CALTECH
Control & Dynamical Systems

# Invariant Manifolds, Material Transport and Space Mission Design

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# Acknowledgements

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- ***Theme***
  - Using dynamical systems theory for understanding solar system dynamics and identifying useful orbits for space missions.

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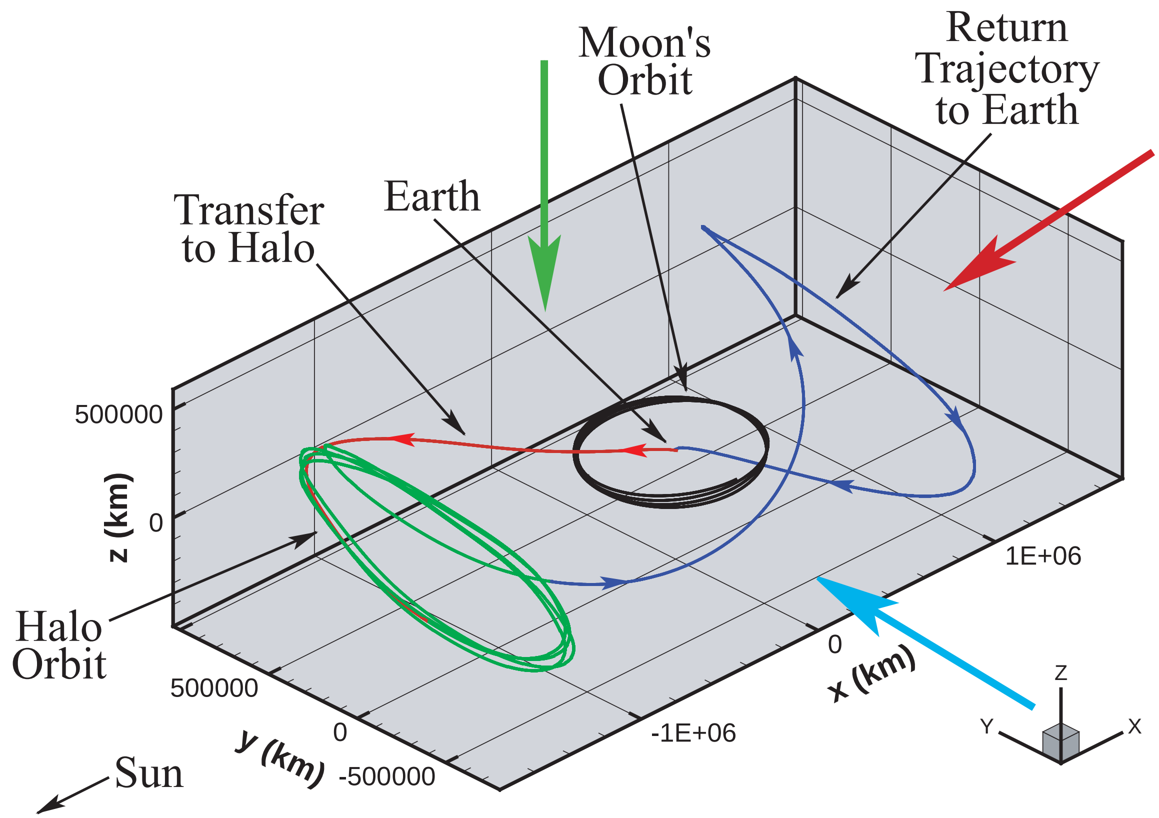
  - Using dynamical systems theory for understanding solar system dynamics and identifying useful orbits for space missions.
- ***Current research importance***
  - development of some NASA mission trajectories, such as the *Genesis Discovery Mission* to be launched Monday
  - of current astrophysical interest for understanding the transport of solar system material (eg, how ejecta from Mars gets to Earth, etc.)

# Genesis Discovery Mission

- Genesis will collect solar wind samples at the Sun-Earth L1 and return them to Earth.
- It was the first mission designed start to finish using dynamical systems theory.



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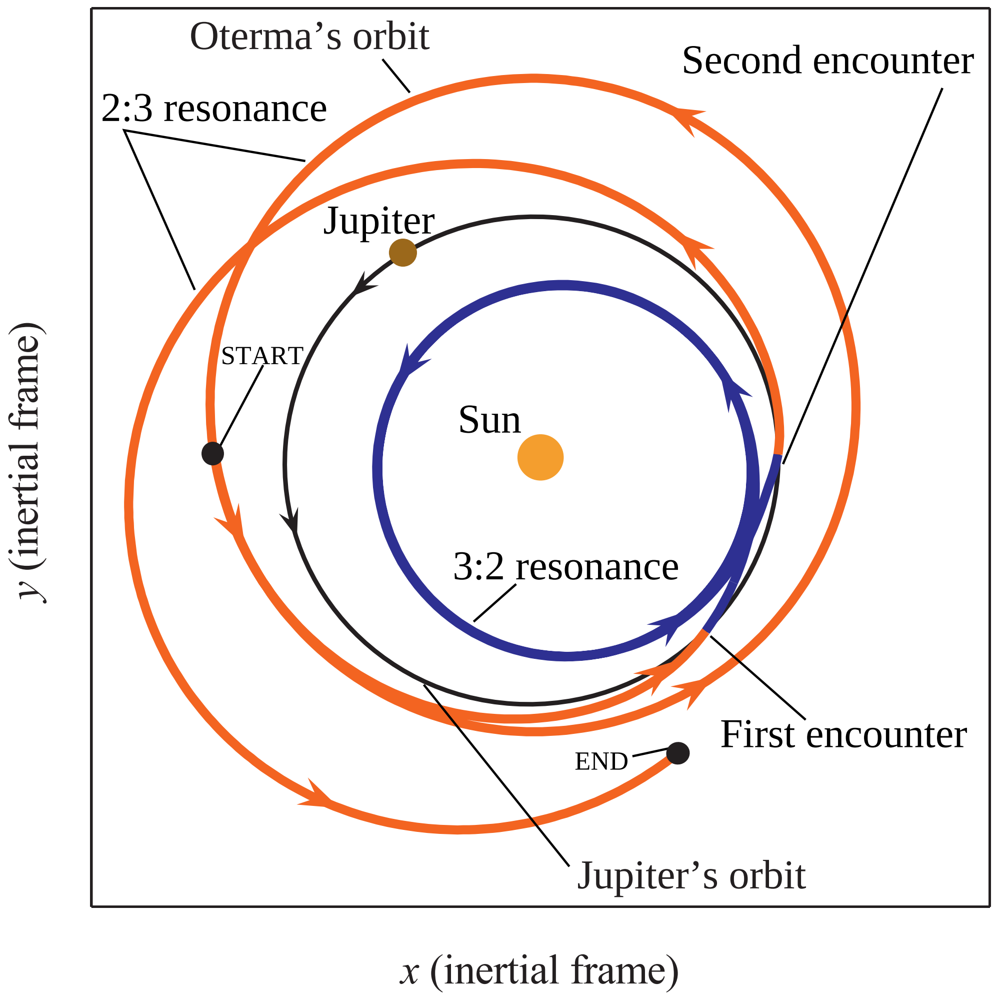
    - Earth-Moon L1 Gateway station
  - optimal control
    - trajectory correction maneuvers for *Genesis*

# Jupiter Comets

- We consider the historical record of the comet Oterma from 1910 to 1980
  - first in an inertial frame
  - then in a rotating frame
  - a special case of pattern evocation
- similar pictures exist for many other comets

