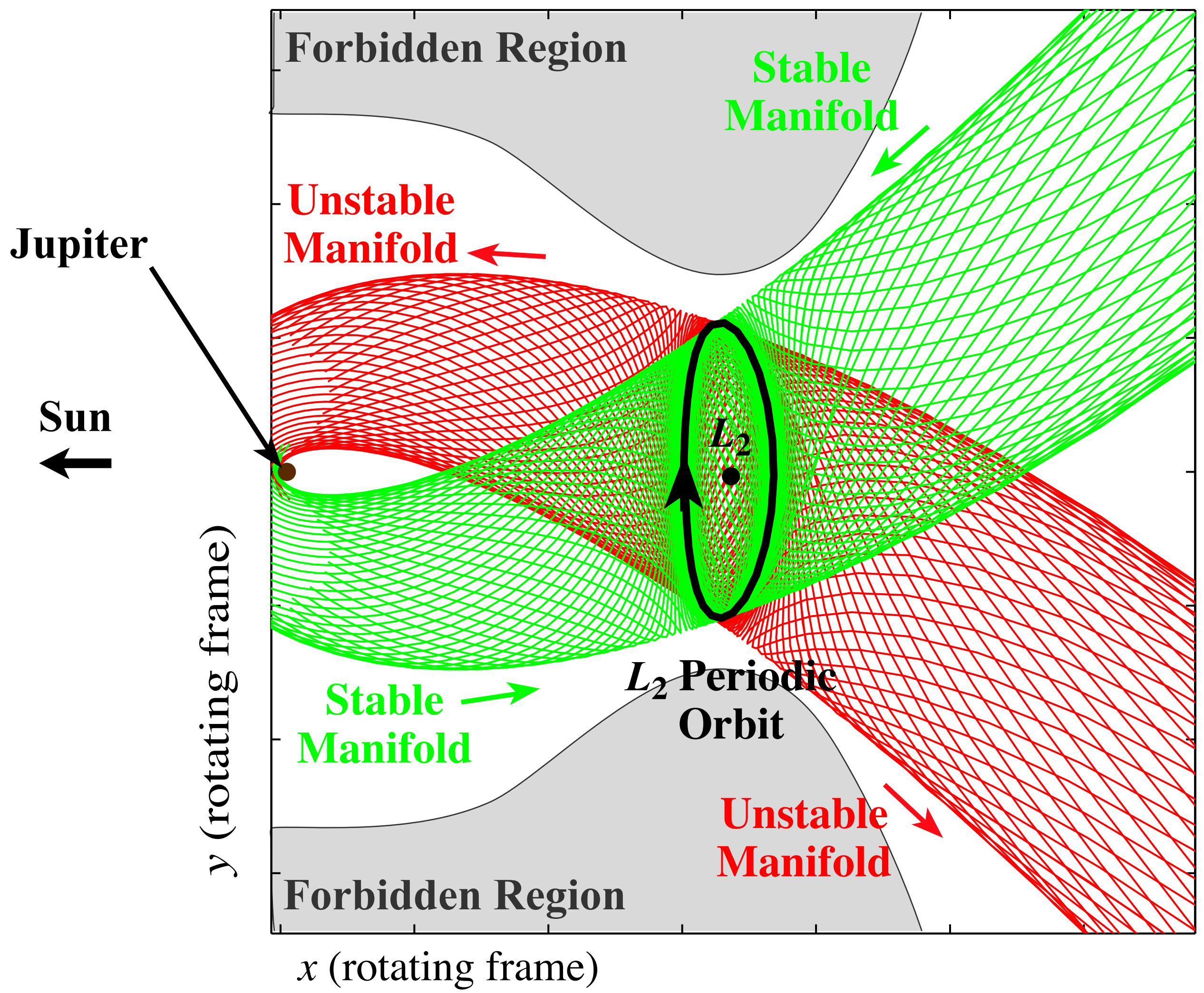
Equilibrium Region

Equilibrium region and tubes

- Eigenvalues of linearized equations: $\pm\lambda$, $\pm i\nu$
- Equilibrium region has a saddle $\times$ center geometry
- For each energy, there is one periodic orbit
- Its stable & unstable manifolds are cylindrical $\simeq S^1 \times \mathbb{R}$
- Locally obtained analytically via normal form expansion
- Can be globalized, numerically extended under the flow
- We call them **tubes**

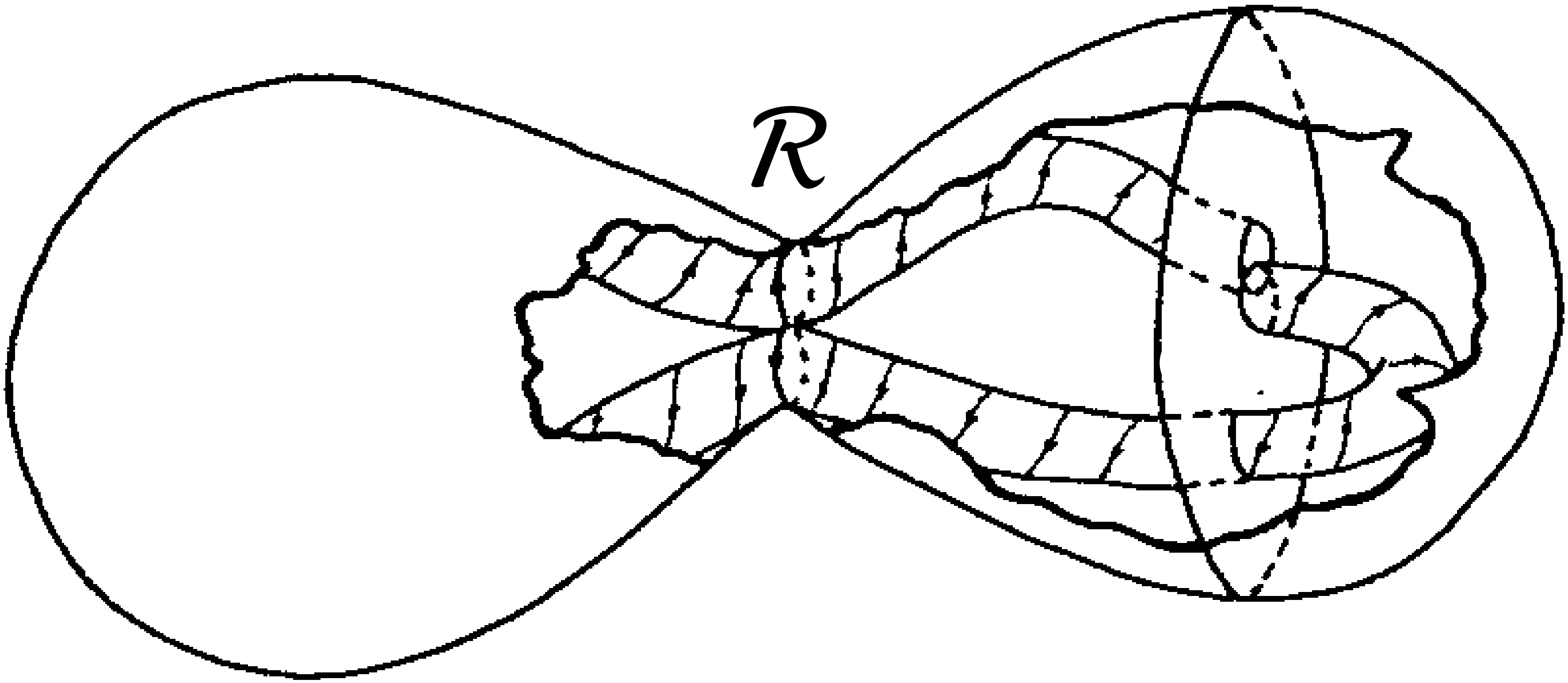
Tubes in the 3-body problem

- **Stable** and **unstable** **manifold tubes**
  - Control transport through the potential barrier.



Tube dynamics

- All motion between realms connected by equilibrium neck regions $\mathcal{R}$ must occur through the interior of the cylindrical stable and unstable manifold **tubes**



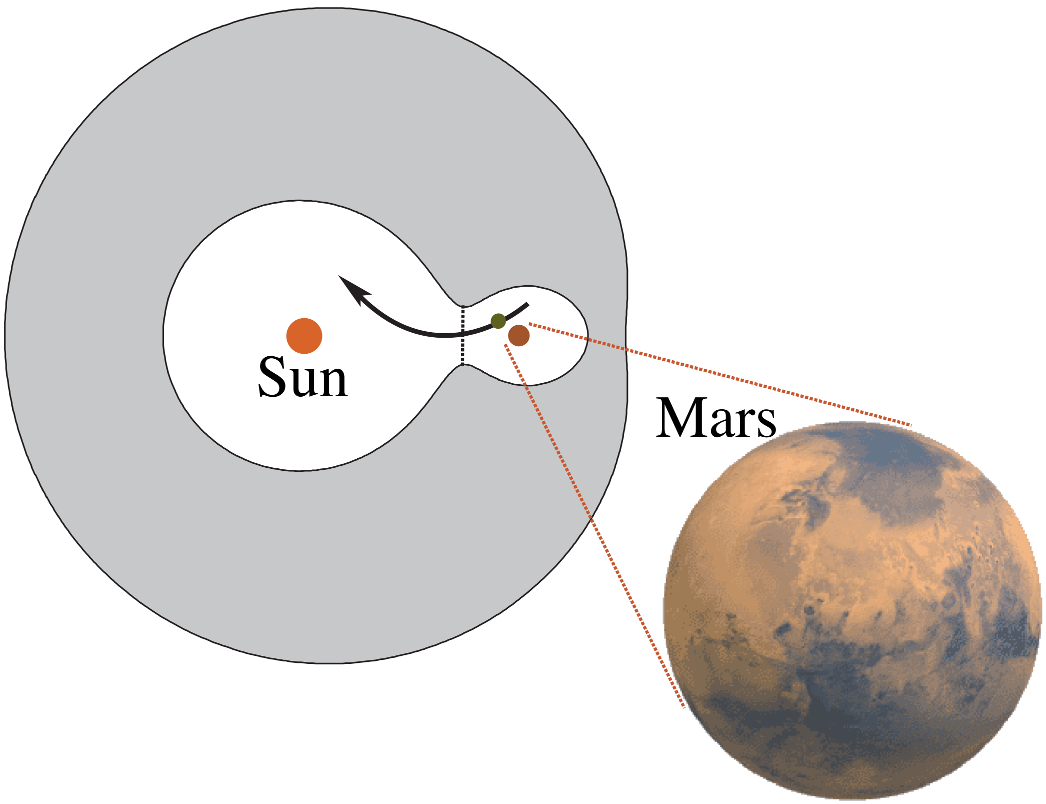
Some remarks

- Tubes are generic consequence of rank 1 saddle
  – saddle $\times$ center $\times \cdots \times$ center
- Tubes exist in 3 dof rest. 3-body problem ($\simeq S^3 \times \mathbb{R}$)
- Tubes persist
  – when primary bodies' orbit is eccentric
  – in presence of 4th massive body
- Observed in the solar system!

Koon, W. S., M. W. Lo, J. E. Marsden, and S. D. Ross [2000], Heteroclinic connections between periodic orbits and resonance transitions in celestial mechanics, *Chaos*, **10**, 427–469, Gómez, G., W. S. Koon, M. W. Lo, J. E. Marsden, J. Masdemont, and S. D. Ross [2004], Connecting orbits and invariant manifolds in the spatial three-body problem, *Nonlinearity*, **17**, 1571–1606, and Yamato, H. and D. B. Spencer [2003], Numerical investigation of perturbation effects on orbital classifications in the restricted three-body problem. In *AAS/AIAA Space Flight Mechanics Meeting*, Ponce, Puerto Rico. Paper No. AAS 03-235.

Escape and capture rates

- Consider Mars ejecta with enough energy to escape sunward. Using a **statistical approach** used in transition state theory (developed by chemists), the rate of escape can be estimated.[3]



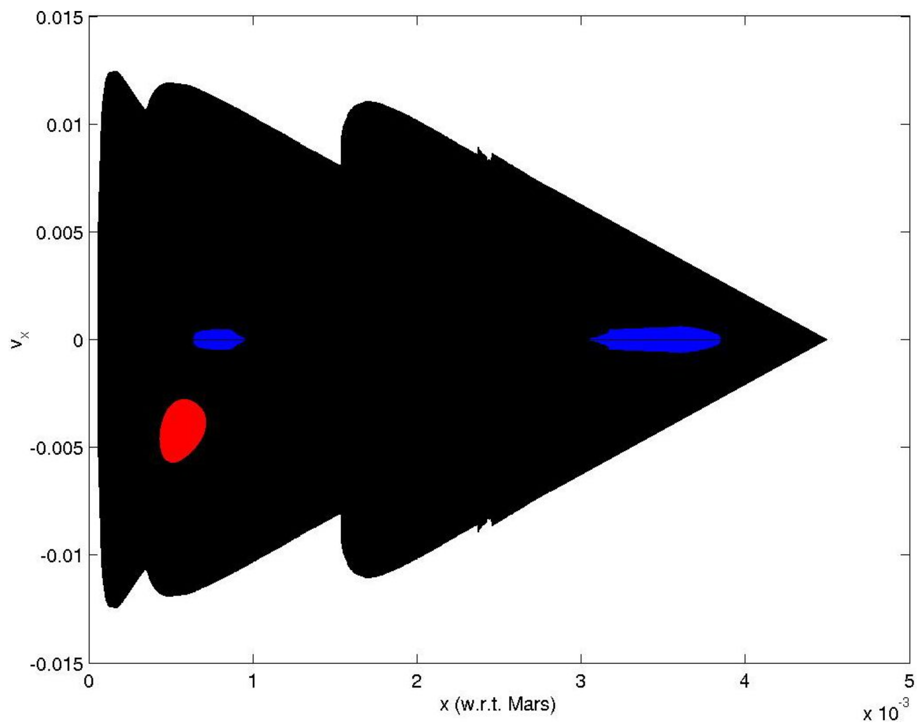
[3] Jaffé, Ross, Lo, Marsden, Farrelly, and Uzer [2002]

Escape and capture rates

- **Mixing assumption:** all asteroids in the chaotic sea surrounding Mars are **equally likely to escape**.

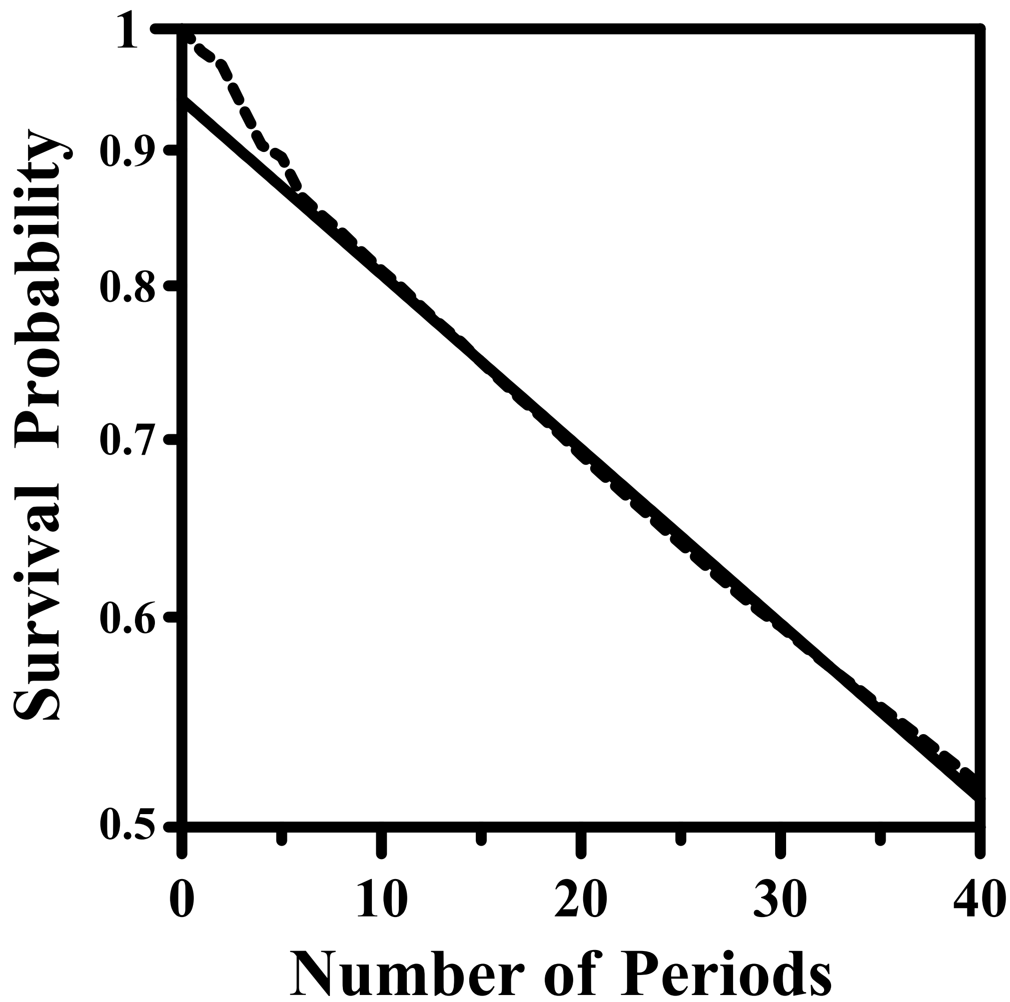
  Escape rate constant $= k_{esc} = -\log(1 - p_{esc})$, where

$$p_{esc} = \frac{\text{Volume of exit sunward (red)}}{\text{Volume of chaotic sea (black)}}$$



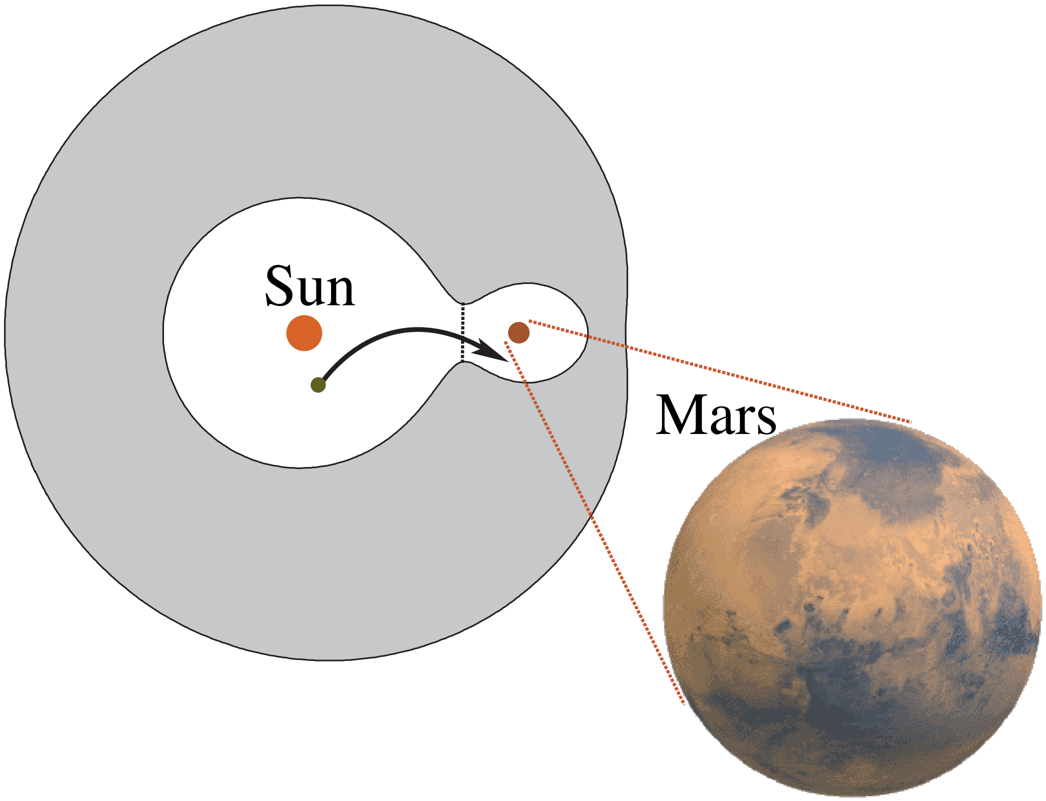
Escape and capture rates

- Theory and numerical simulations agree well
  - Monte Carlo simulation (dashed) and theory (solid)



Escape and capture rates

- Similarly, can estimate the probability of a rogue asteroid encountering Mars.