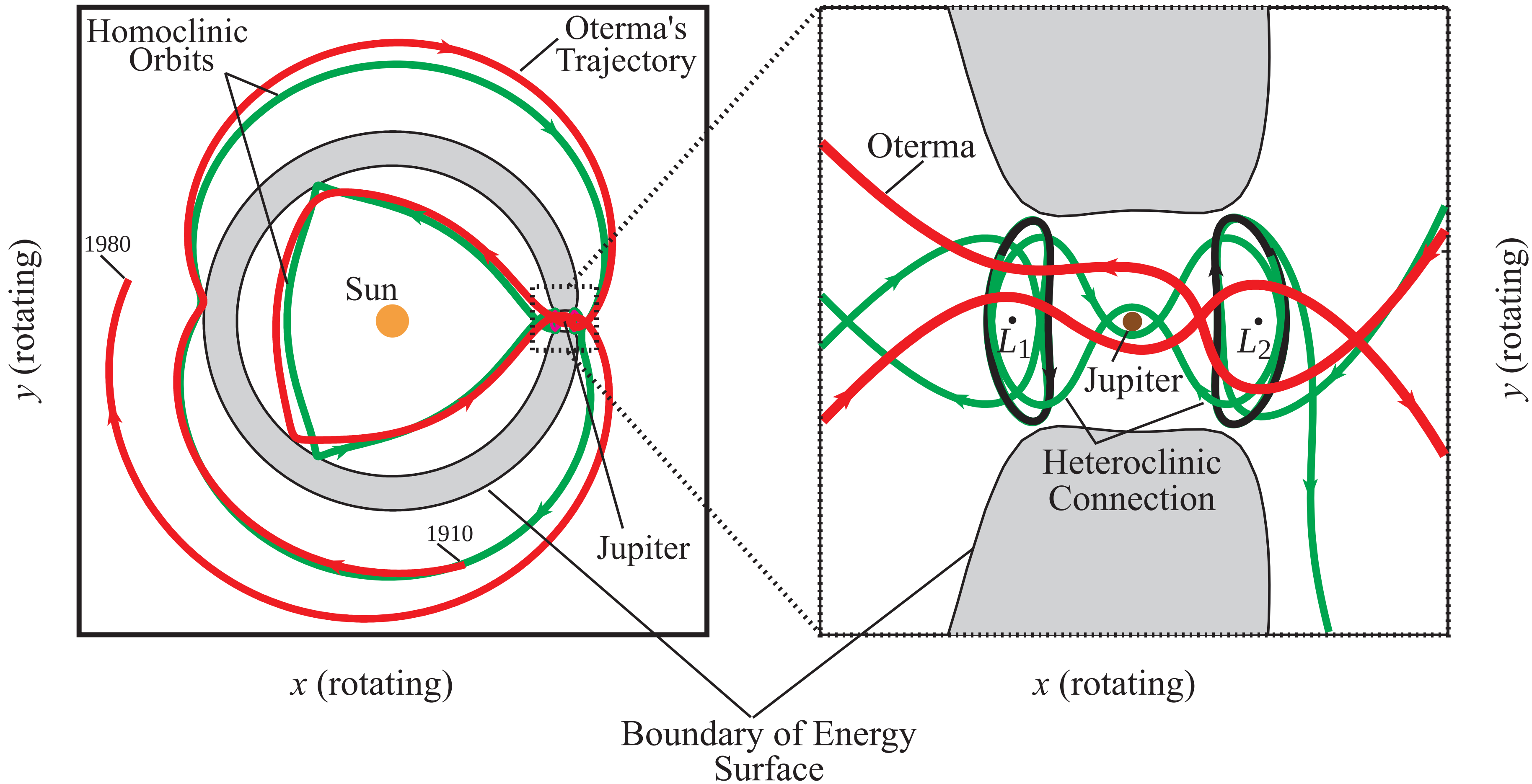
# Jupiter Comets

- Rapid transition: outside to inside Jupiter's orbit.
  - Captured temporarily by Jupiter during transition.
  - Exterior (2:3 resonance) to interior (3:2 resonance).



# Viewed in Rotating Frame

- Oterma's orbit in rotating frame with some invariant manifolds of the 3-body problem superimposed.



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*oterma-rot.qt*

# Three-Body Problem

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# Three-Body Problem

## *Circular restricted problem*

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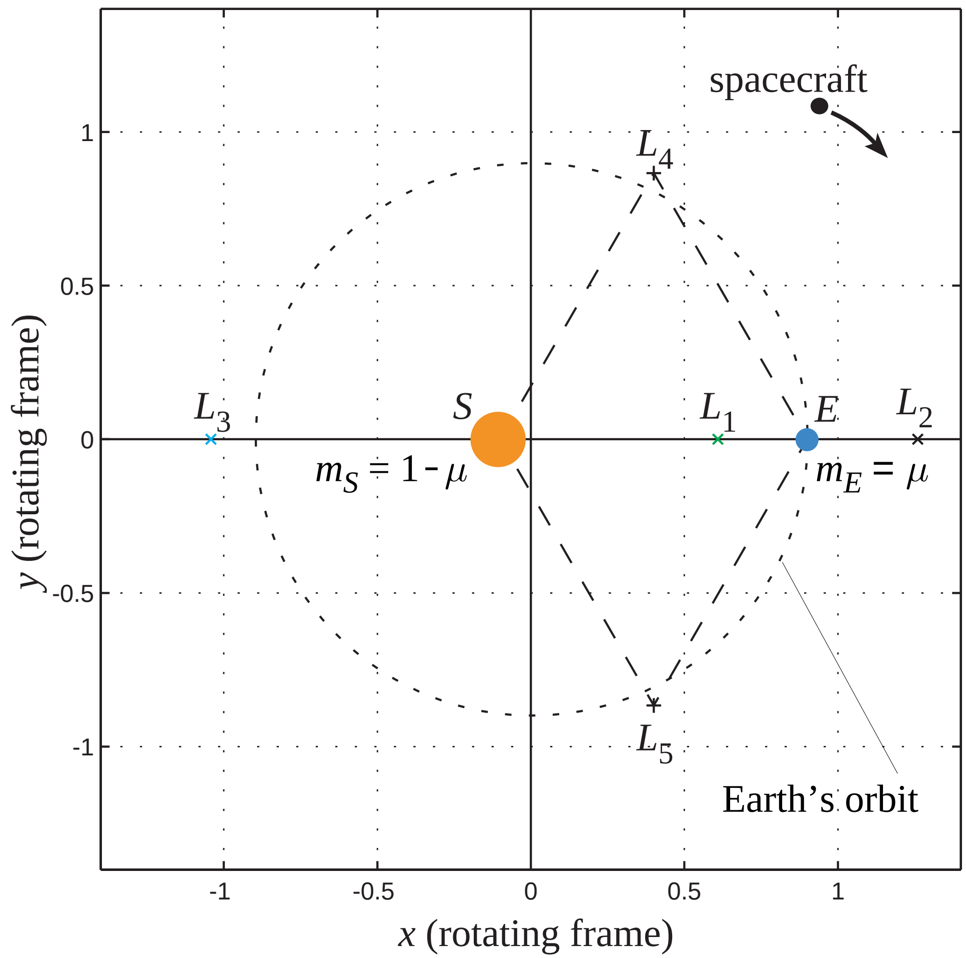
- the two primaries could be the Sun and Earth, or Earth and Moon, or the Sun and Jupiter, etc.
- the smaller body could be a spacecraft, comet, or asteroid
- we consider the planar and spatial problems
- there are five equilibrium points in the rotating frame, places of balance which generate interesting dynamics

# Three-Body Problem

- 3 unstable points on line joining two main bodies – $L_1, L_2, L_3$
- 2 stable points at $\pm 60°$ along the circular orbit – $L_4, L_5$




Equilibrium points

# Three-Body Problem

- orbits exist around $L_1$ and $L_2$; both periodic and quasi-periodic
  - Lyapunov, halo and Lissajous orbits
- one can draw the invariant manifolds assoicated to $L_1$ (and $L_2$) and the orbits surrounding them
- these invariant manifolds play a key role in what follows

# Three-Body Problem

- Equations of motion:

$$\boxed{\ddot{x} - 2\dot{y} = -U_x^{\text{eff}}, \qquad \ddot{y} + 2\dot{x} = -U_y^{\text{eff}}}$$

where

$$U^{\text{eff}} = -\frac{(x^2 + y^2)}{2} - \frac{1-\mu}{r_1} - \frac{\mu}{r_2}.$$

- Have a first integral, the Hamiltonian energy, given by

$$E(x, y, \dot{x}, \dot{y}) = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) + U^{\text{eff}}(x, y).$$

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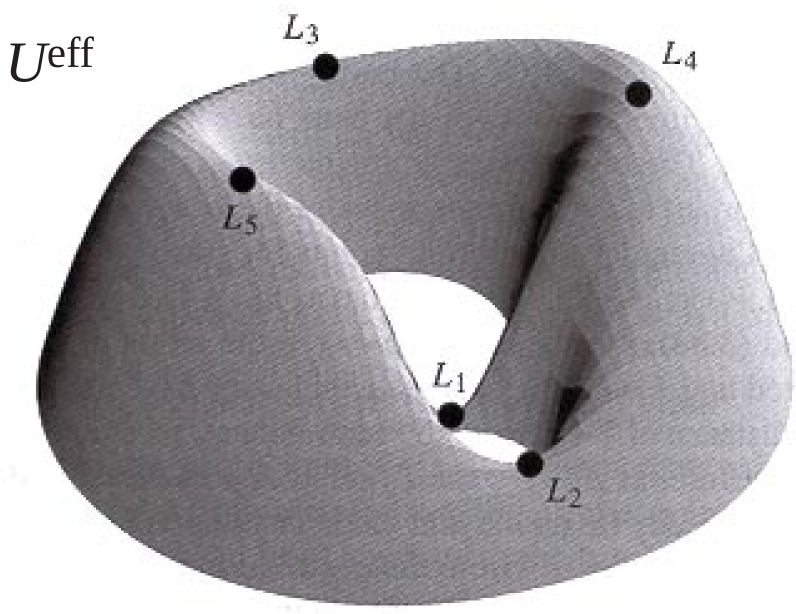
- Energy manifolds are 3-dimensional surfaces foliating the 4-dimensional phase space.
- This is for the planar problem, but the spatial problem is similar.

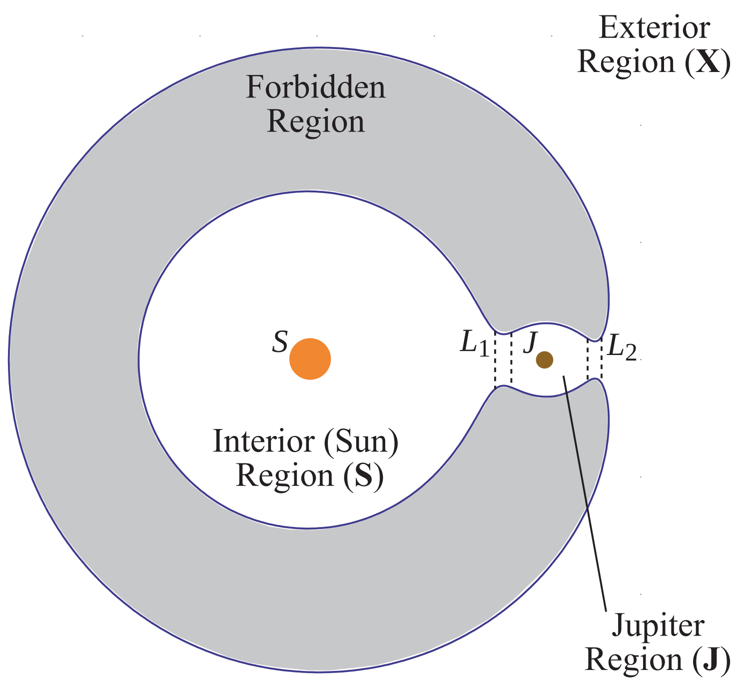
# Regions of Possible Motion

## *Effective potential*

- In a rotating frame, the equations of motion describe a particle moving in an effective potential plus a magnetic field (goes back to work of Jacobi, Hill, etc).



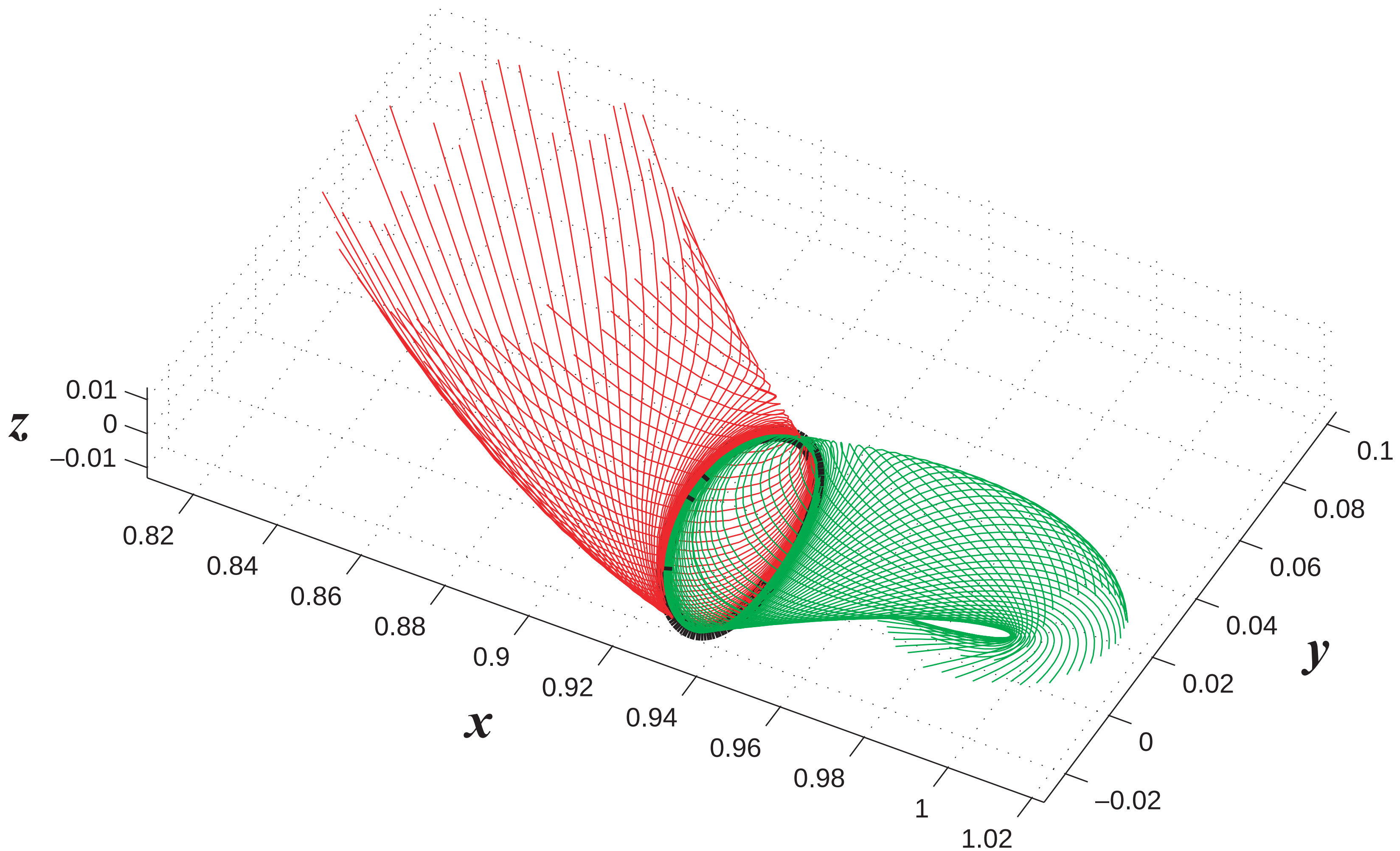

Effective potential




Level set shows accessible regions

# Transport Between Regions

- ***Invariant manifolds of $L_1/L_2$ orbits***
  - red = unstable, green = stable