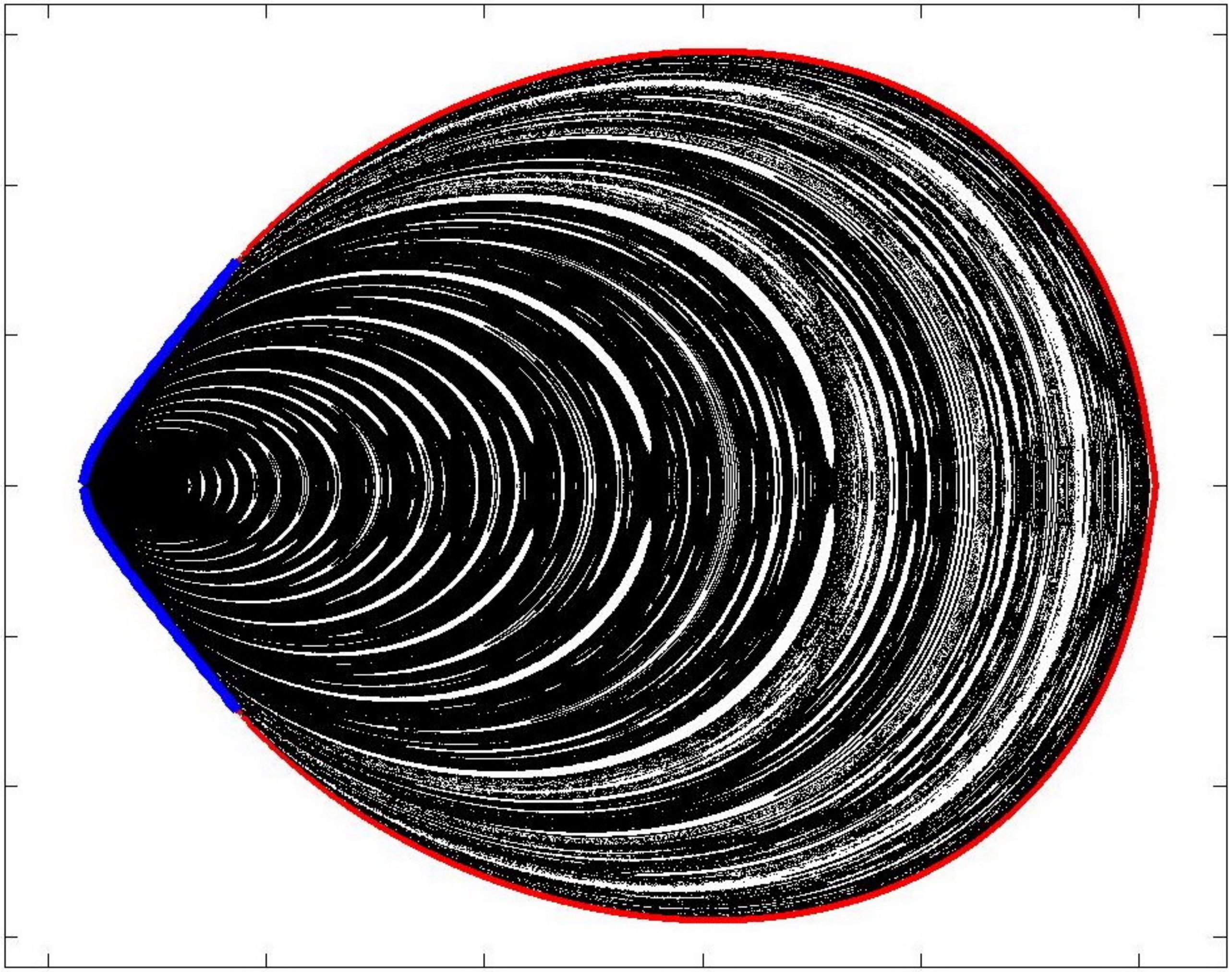
Escape and capture rates

- Same **mixing assumption**, i.e., ignoring lobe dynamics and resonances.

  Capture rate constant $= k_{cap} = -\log(1 - p_{cap})$, where

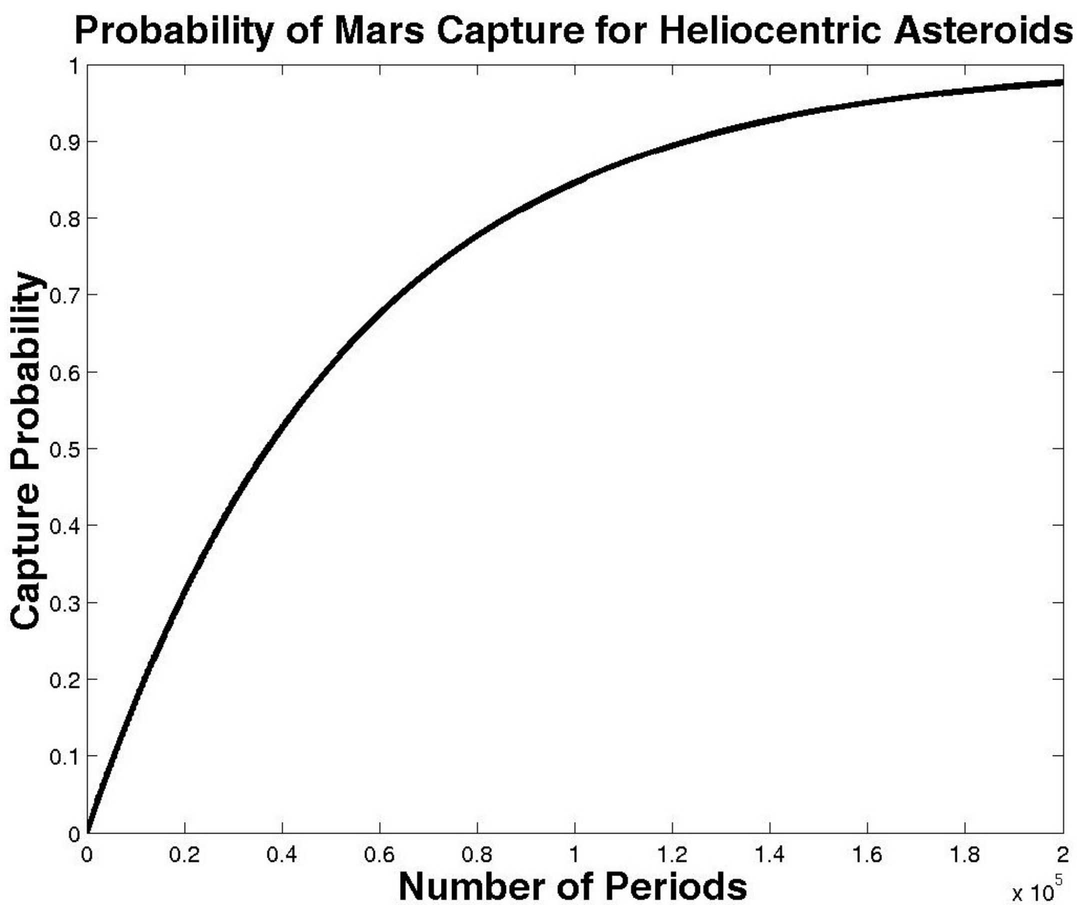
$$p_{cap} = \frac{\text{Volume of exit Marsward (same)}}{\text{Volume of interior chaotic sea (larger)}}$$

Escape and capture rates



- Bounded chaotic sea
- Sunward edge can reach Earth – consider restricted 4-body system

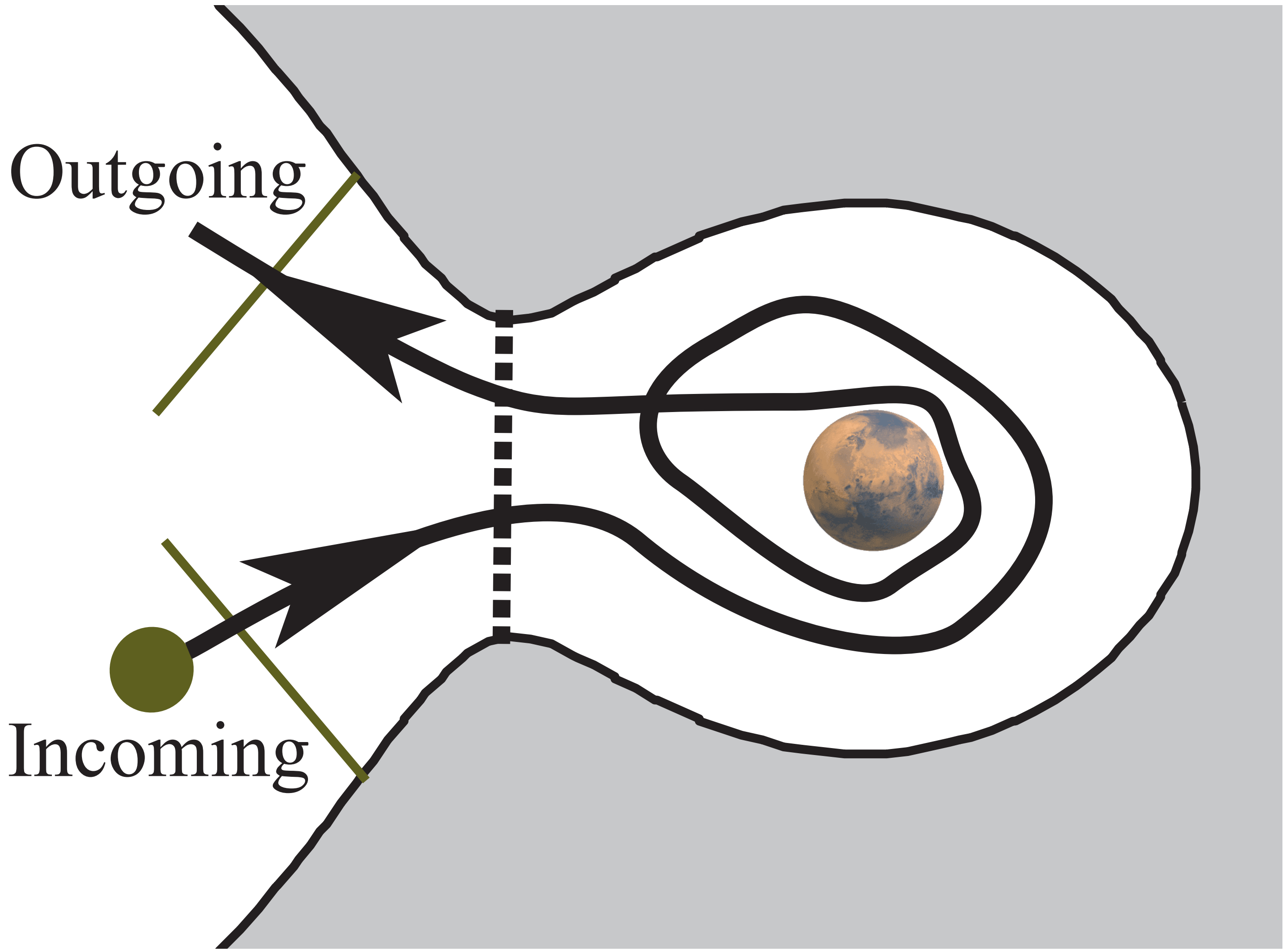
Escape and capture rates



- "Half-life" until capture $\sim 10^5$ years
- Overestimate due to ignoring partial barrier behavior of resonances
- Capture is **temporary** or leads to **collision**

Temporary capture times

- A kind of scattering problem

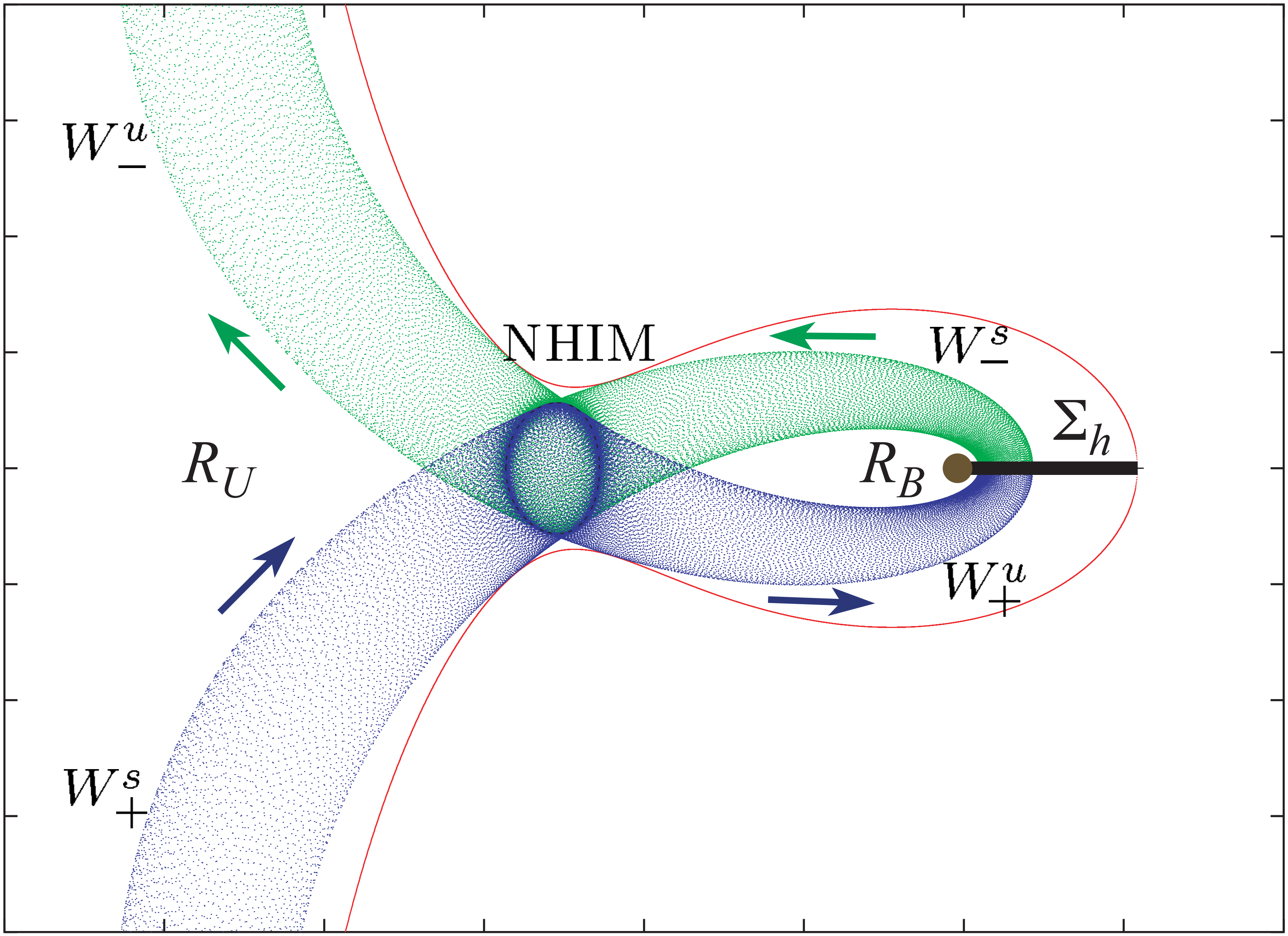


Temporary capture times

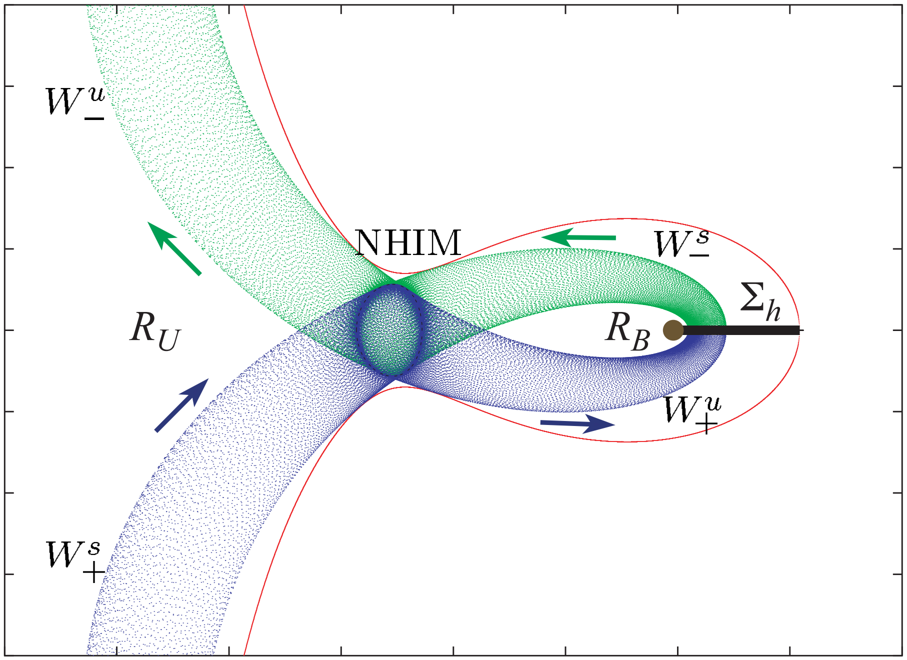
- Related to scattering of an electron by a Rydberg ion in crossed magnetic and electric fields, a recently solved problem[4]
- Earlier assumption, transition state theory, not adequate
- Scattering profile is **structured**, not exponential
- Scattering completely determined by **tube dynamics**

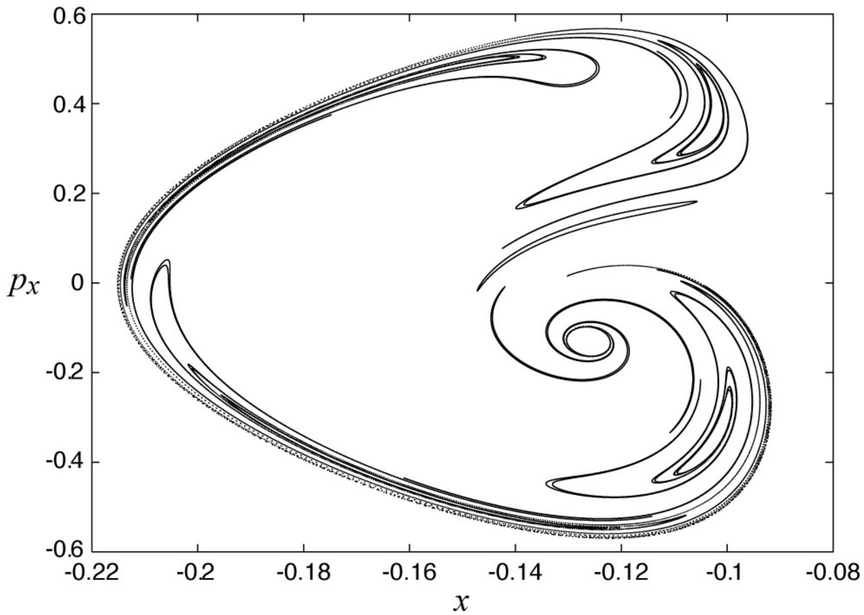
[4] F. Gabern, W.S. Koon, J.E. Marsden and S.D. Ross [2005] Theory and computation of non-RRKM lifetime distributions and rates in chemical systems with three or more degrees of freedom, submitted for publication.