# Transport Between Regions

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- and what bounces back (non-transit orbits)
- transit possible for objects "inside" the tube, otherwise no transit — this is important for transport issues

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- These manifold tubes play an important role in what passes by Jupiter (transit orbits)
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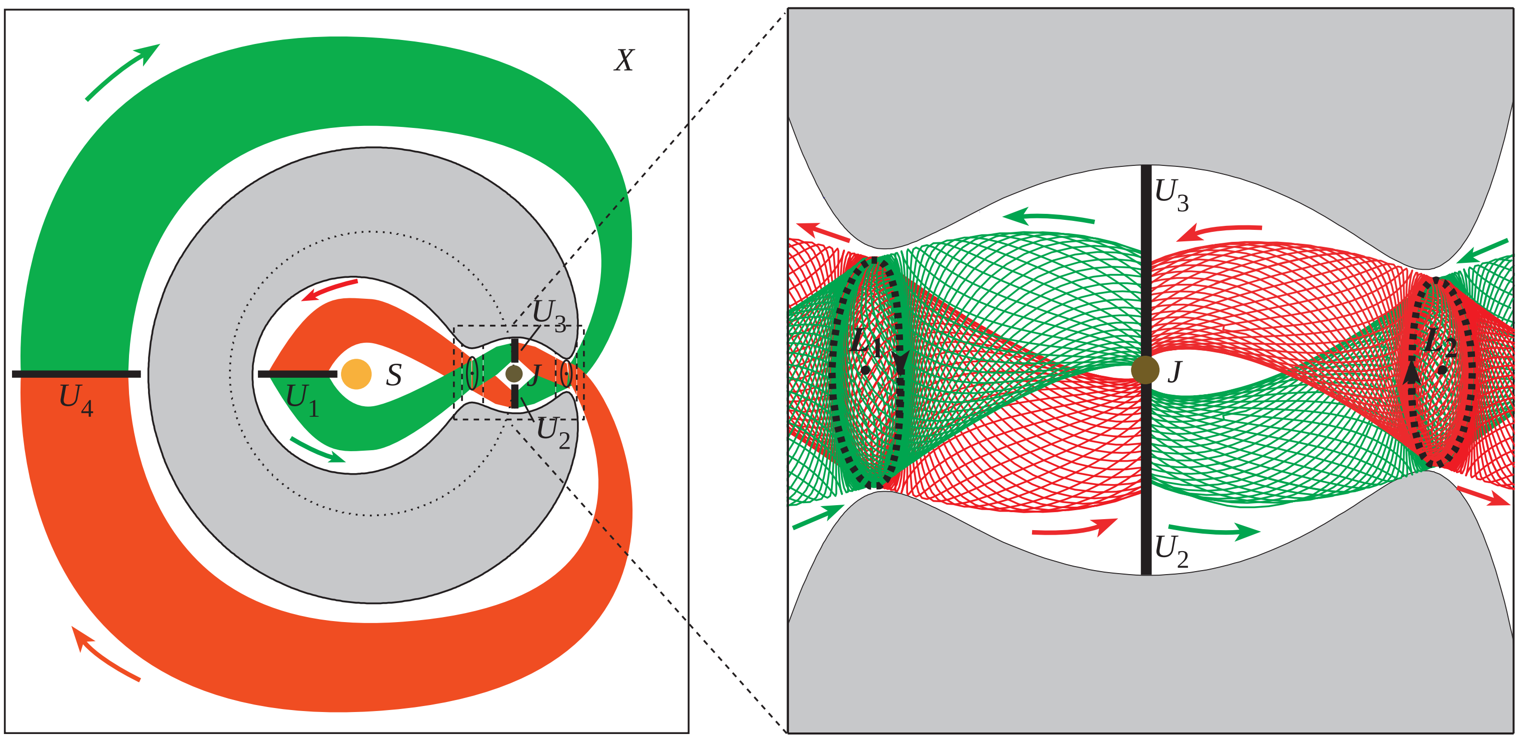
- transit possible for objects "inside" the tube, otherwise no transit — this is important for transport issues
- earlier work in this direction by Conley and McGehee in the 1960's was extended by Koon, Lo, Marsden, and Ross [2000]
- discovery of heteroclinic connection between $L_1$ and $L_2$ orbits was key

# Transport Between Regions

- ***Theorem of global orbit structure***
  - says we can construct an orbit with any itinerary, eg $(\ldots, J, X, J, S, J, S, \ldots)$, where $X, J$ and $S$ denote the different regions (symbolic dynamics)

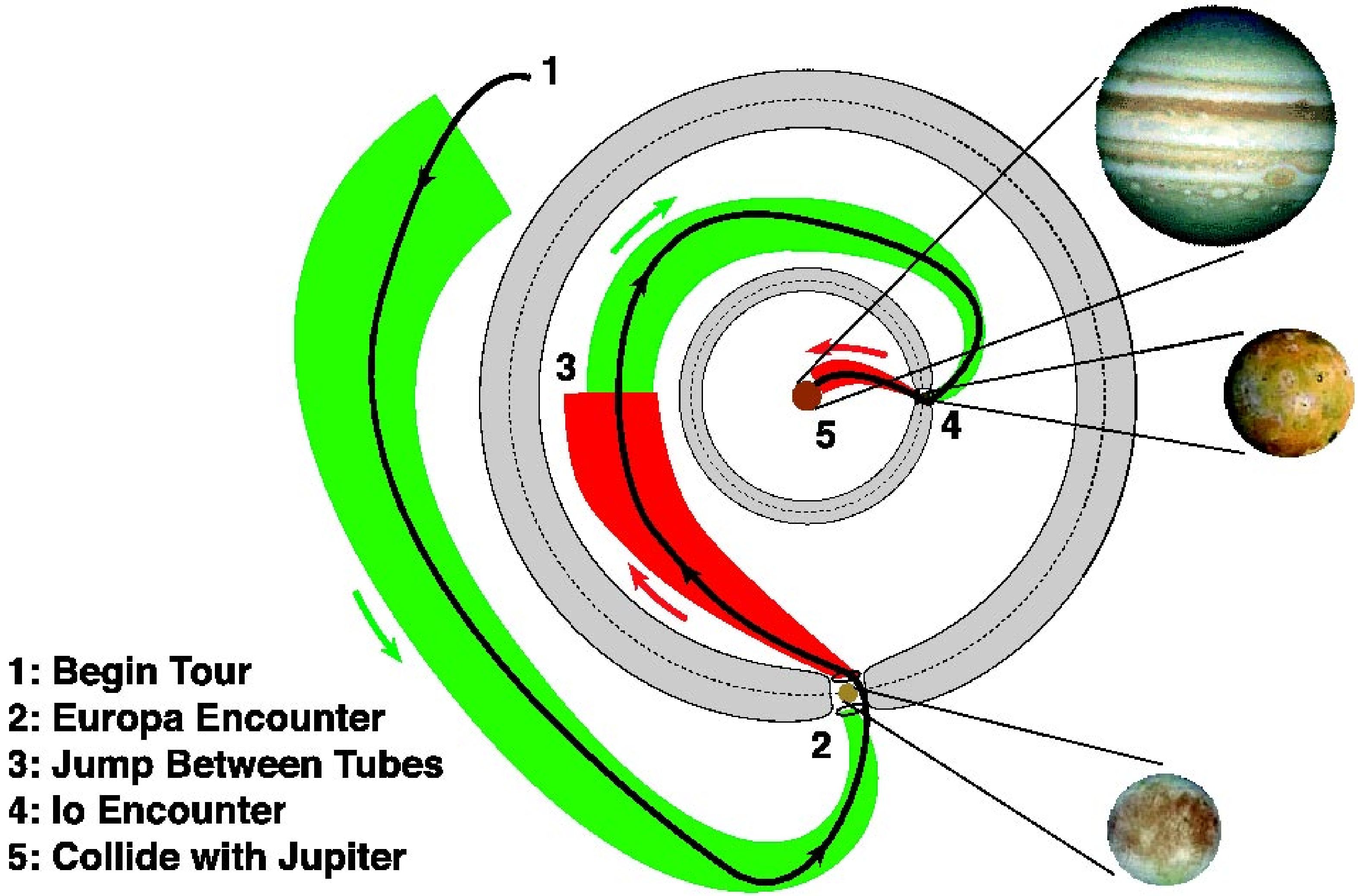


# Construction of Trajectories

- One can systematically construct new trajectories, which use little fuel.
  - by linking stable and unstable manifold tubes in the right order
  - and using Poincaré sections to find trajectories “inside” the tubes
- One can construct trajectories involving multiple 3-body systems.

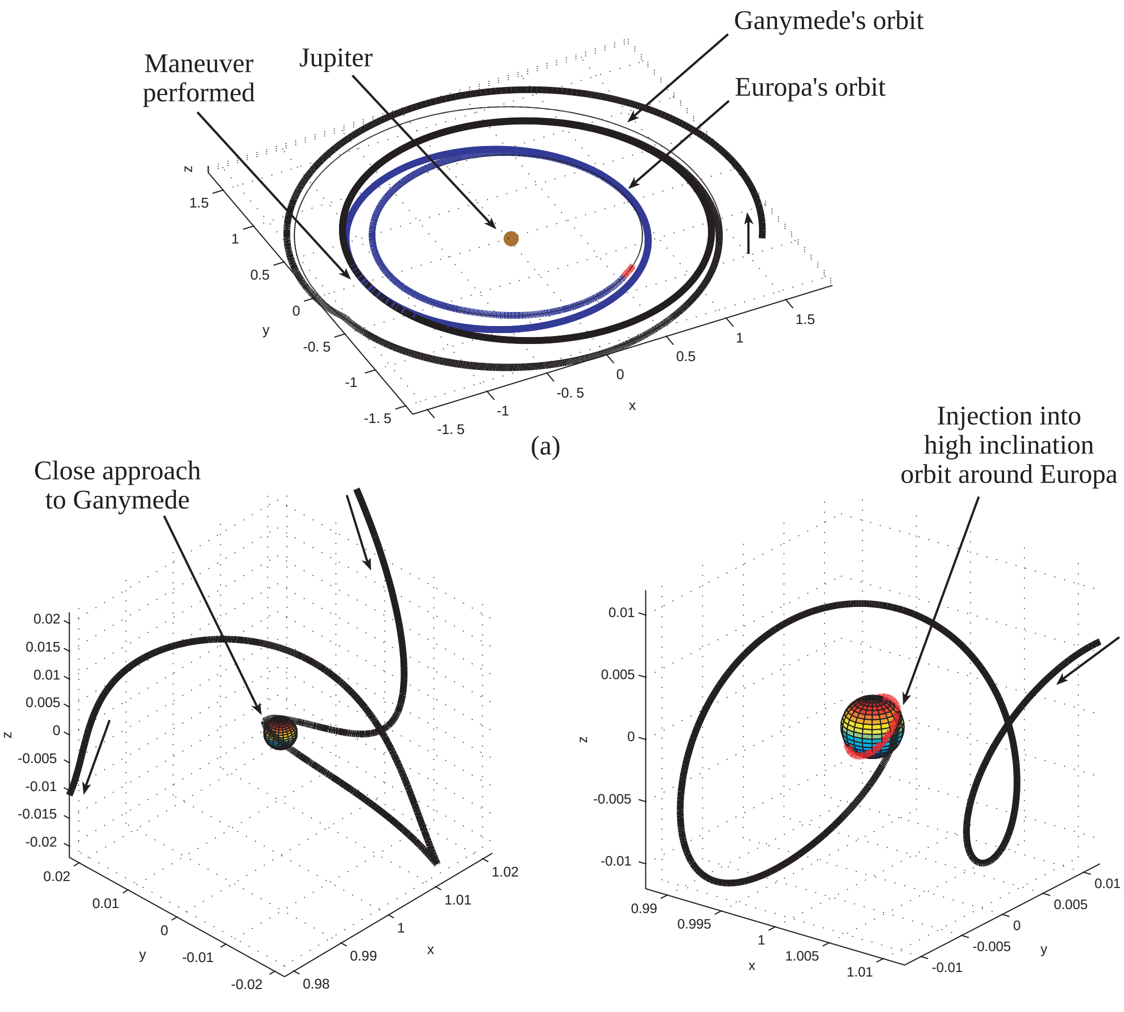
# Tour of Jupiter's Moons

- ***Tours of planetary satellite systems.***
  - *Example 1:* Europa → Io → Jupiter



# Tour of Jupiter's Moons

- *Example 2:* Ganymede → Europa → injection into Europa orbit




(a) (b) (c)

# Tour of Jupiter's Moons

*pgt-3d-orbit-eu.qt*

# Earth to Moon Transfer

- In 1991, a Japanese mission was "saved" by using a more fuel efficient way to the Moon (Miller and Belbruno)
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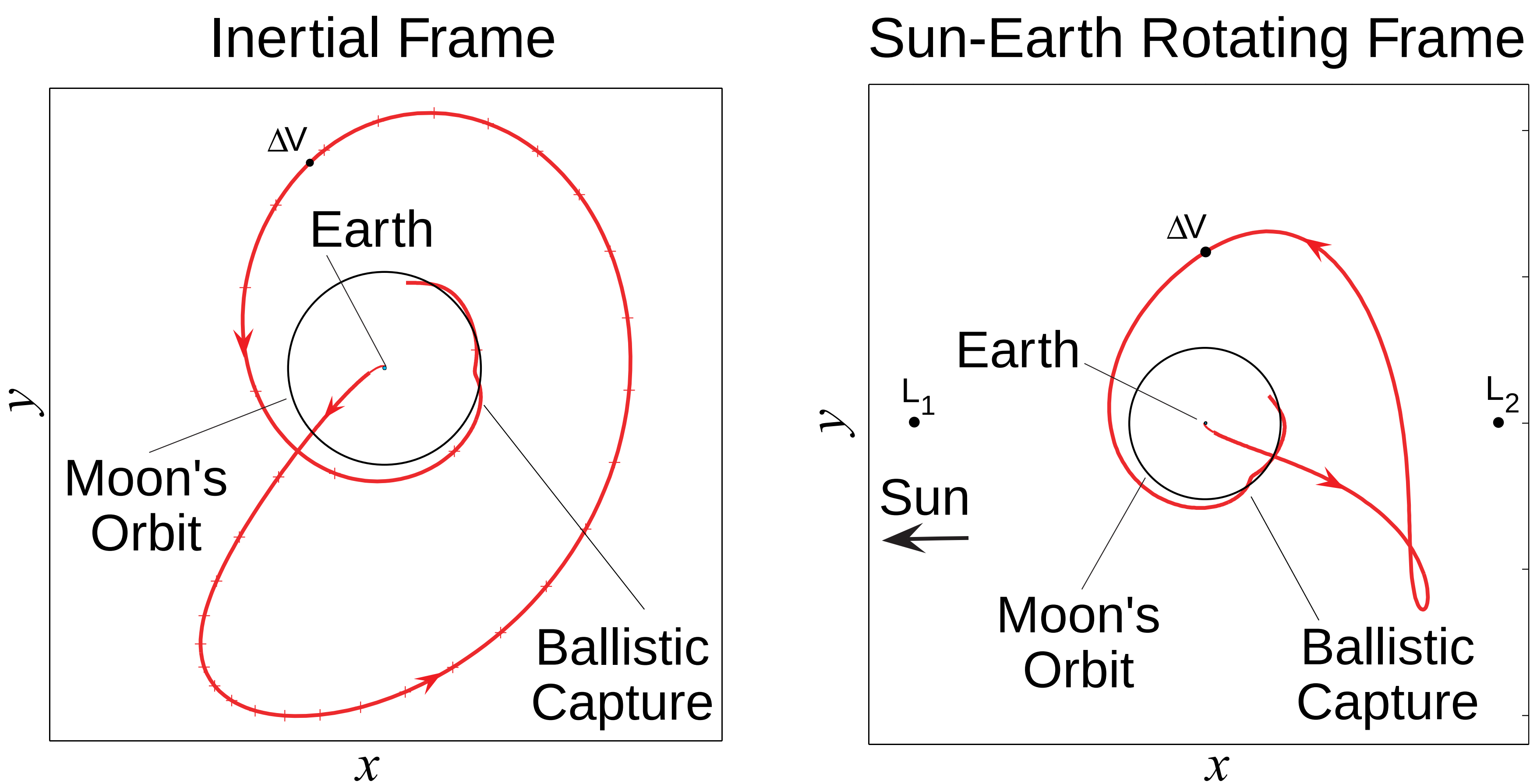
  - systematically implementing the view that the Sun-Earth-Moon-spacecraft 4-body system can be regarded as two coupled 3-body systems
  - and using invariant manifold ideas
- we transfer from
  - the Sun-Earth-spacecraft system to
  - the Earth-Moon-spacecraft system