Temporary capture times



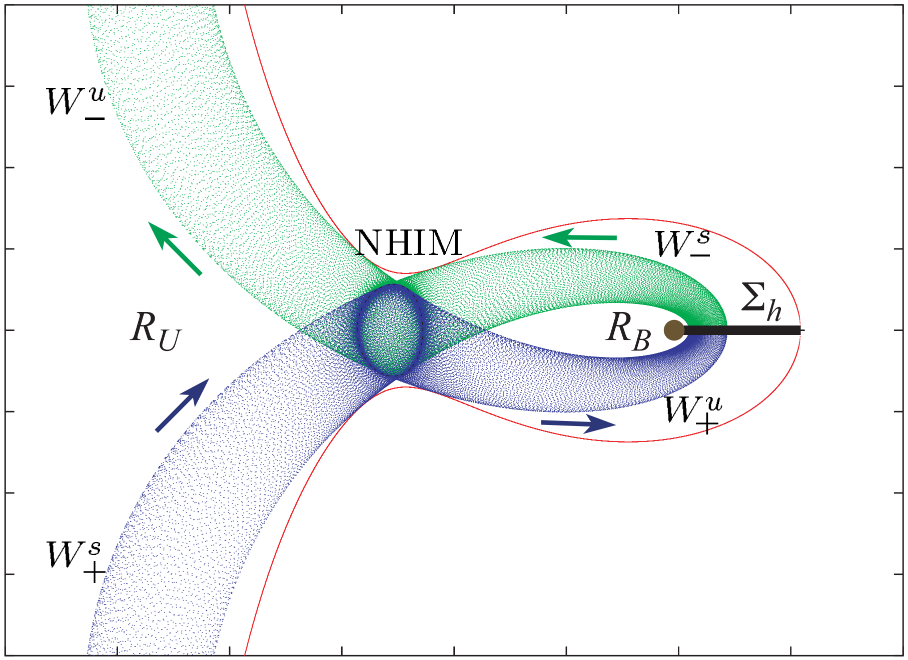
Scattering determined by tube dynamics

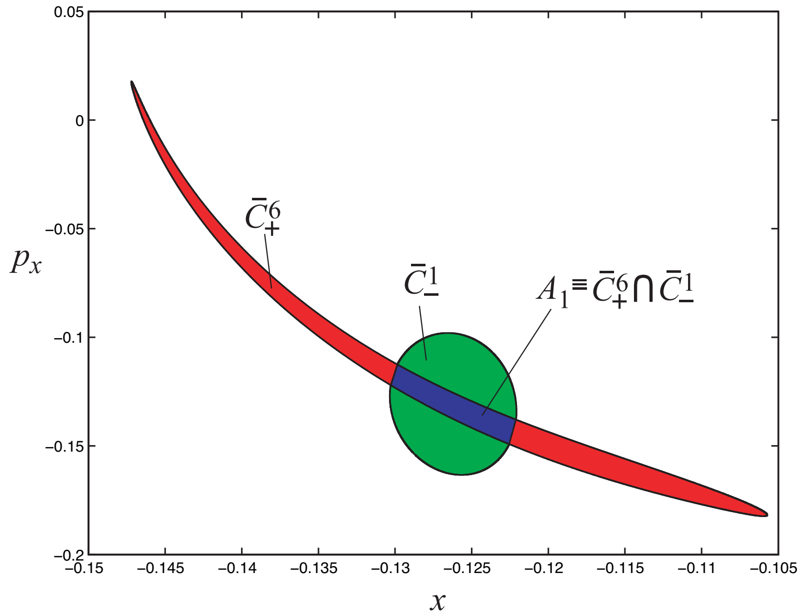




- Intersection of incoming and outgoing tubes as they wind around is the mechanism of scattering.

Scattering determined by tube dynamics

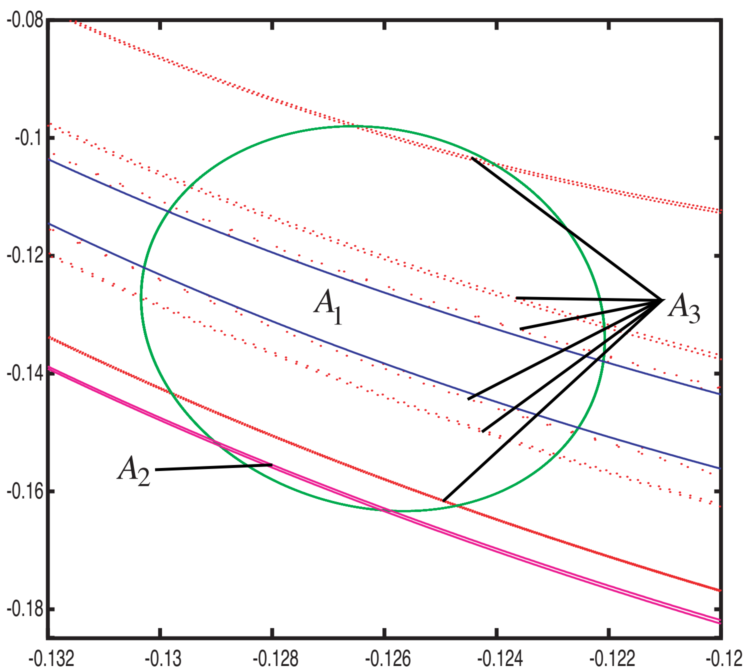


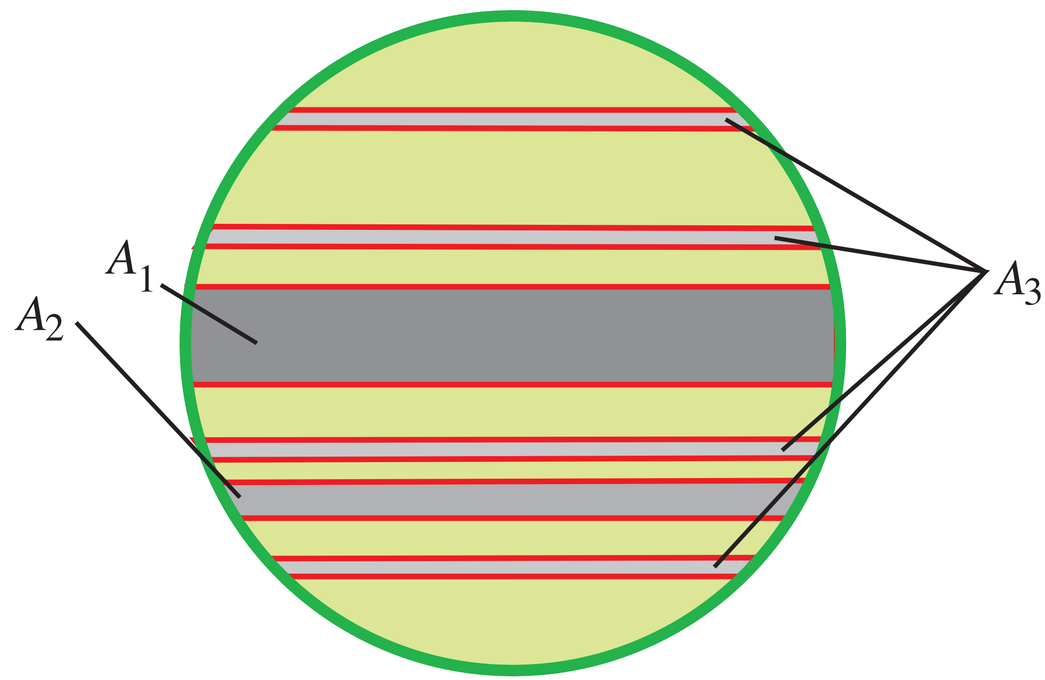


- ☐ Let first intersection of incoming tube be the entrance, similarly define the exit
- ☐ Intersections of images of entrance with exit determine scattering profile

Scattering determined by tube dynamics

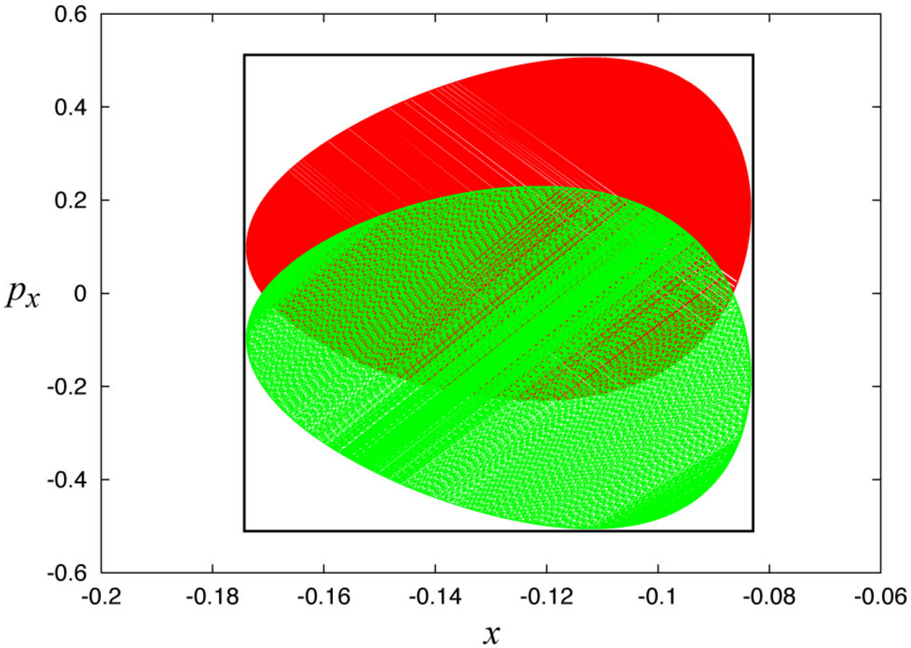
- Fractal tiling of the exit



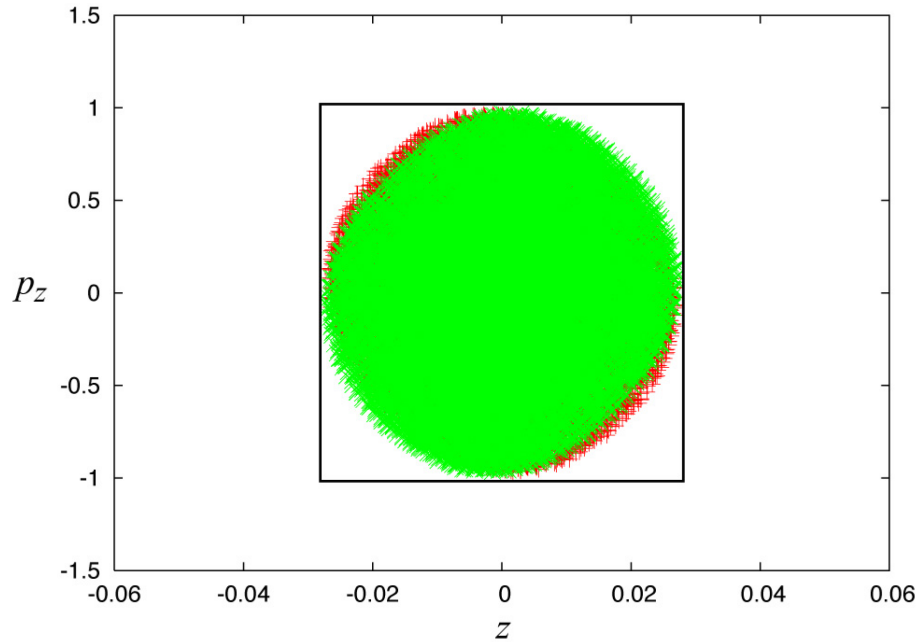


Scattering determined by tube dynamics

- 4D intersection volumes computed using Monte Carlo method.



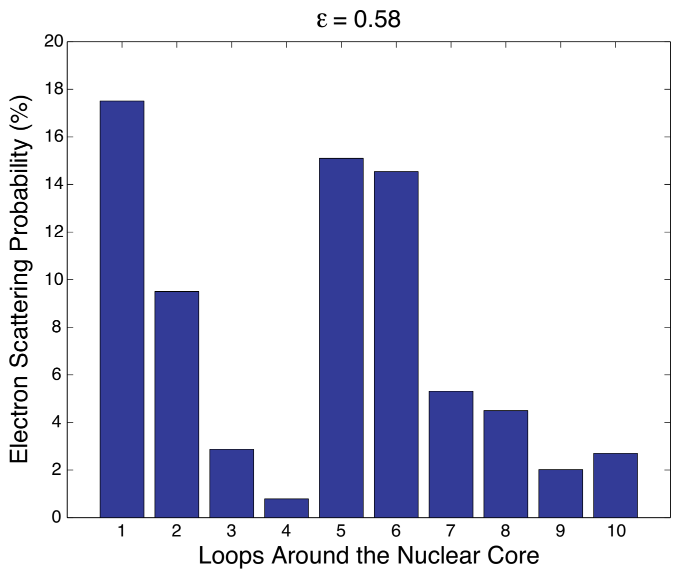
(a)

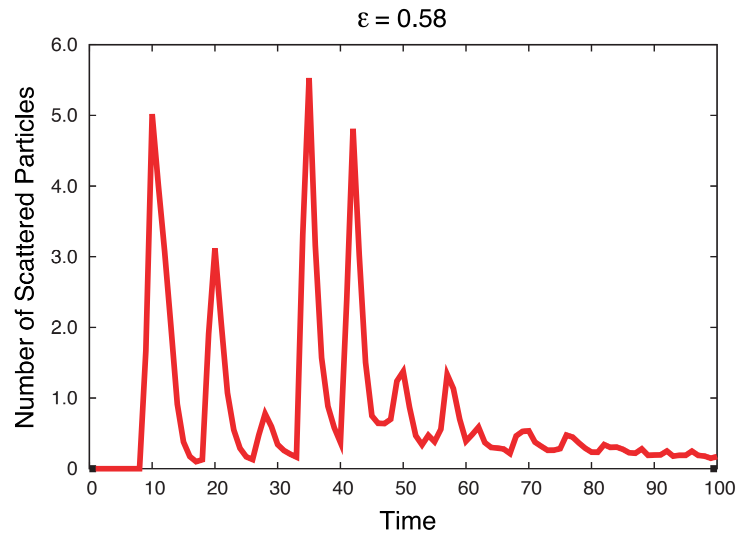


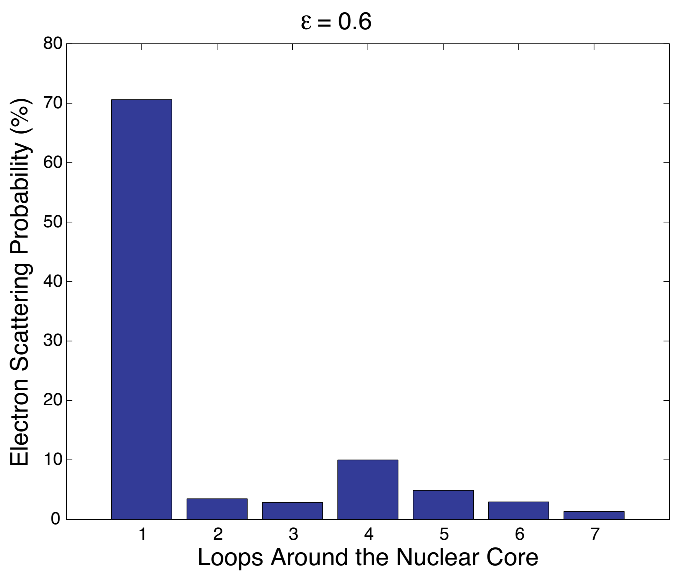
(b)