# Earth to Moon Transfer

- 20% more fuel efficient than Apollo-like transfer
  - but takes longer; a few months compared to a few days



# Earth to Moon Transfer

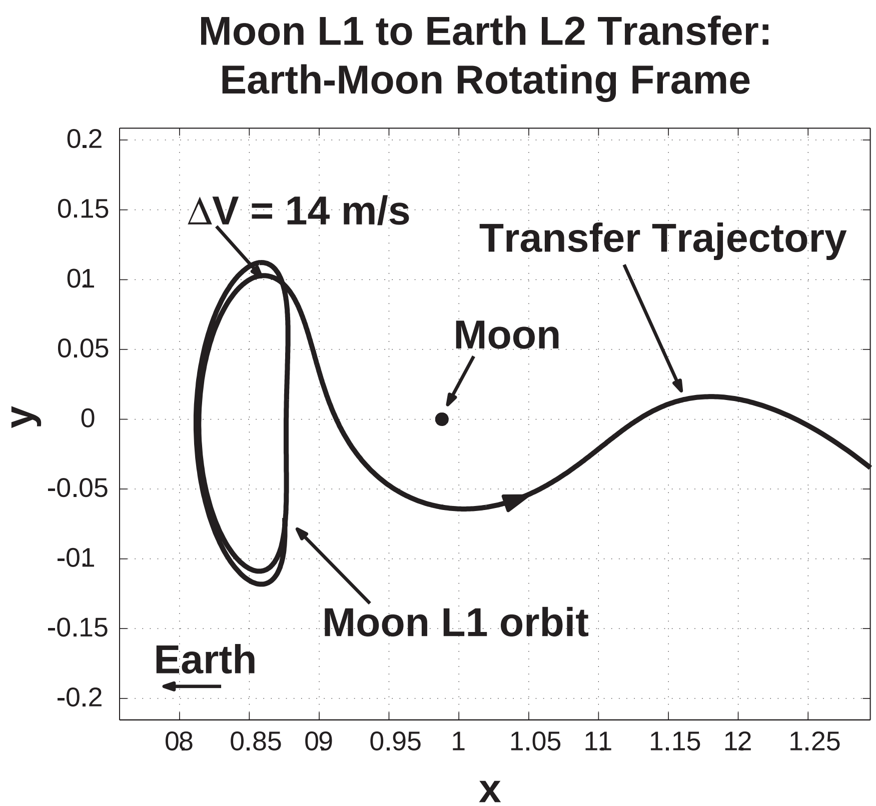
*shootthemoon-rotating.qt*

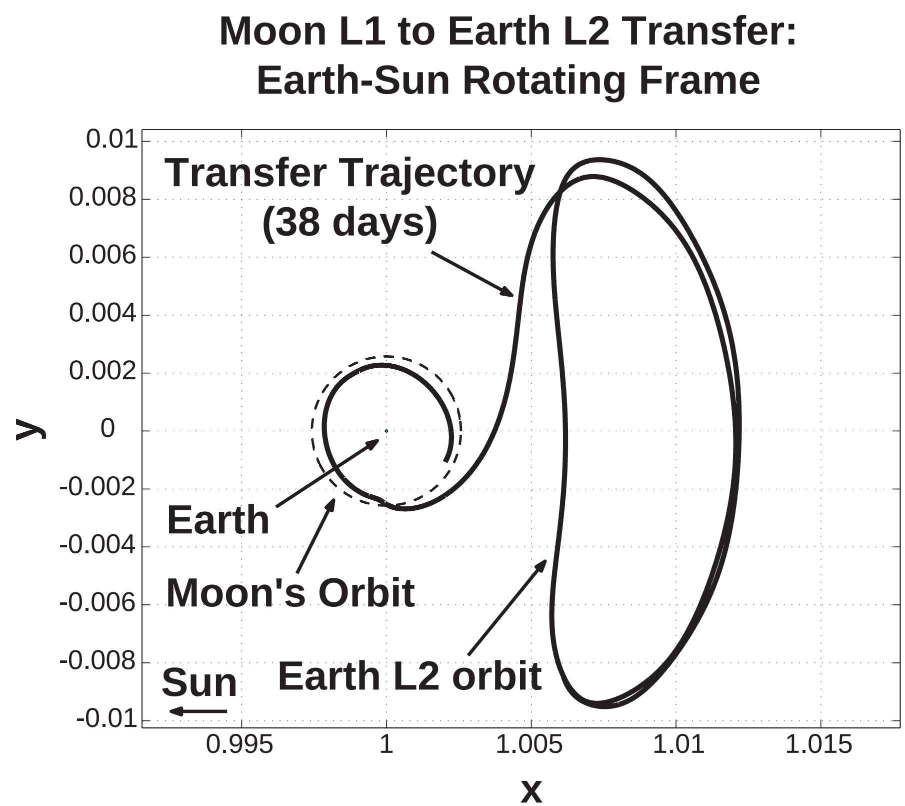
# L1 Gateway Station

- The Earth-Moon $L_1$ point is of interest as a permanent manned site.
- could operate as a transportation node for going to the moon, asteroids and planets
- could provide servicing for telescopes at Sun-Earth $L_2$ point
- Efficient transfers can be created using the 3-body and invariant manifold techniques discussed

# L1 Gateway Station

- Below is a near-optimal transfer between the $L_1$ Gateway station and a Sun-Earth $L_2$ orbit