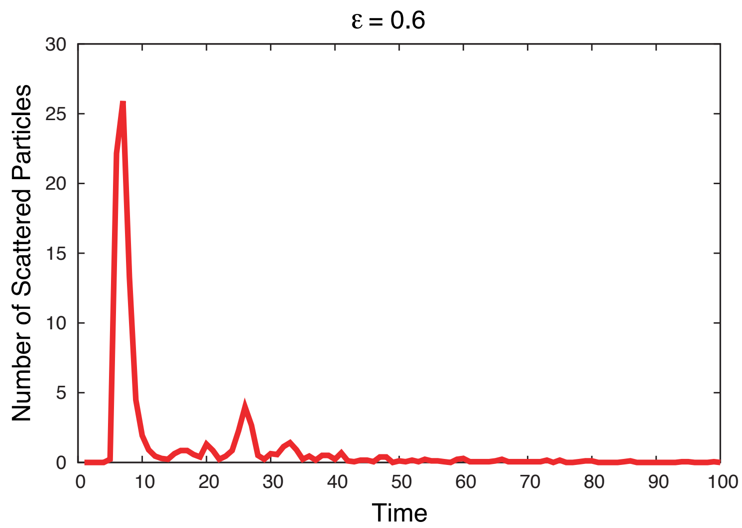
Scattering determined by tube dynamics

- Scattering/capture time profiles are structured



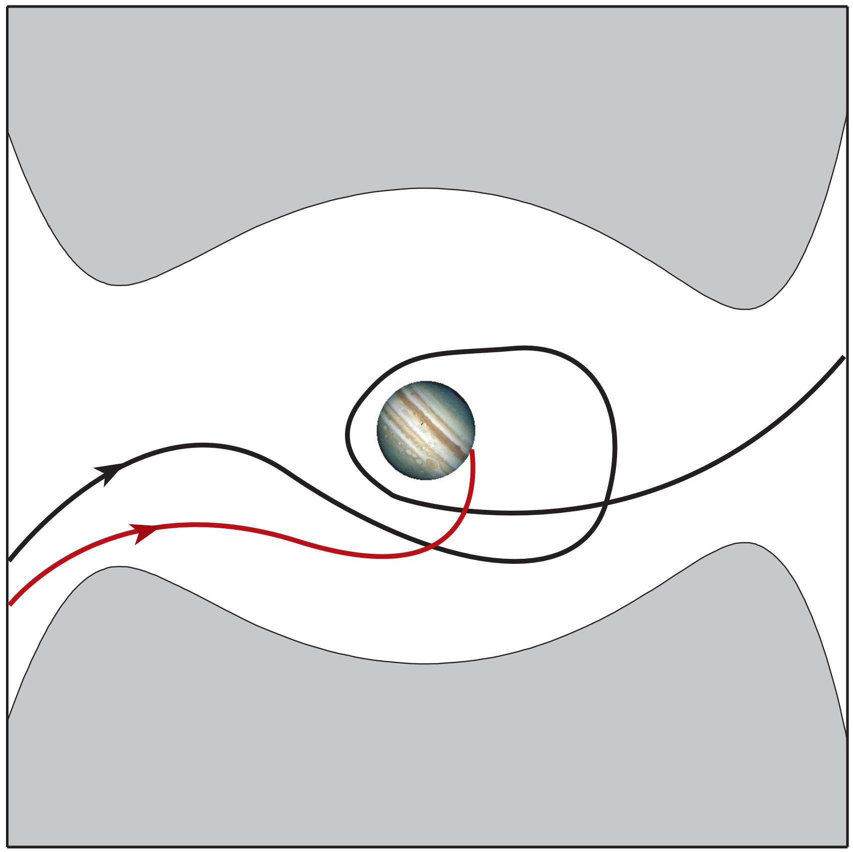






Transition and collision

- ***Full picture even more complicated!***[5]



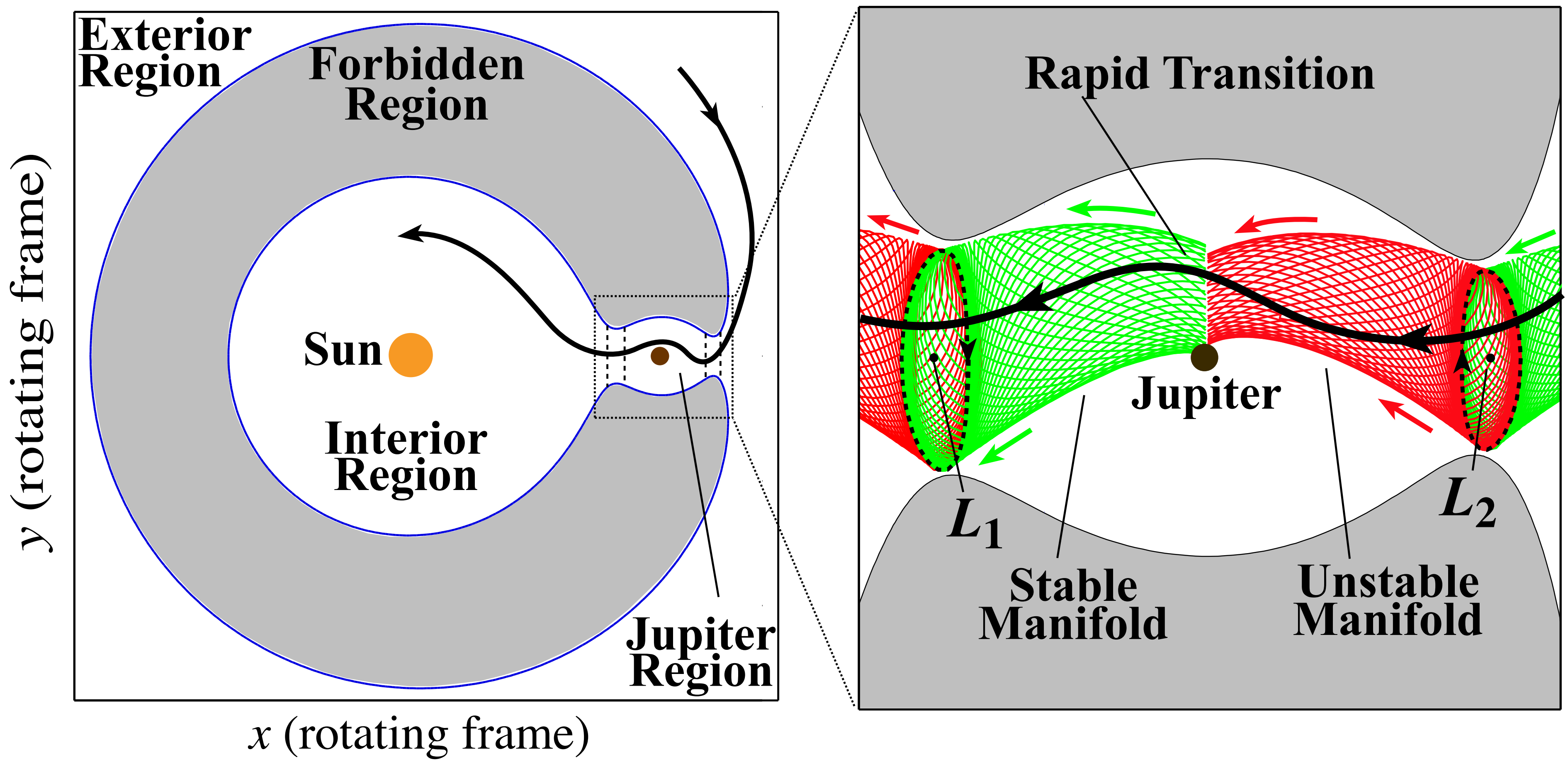
- Transition to other realms and collision possible.

[5] Ross [2003] Statistical theory of interior-exterior transition and collision probabilities for minor bodies in the solar system, *Libration Point Orbits and Applications*, World Scientific, pp. 637-652.

Transition probabilities

Transition probabilities

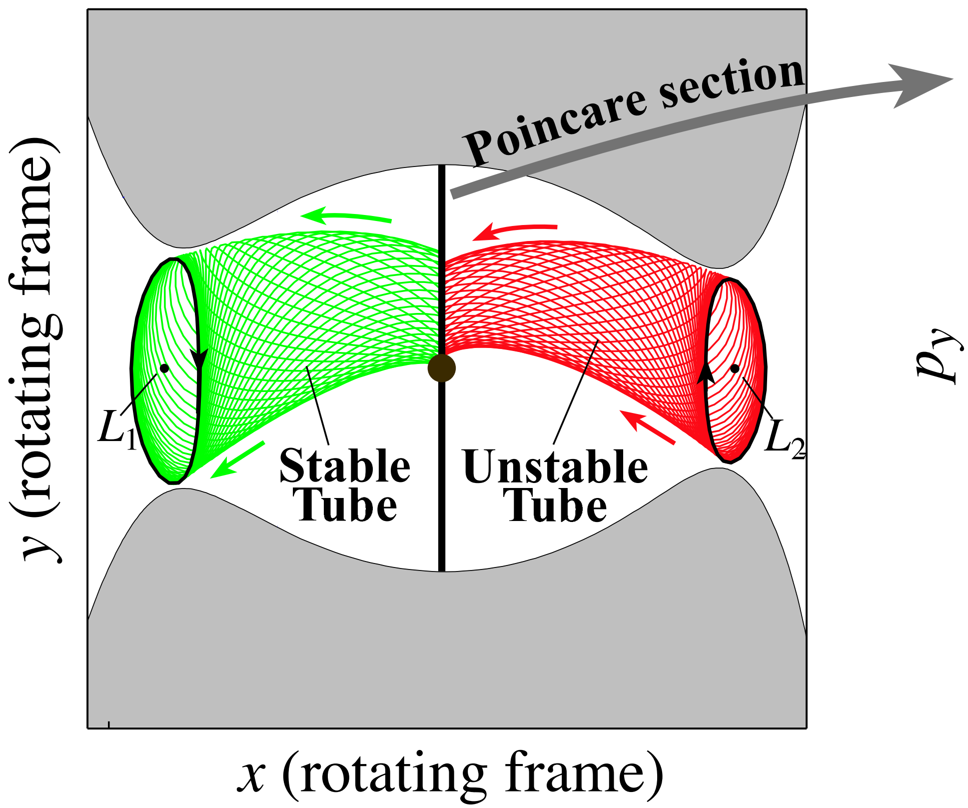
- Example: Comet transport between outside and inside of Jupiter (i.e., **Oterma**-like transitions)



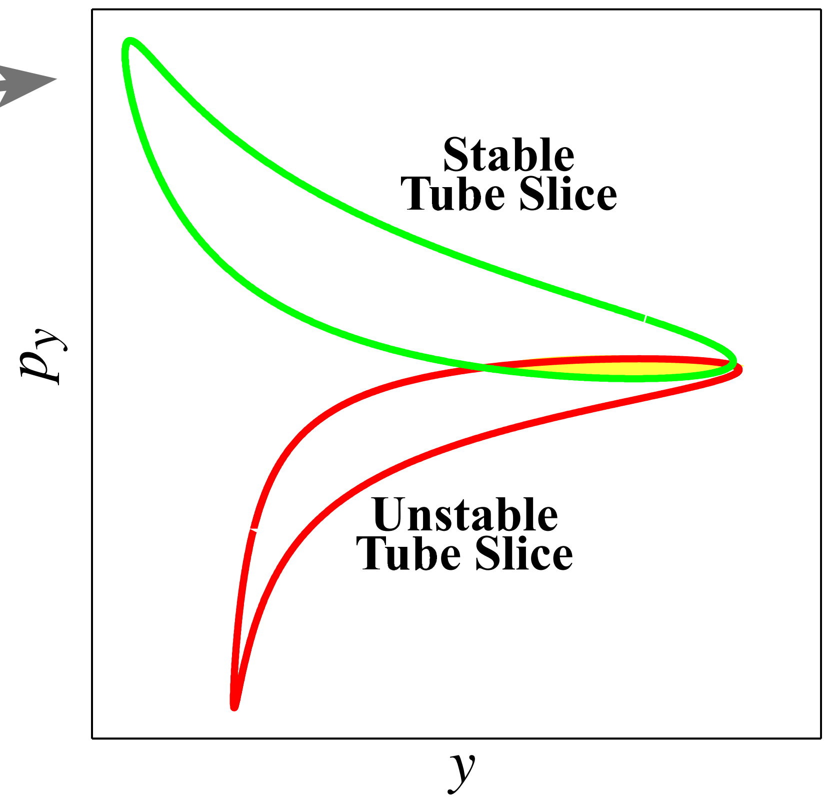
(a)

Transition probabilities

- Consider Poincaré section intersected by both tubes.



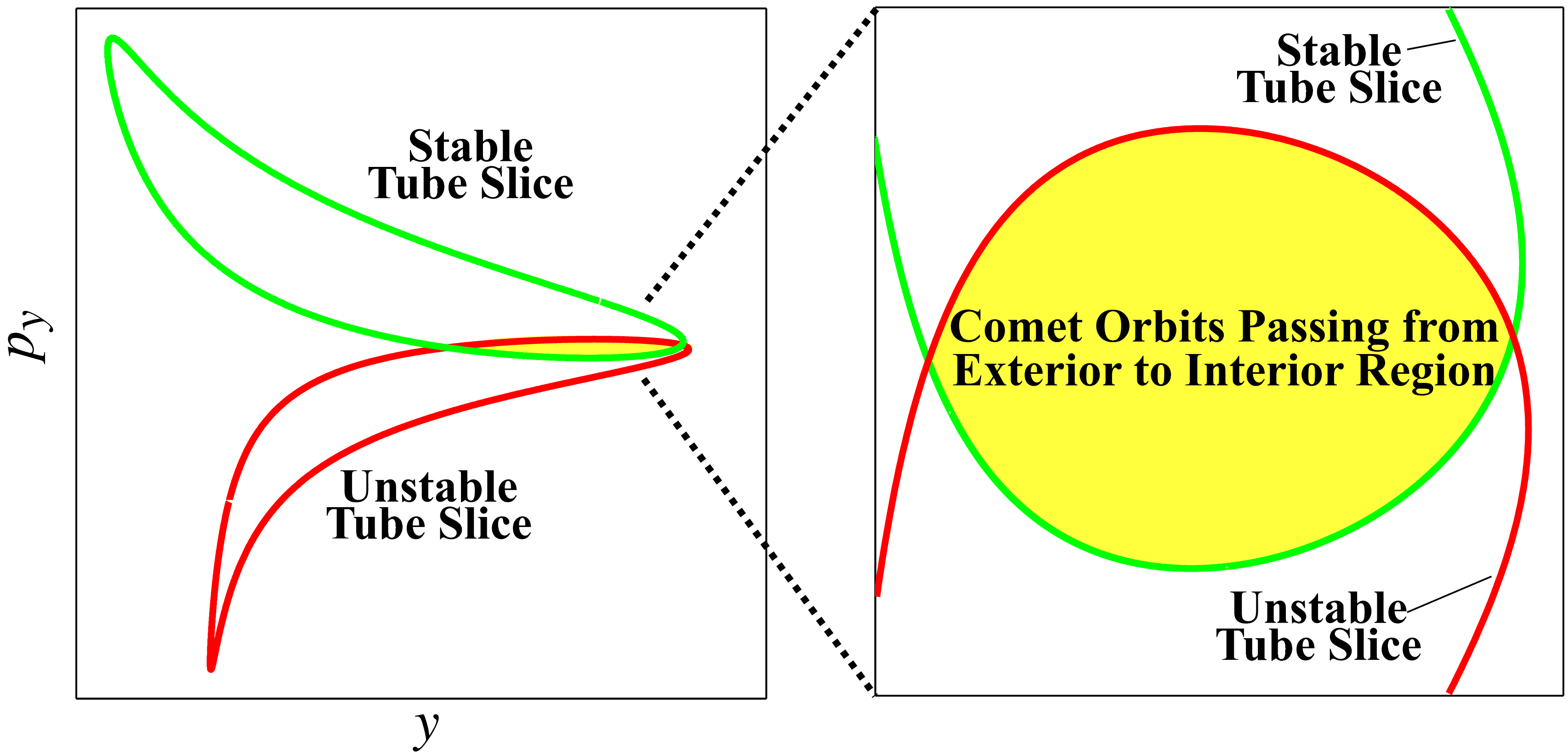
Position Space



Canonical Plane (y, p_y)

Transition probabilities

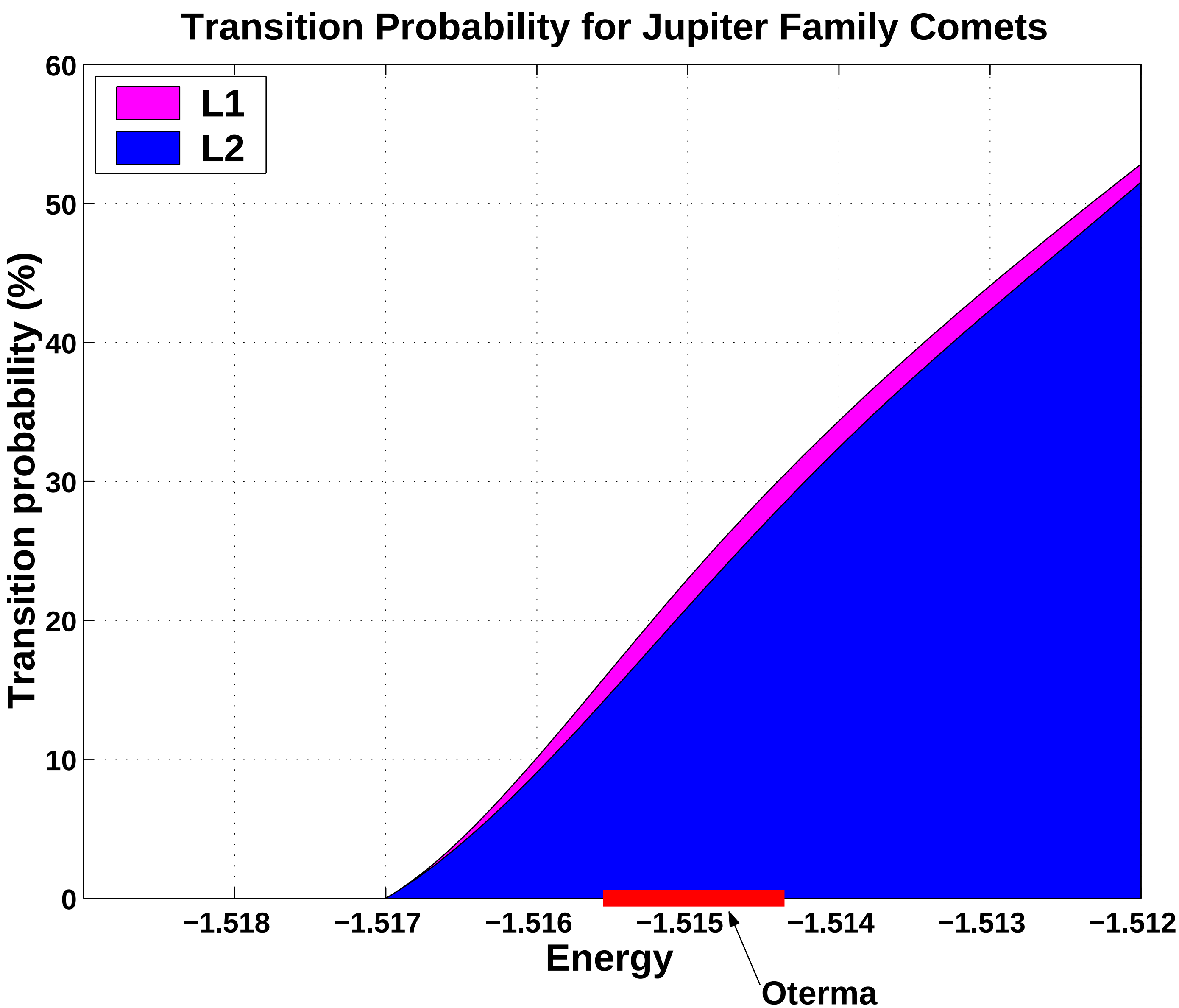
- Canonical area ratio gives the **conditional probability** to pass from **outside** to **inside** Jupiter's orbit.
  - Assuming a well-mixed connected region on the energy mfd.



Canonical Plane (y, p_y)

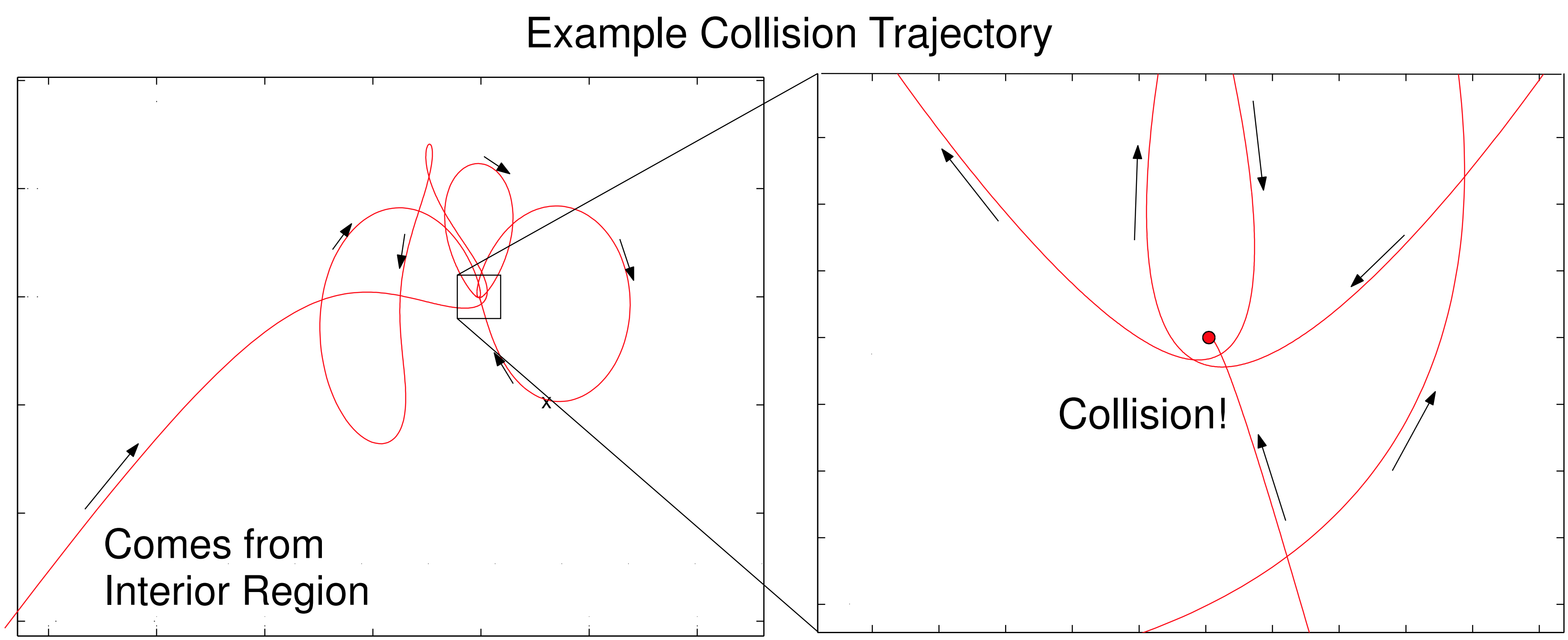
Transition probabilities

- Jupiter family comet transitions: **X** $\rightarrow$ **S**, **S** $\rightarrow$ **X**



Collision probabilities

- Low velocity impact probabilities
- Assume object enters the planetary region with an energy slightly above L1 or L2
  - eg, **Shoemaker-Levy 9** and **Earth-impacting asteroids**

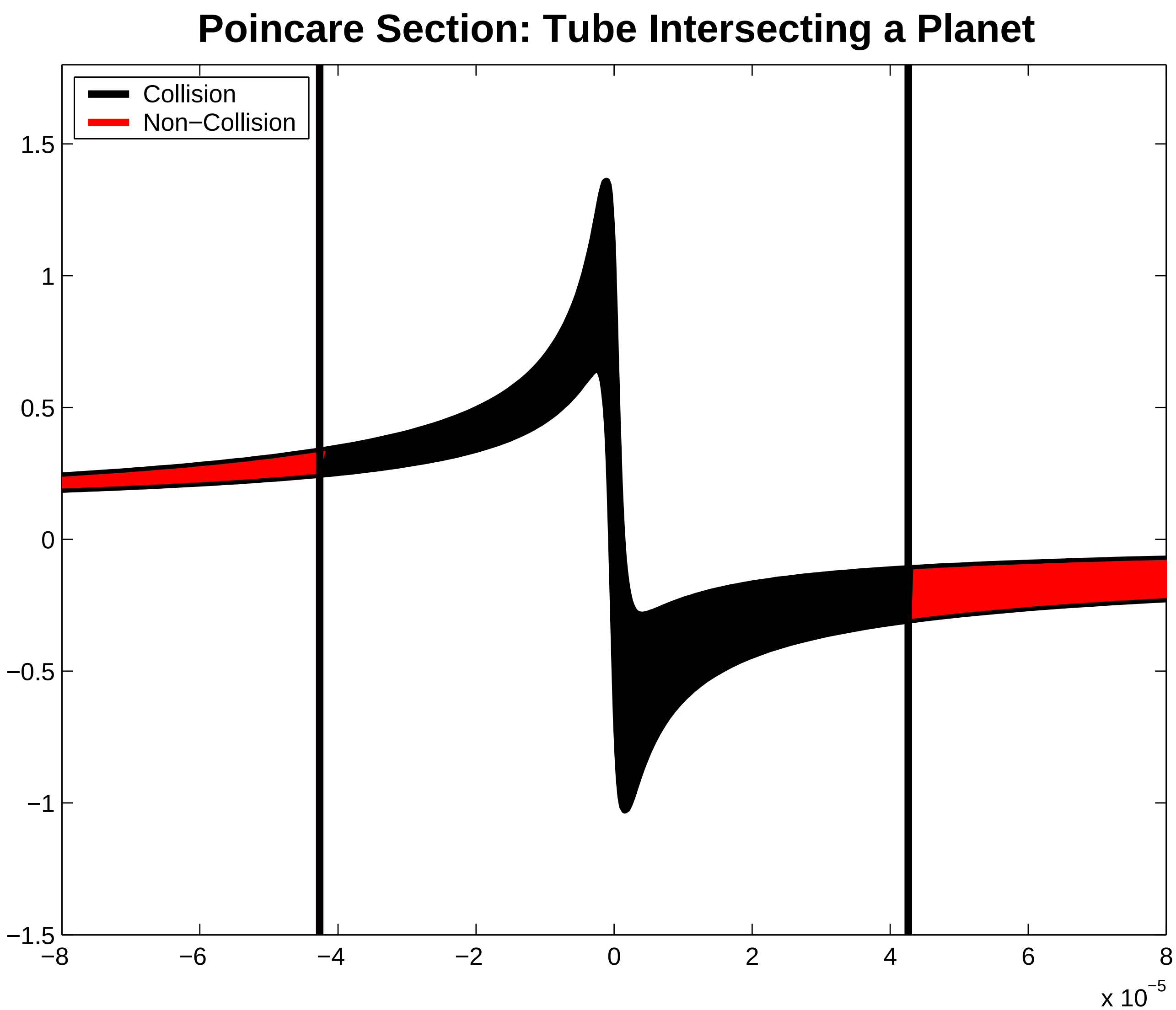


Collision probabilities

- Compute from tube intersection with planet on Poincaré section
- Planetary diameter d is a parameter
- Tube "breaks apart" after each collision, becomes difficult to follow

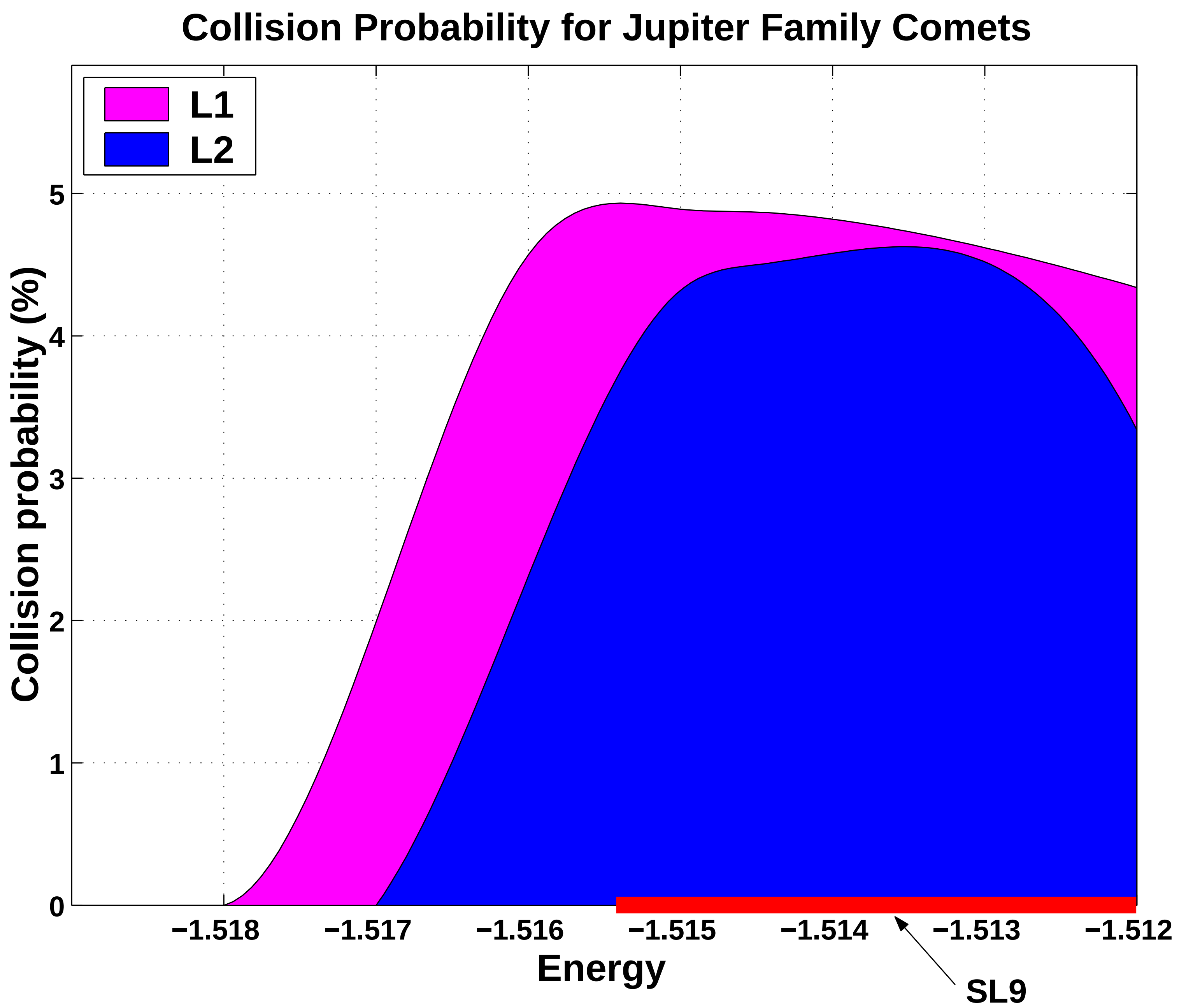
$\longleftarrow$ Diameter of planet $\longrightarrow$

Collision probabilities

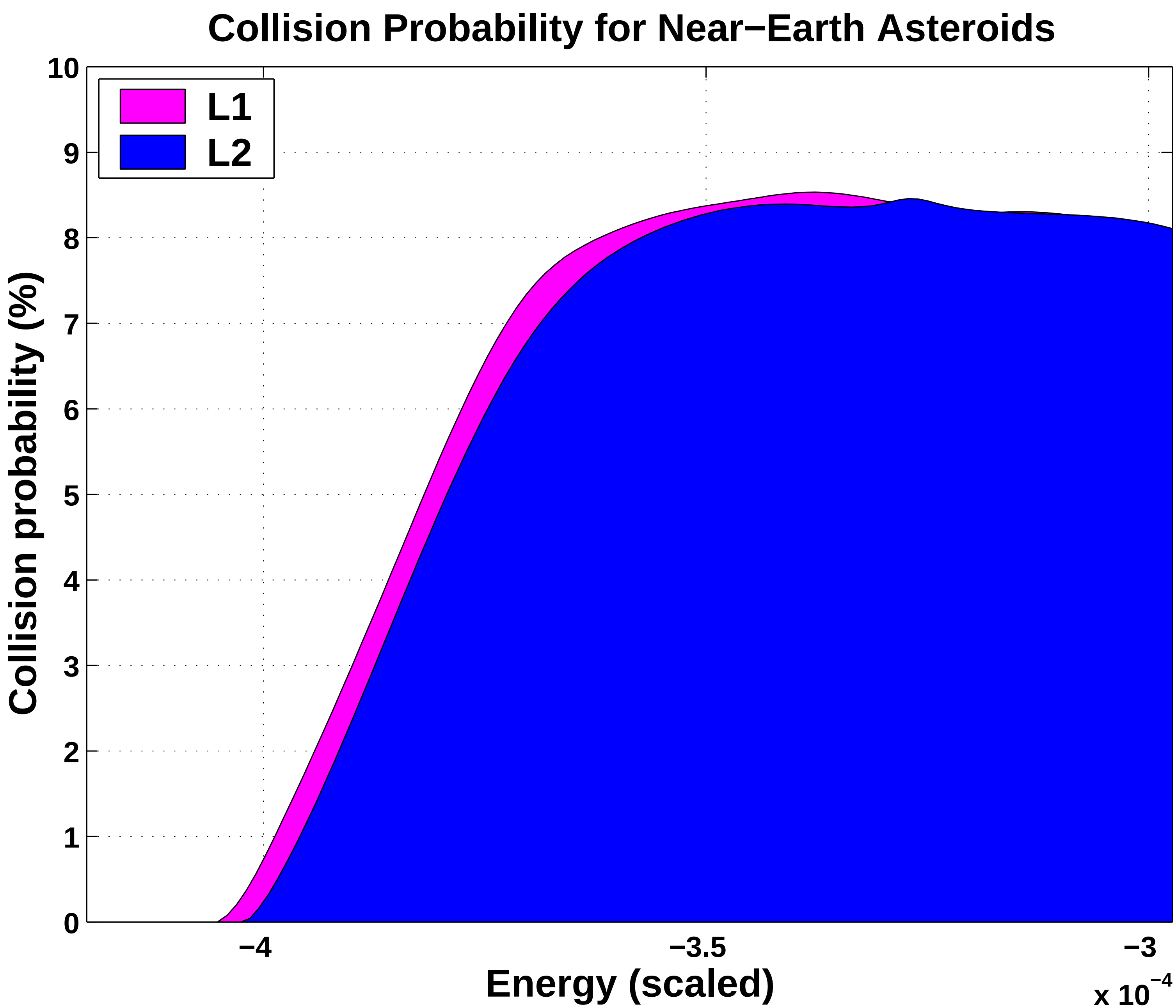


← Diameter of planet →

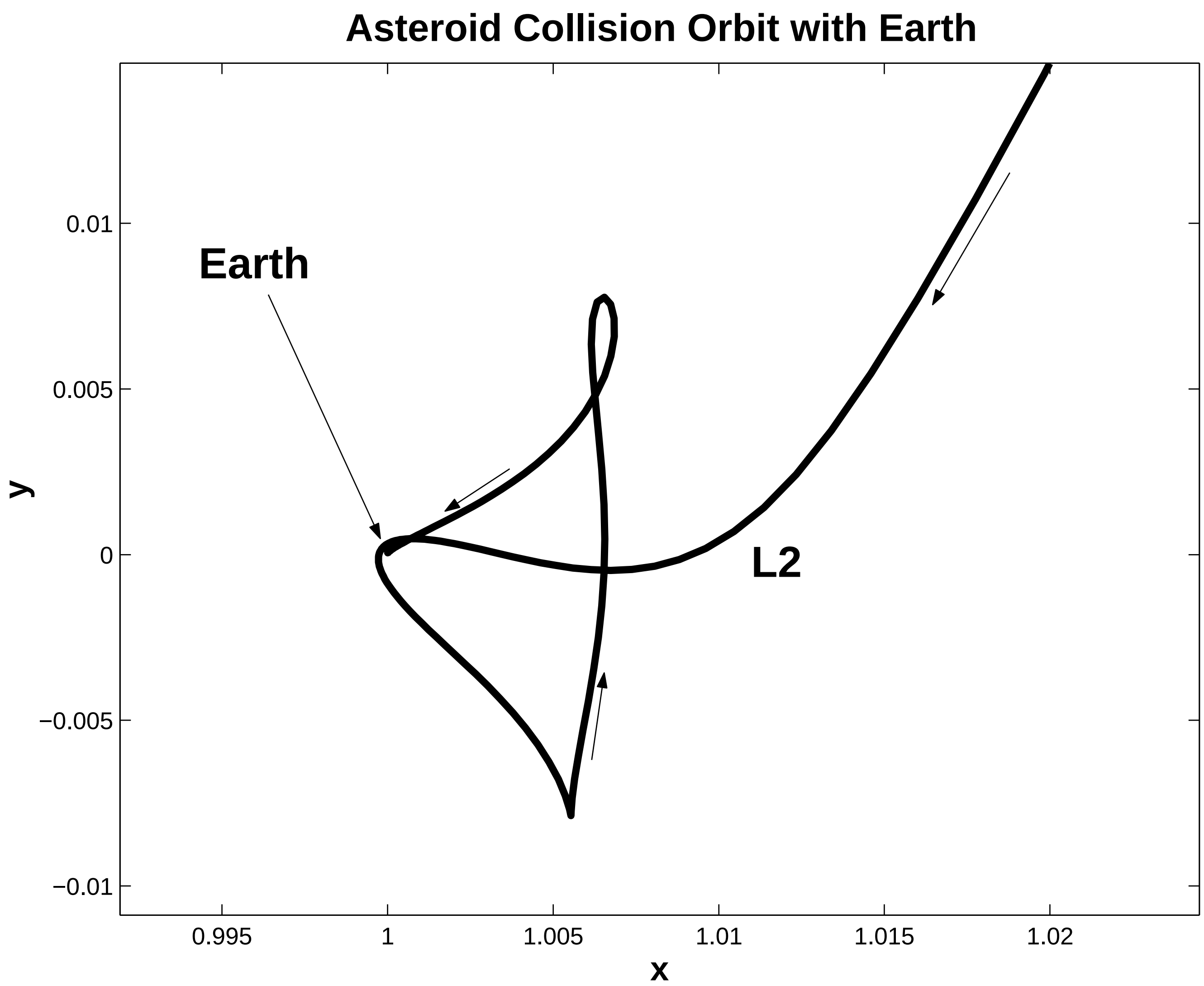
Probability for comet collision with Jupiter