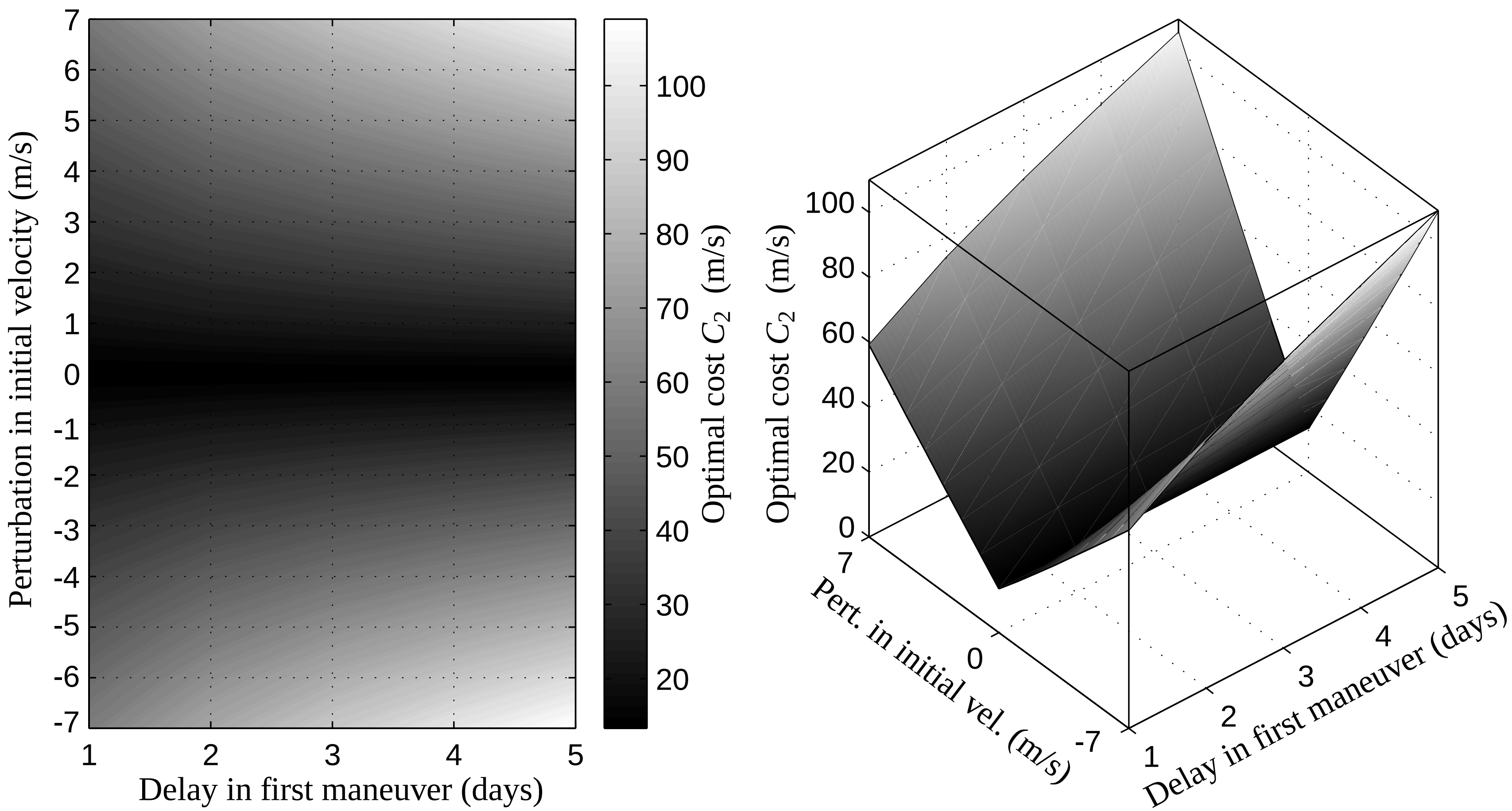
# Optimal Control

## *Halo Orbit Insertion*

- After launch, the *Genesis Discovery Mission* will get onto the stable manifold of its eventual periodic orbit around $L_1$
- Launch velocity errors necessitate corrective maneuvers
- The software COOPT has been used to determine the necessary corrections (burn sizes and timing) systematically for a variety of launch conditions
- It gets one onto the orbit at the right time, while minimizing fuel consumption

# Optimal Control

- A very nice mixture of dynamical systems (providing guidance and first guesses) and optimal control




see Serban, Koon, Lo, Marsden, Petzold, Ross, and Wilson [2001]

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    - continuous (low) thrust

# Proposed Research

- solar system dynamics
  - probabilities of transition, capture, collision
    - comets between planets / Kuiper Belt
    - *Shoemaker-Levy 9* type collisions (Chodas, *et al.*)
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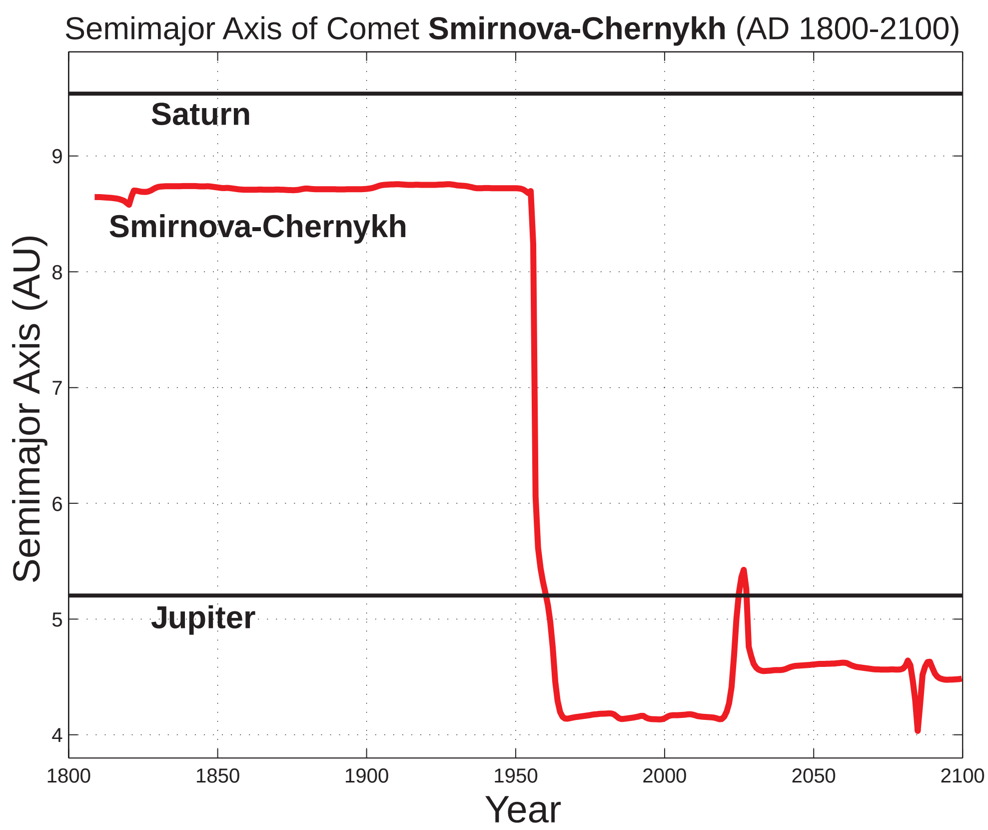
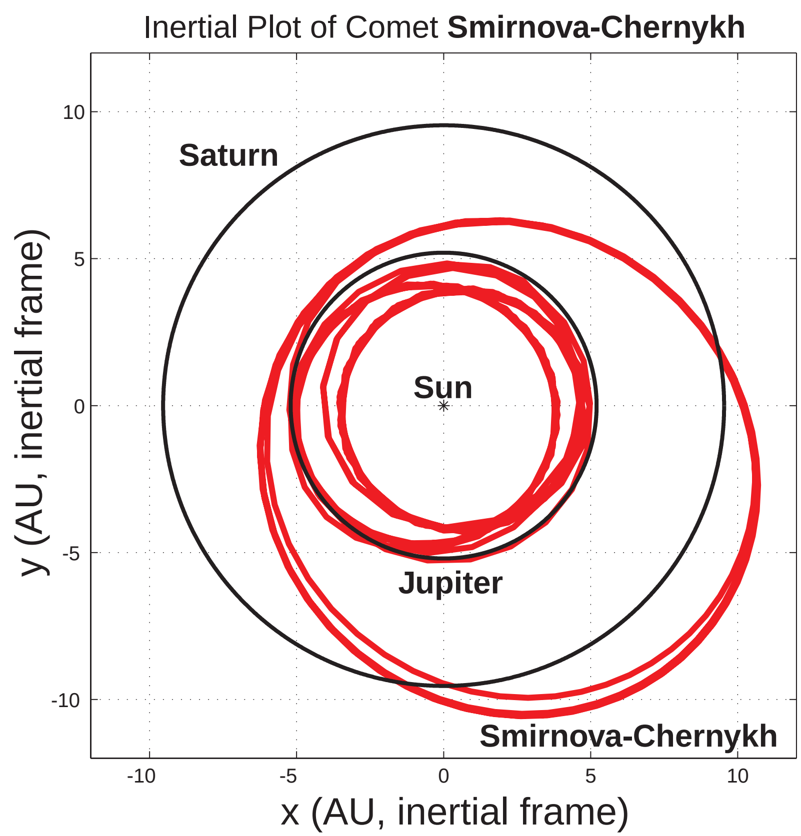
  - large scale structure
    - dust clouds around stellar systems (for *TPF*)