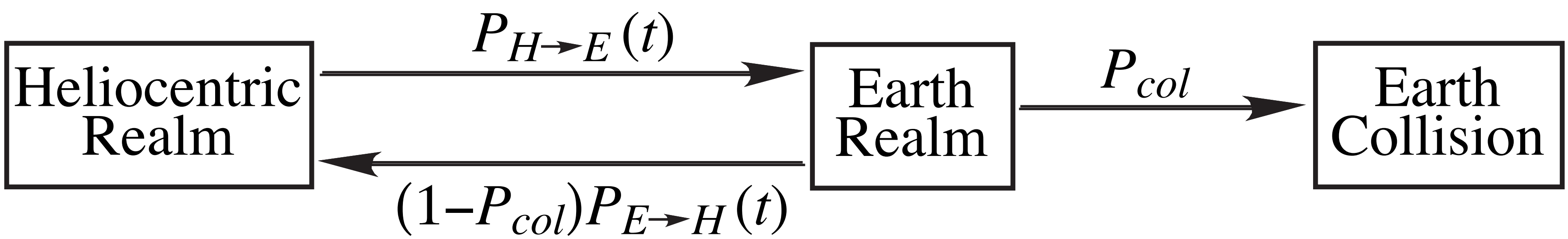
Probability for NEA collision with Earth



Typical collision orbit

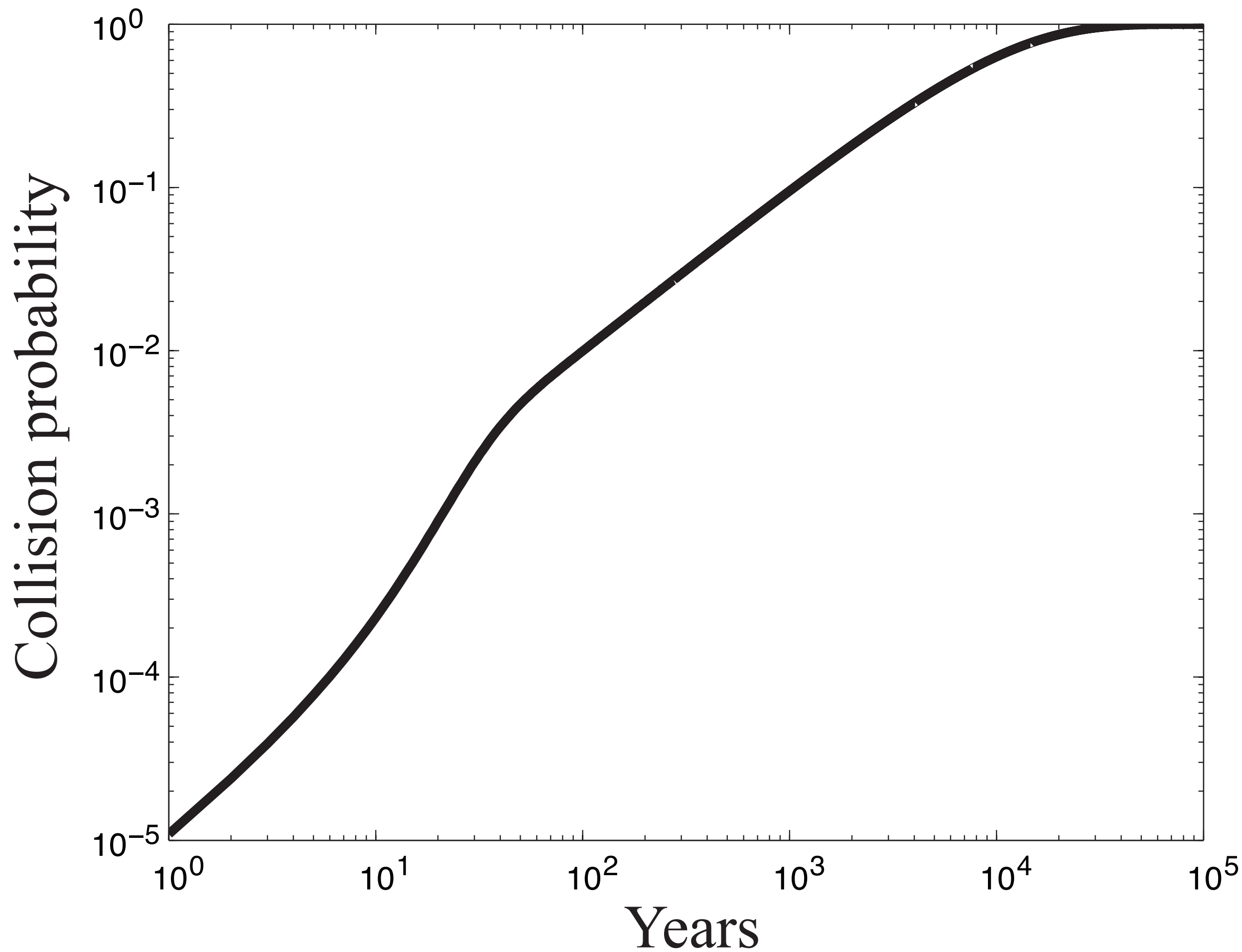


Simple kinetic mechanism for Earth collision



- Inspired by chemical reaction kinetic mechanisms (Markov process)

Simple kinetic mechanism for Earth collision



◦ Typical NEA strikes Earth within $\tau_{col} \sim 10^4 - 10^5$ years

– energy of 2004 MN4, potentially hazardous asteroid

Collisions in other systems

- timescale of collision

$$\tau_{col} \propto t_{orb} \left(\frac{m_2}{m_1}\right)^{k_m} E^{k_E} \, d^{k_d}$$

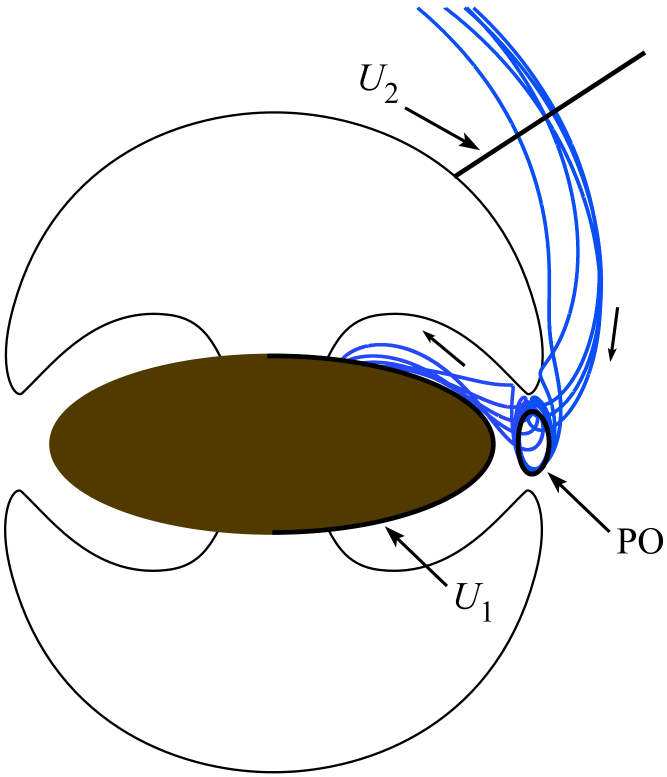
Binary asteroids

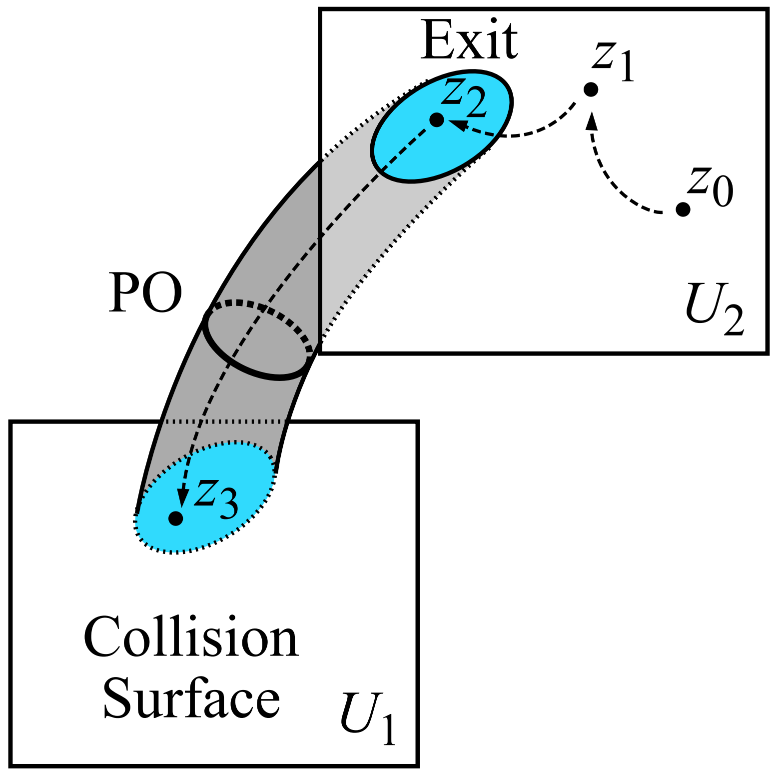
- Apply transport calculations to asteroid pairs to calculate, e.g., capture & escape rates.
  - example of full body problem (rotational-translational coupling)

Dactyl in orbit about Ida, discovered in 1994 during the *Galileo* mission.

Binary asteroids

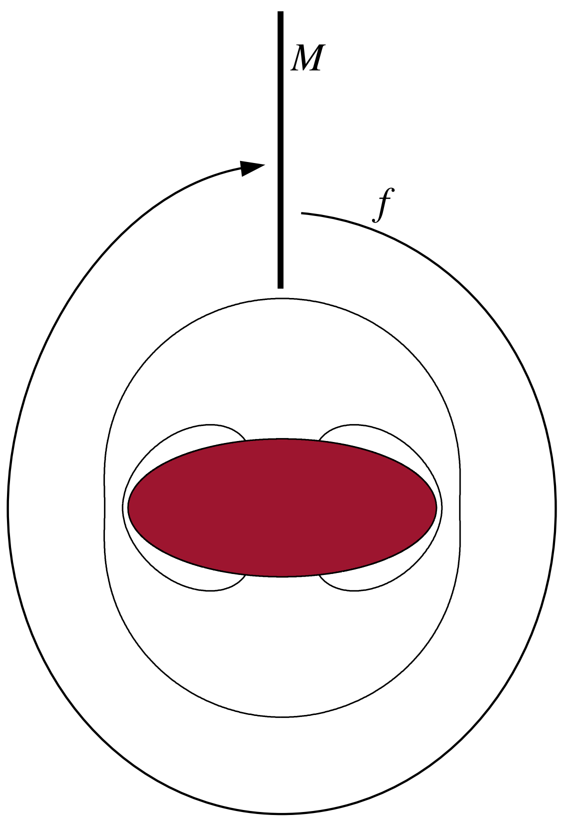
- Slices of energy surface: Poincaré sections U_i
- Tube dynamics: evolution **between** U_i
- Lobe dynamics: evolution **on** U_i



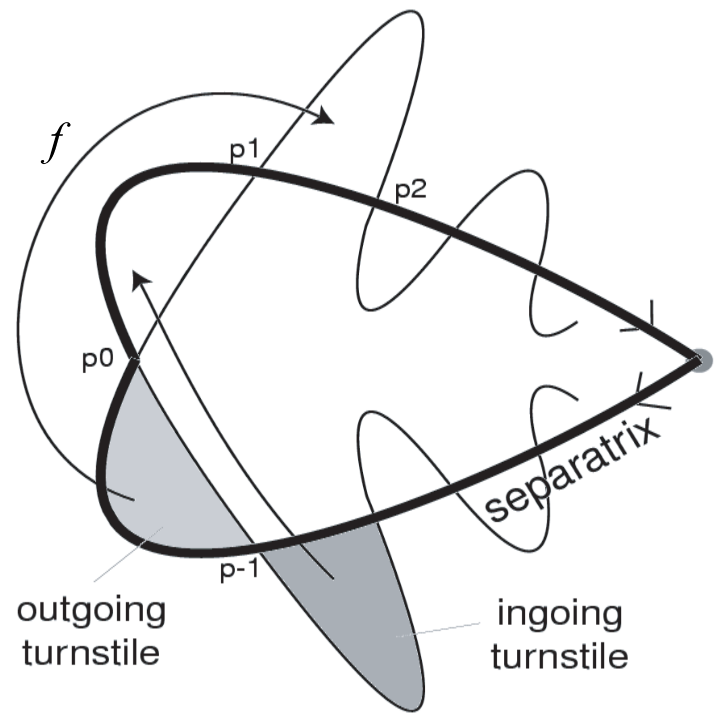


Lobes of ejection

- Smaller body **ejected** if within lobes bounded by manifolds of a hyperbolic fixed point at ∞ – similar to van der Waals complex formation

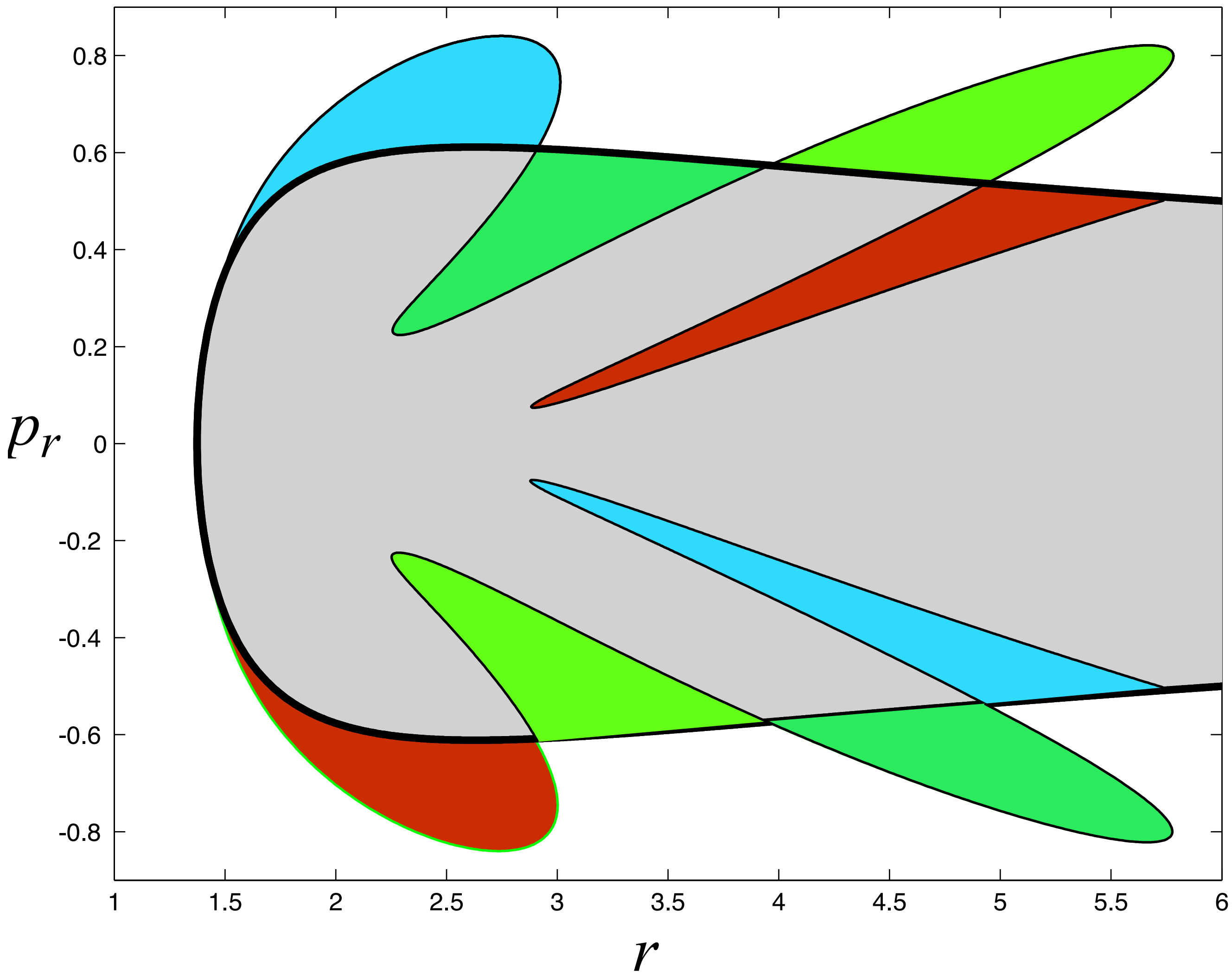


Position space projection



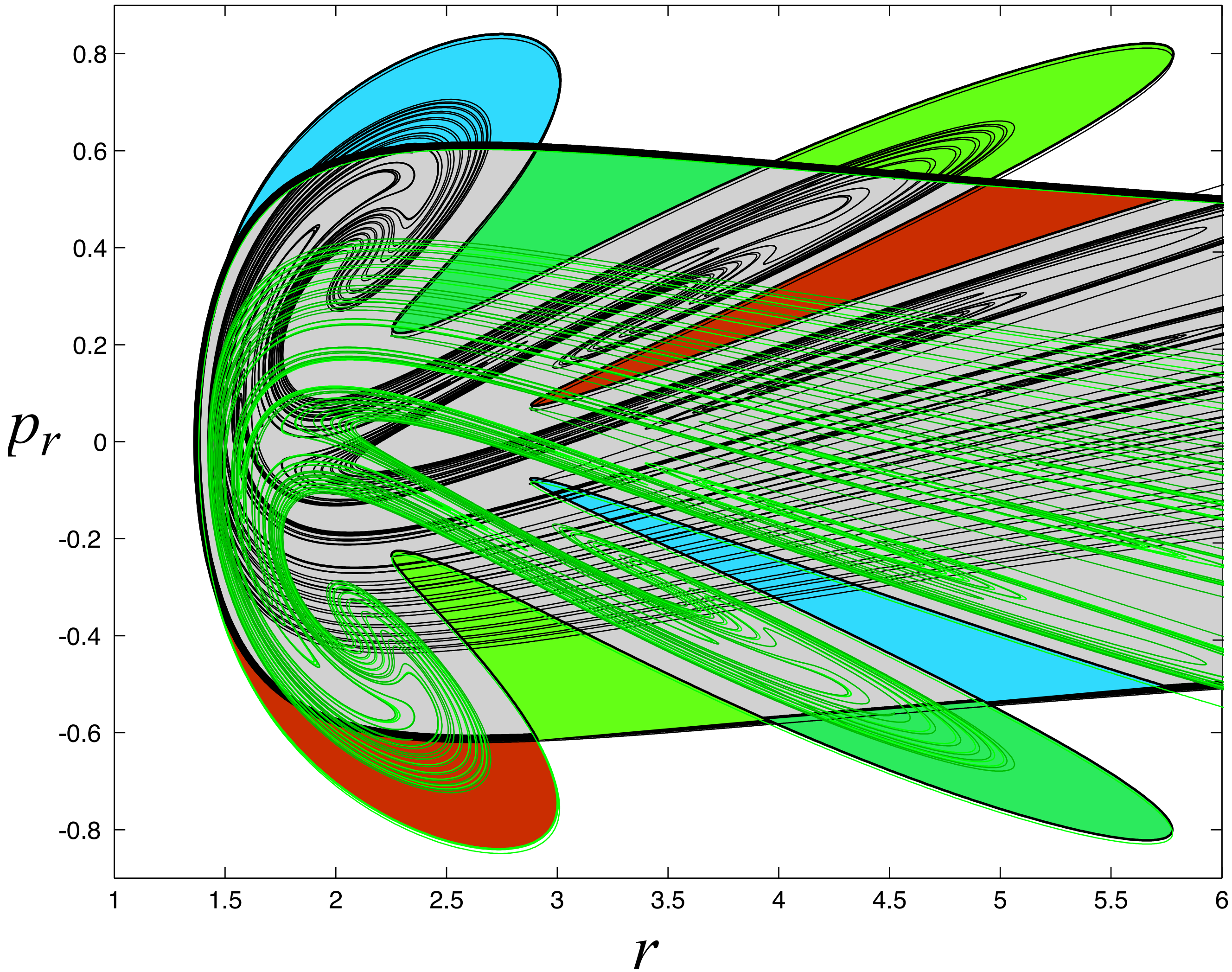
Lobe turnstile mechanism

Lobes of ejection



Numerical simulation using MANGEN

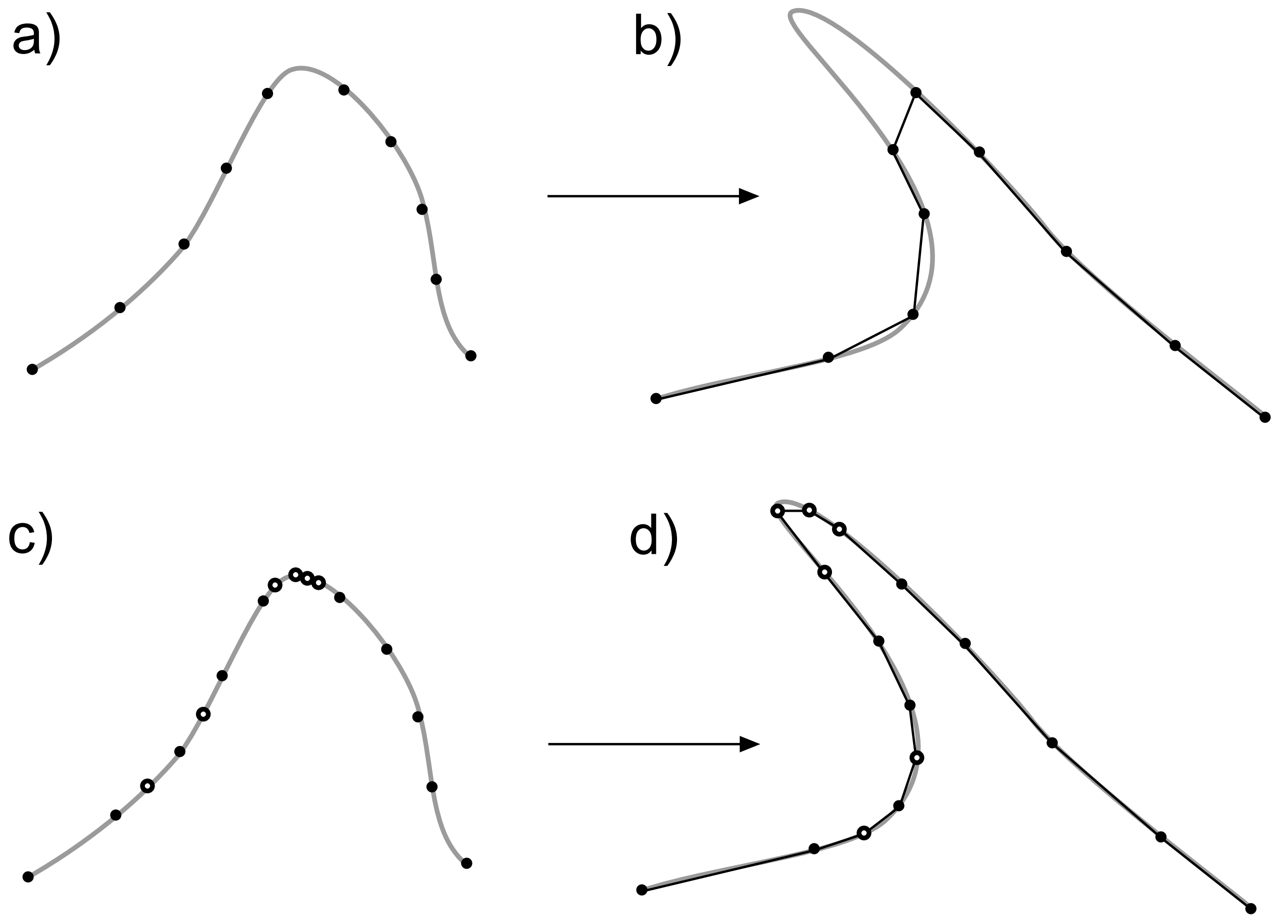
Lobes of ejection



Curves can be followed to very high accuracy

MANGEN description

- Simulations use MANGEN[6]
- Adaptive conditioning of surfaces based on curvature
  – for chaotic low dimensional systems of arbitrary time dependence



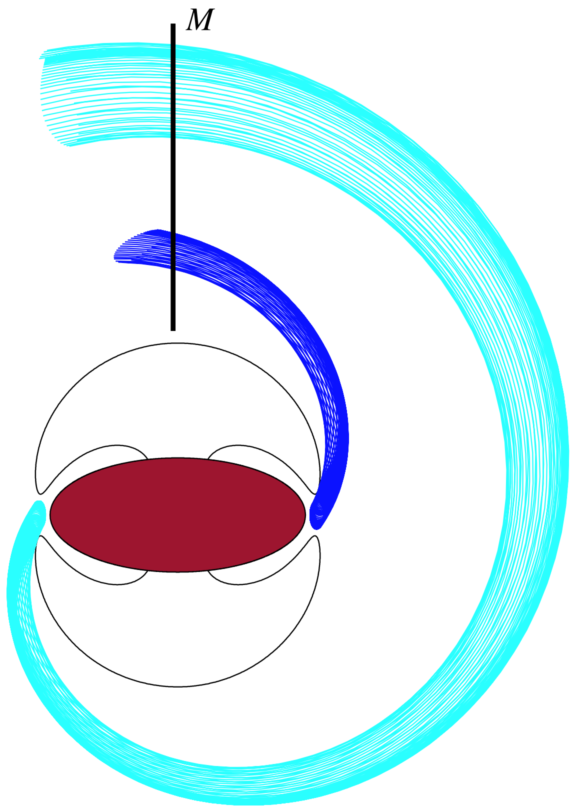
[6] Lekien, Coulliete, Marsden & Ross [2005], preprint

Tube + lobe dynamics

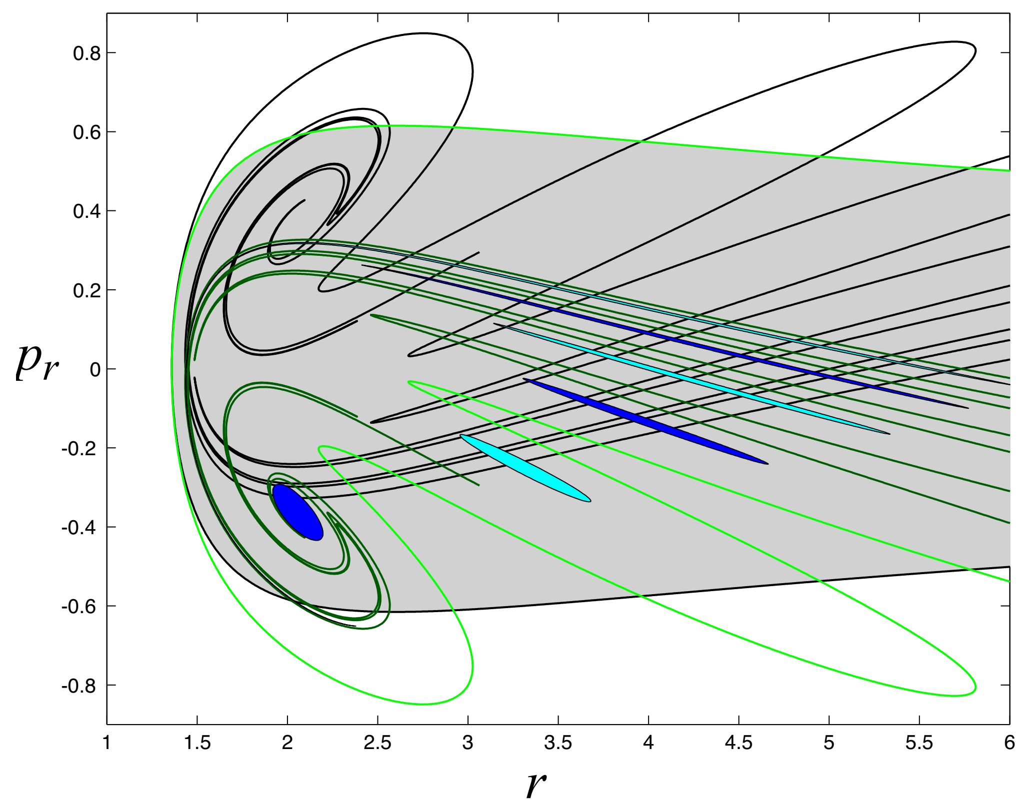
- Suppose energy above collision threshold
- Exterior and asteroid realms **connected via tubes**
- In exterior realm, some tubes lead to collision (others lead away from collision – liberation)
- **Tube + lobe dynamics =**
  Alternate fates of collision and ejection are intimately intermingled.

Tube + lobe dynamics

- Tubes leading to collision with asteroid



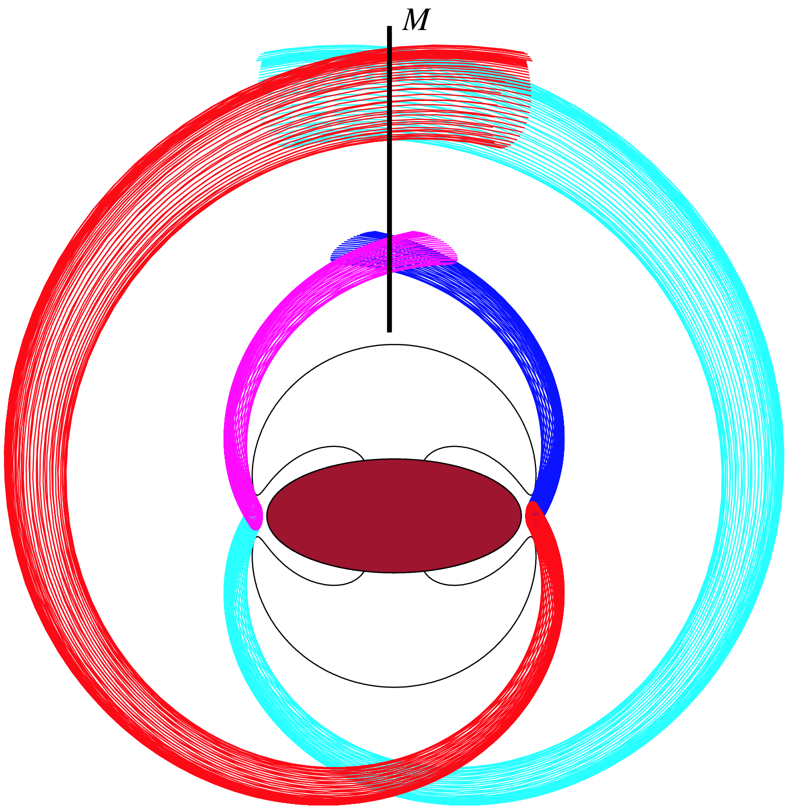
Position space projection



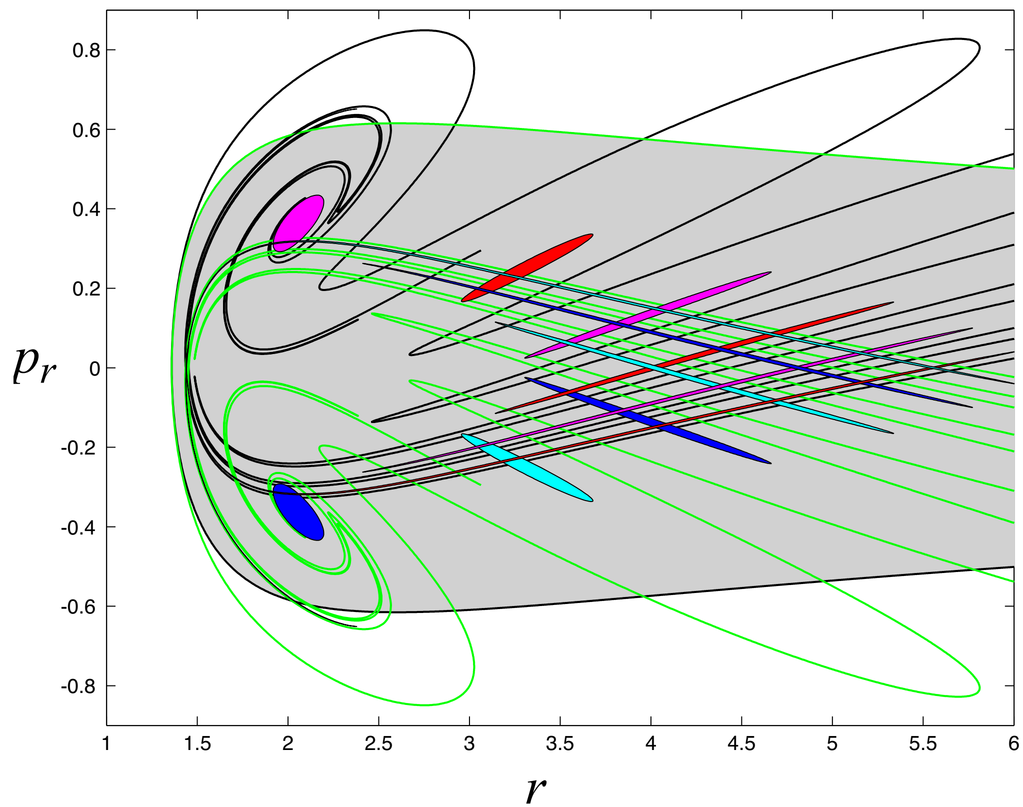
Motion on M

Tube + lobe dynamics

- Tubes leading to collision with asteroid **plus** tubes coming from collision, e.g., liberated ejecta



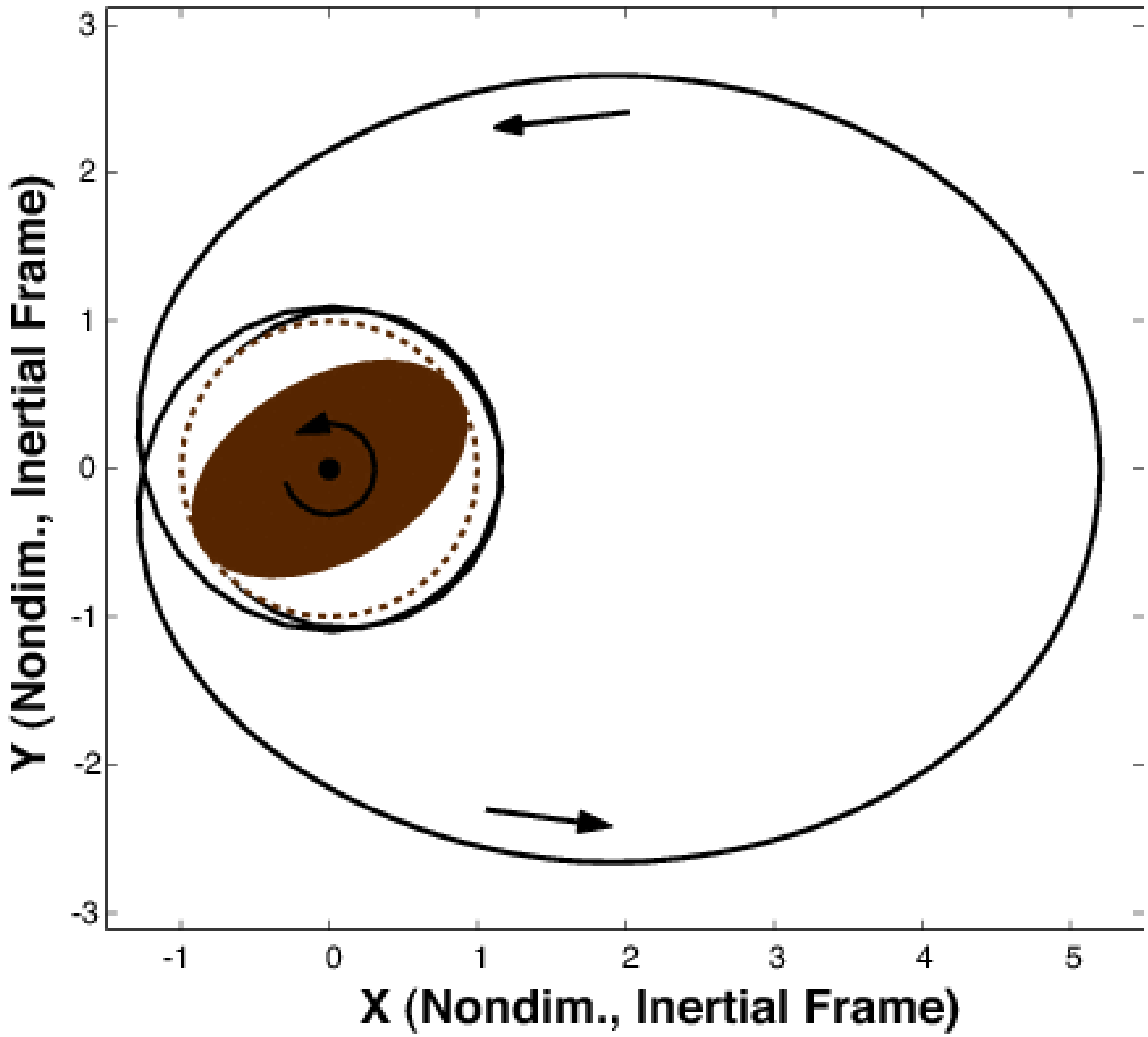
Position space projection



Motion on M

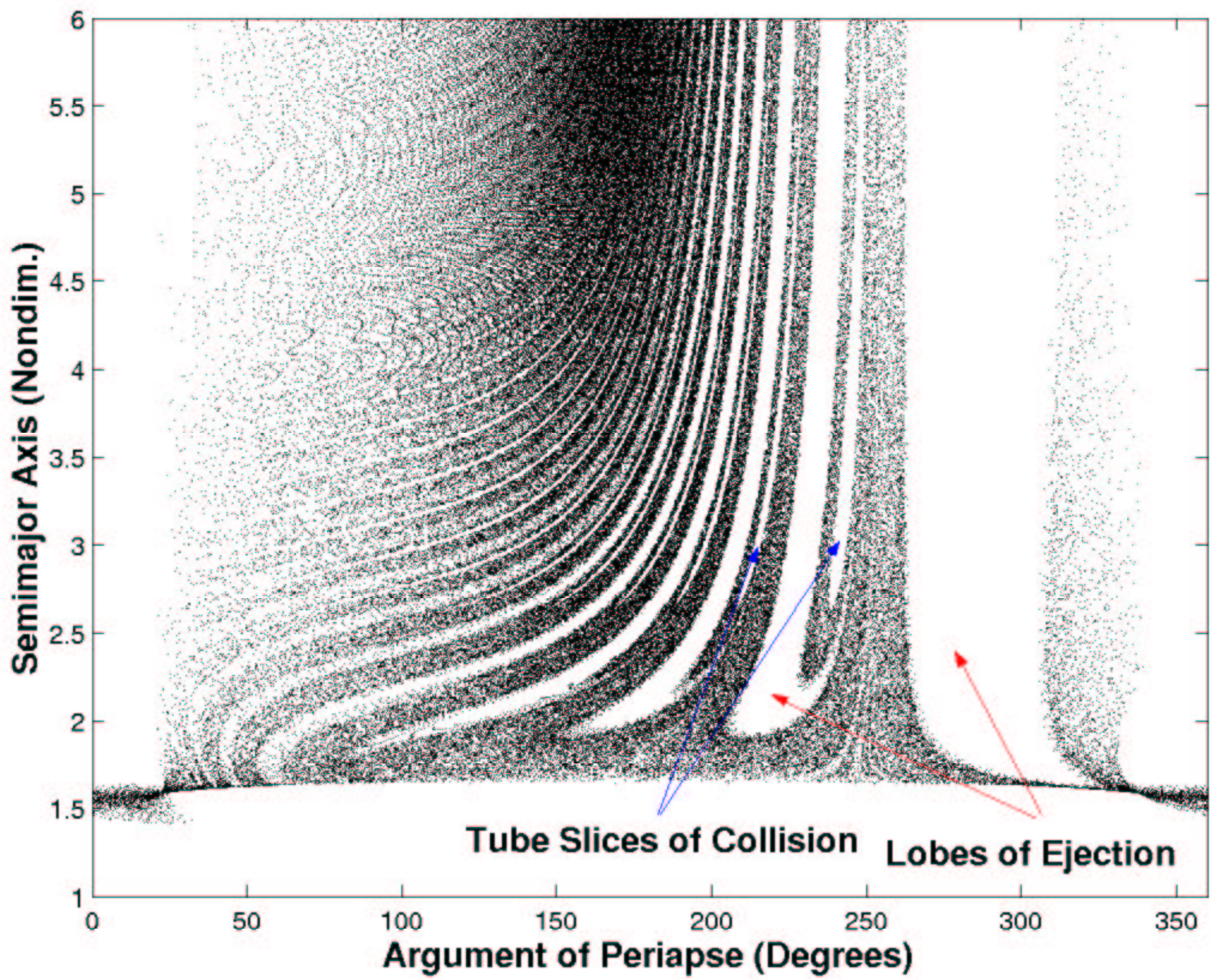
Tube + lobe dynamics

- Escape and re-capture.



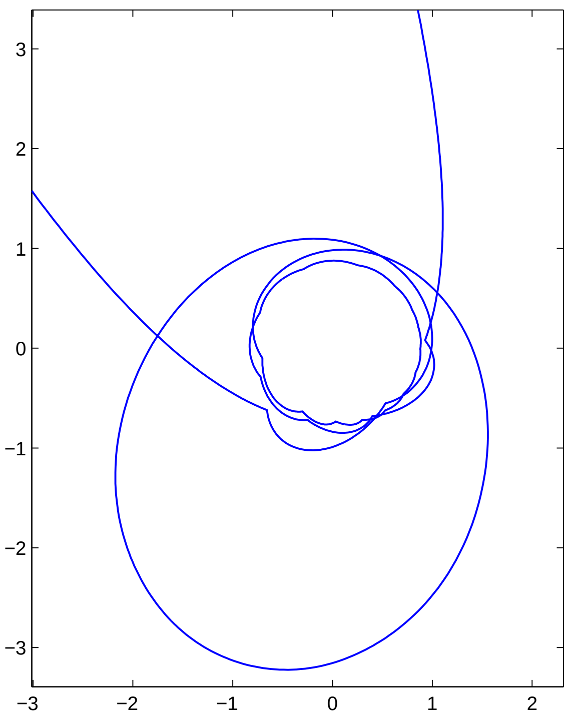
Tube + lobe dynamics

- Alternate fates of ejection and collision intermingled[7]

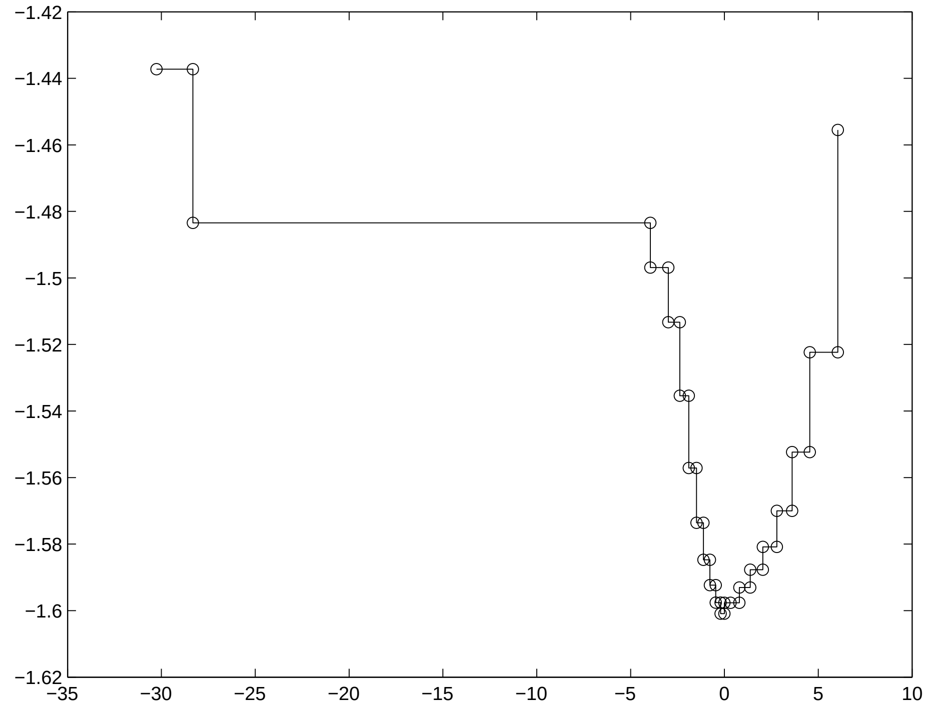


[7] Koon, Marsden, Ross, Lo, and Scheeres [2004] Geometric mechanics and the dynamics of asteroid pairs, *Annals of the New York Academy of Science* 1017, 11–38.

Collisions between rigid bodies



Inertial frame projection



Energy history

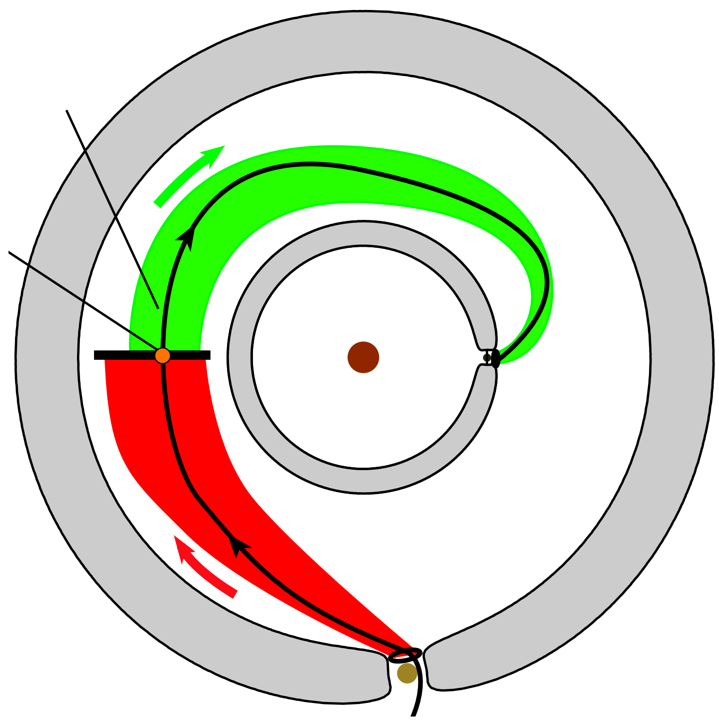
- If bouncing is modeled, dynamics more complicated – bouncing particle moves between energy surfaces

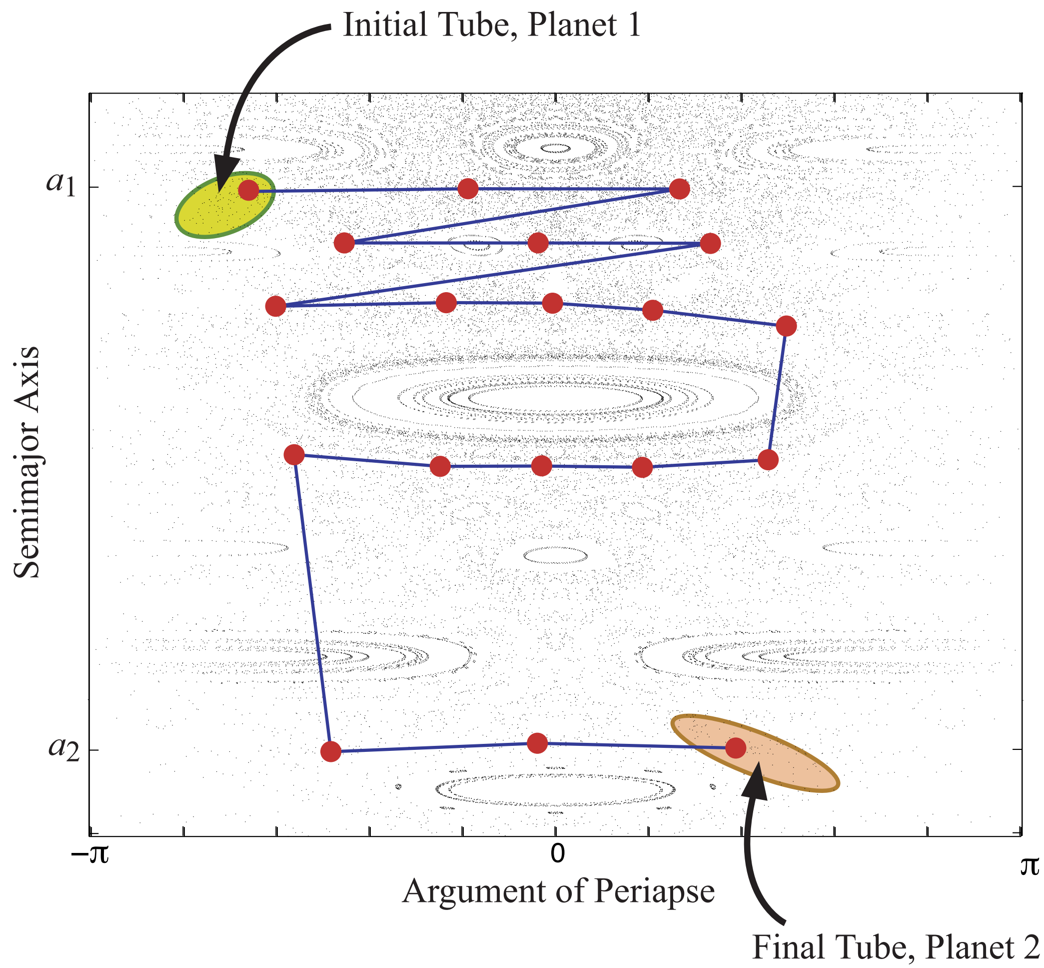
Other situations to explore

- Ejecta transfer between planets
- Dissipative perturbations
- Additional physics, astrophysical situations of interest
  - effect of mass transfer on phase space
  - ideas ???

Ejecta transfer

- Linking multiple 3-body systems



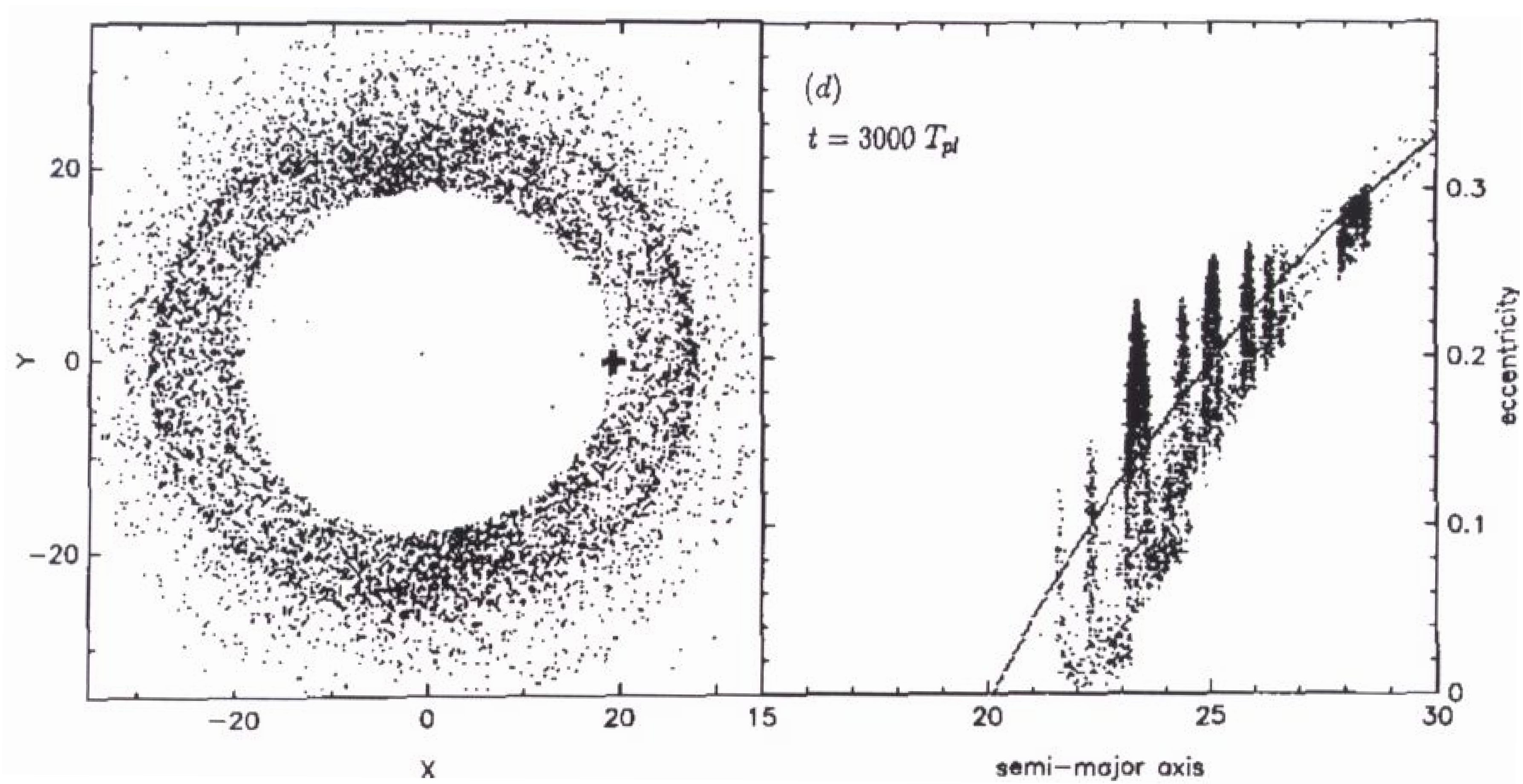


Ejecta transfer

- Earth to moon

Dissipative perturbations

- Dust grains temporarily captured in resonances creating ring structure
  – the circumstellar disk.



Source: Roques, Scholl, Sicardy, and Smith [1994]

Summary and outlook

- Relationship between phase space geometry and statistics for low dimensional systems
  – connected chaotic sets
  – transport via tubes and lobes
  – ejection and collision

- Statistical ideas from chemistry may be useful
  – coarse variables
  – for large N, low-dim manifold may dominate dynamics

Summary and outlook

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