# Transport Between Planets

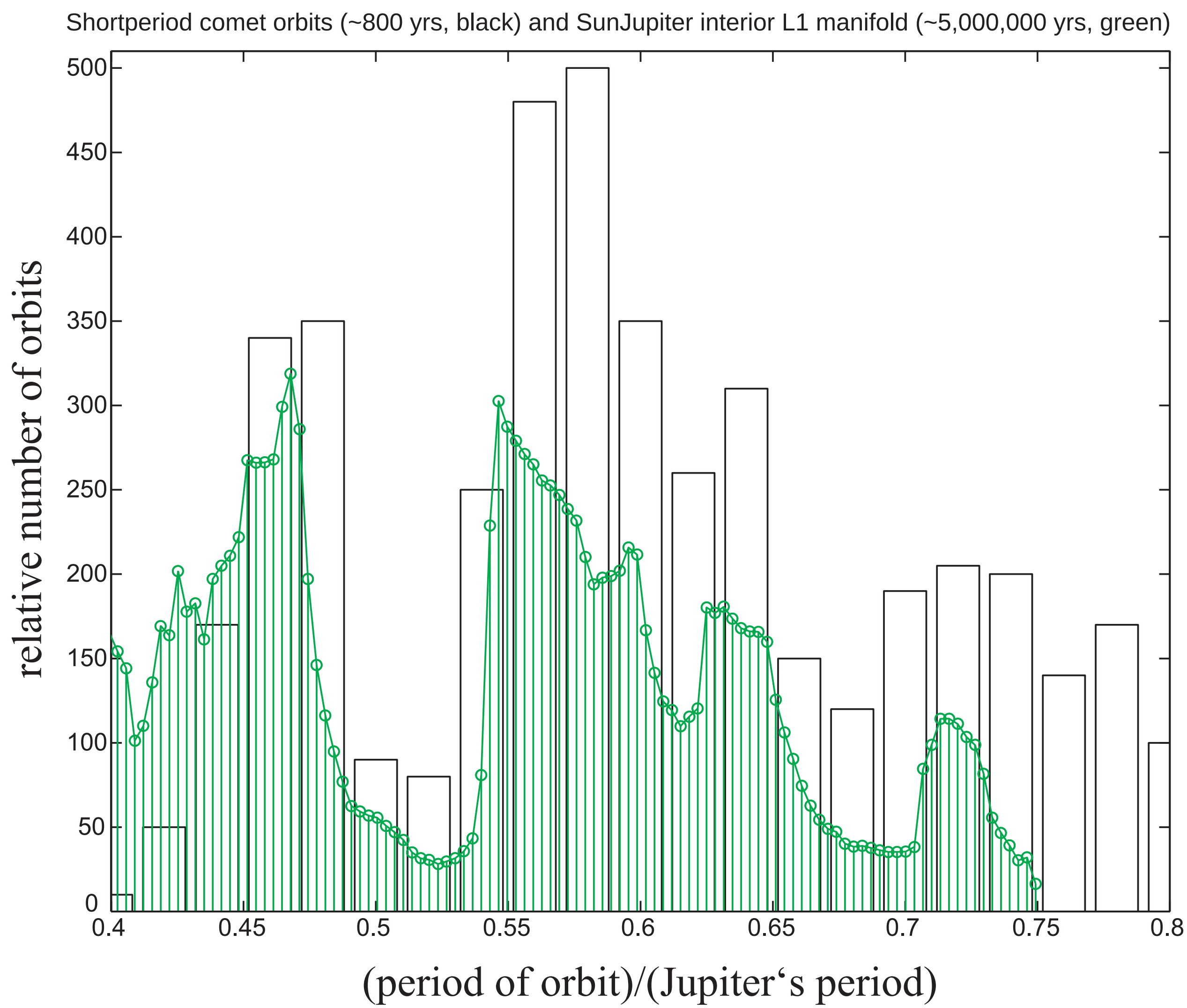
- Comets transfer between the giant planets eg, jumping between "tubes" of Saturn and Jupiter



# Minor Body Statistics

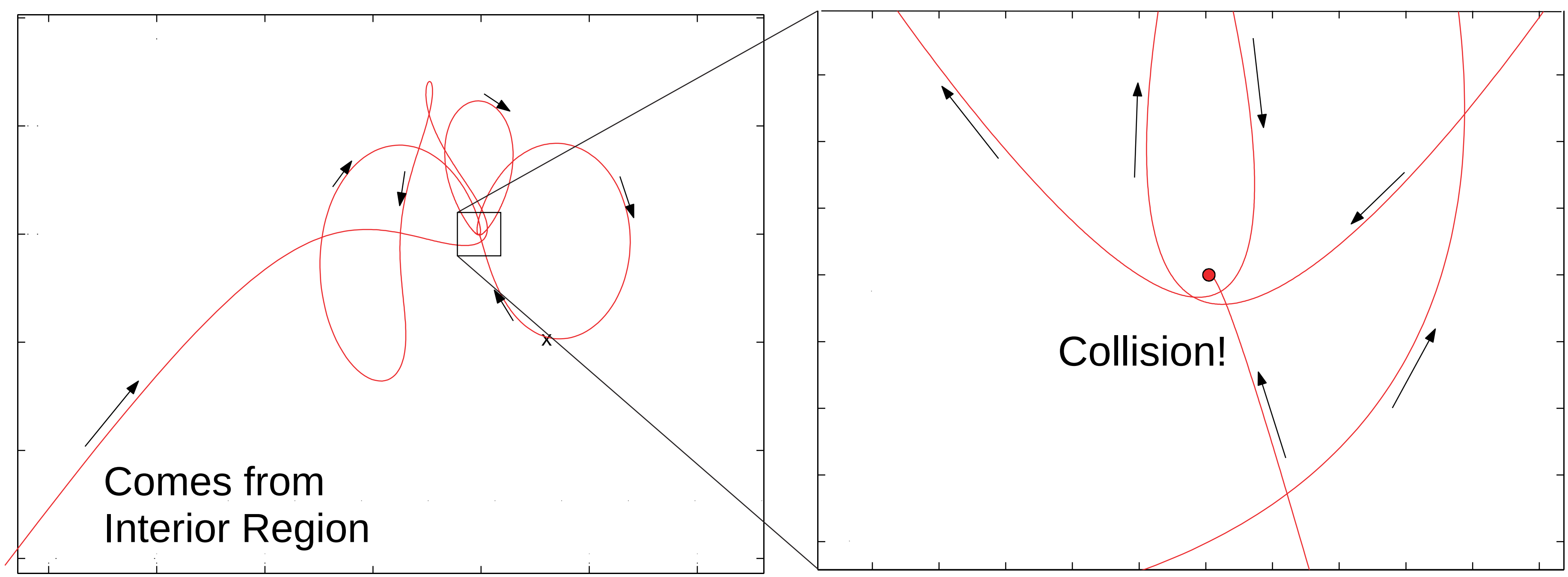
- Computation of long term statistics is possible
  - Compare manifold computation (green) with comet data



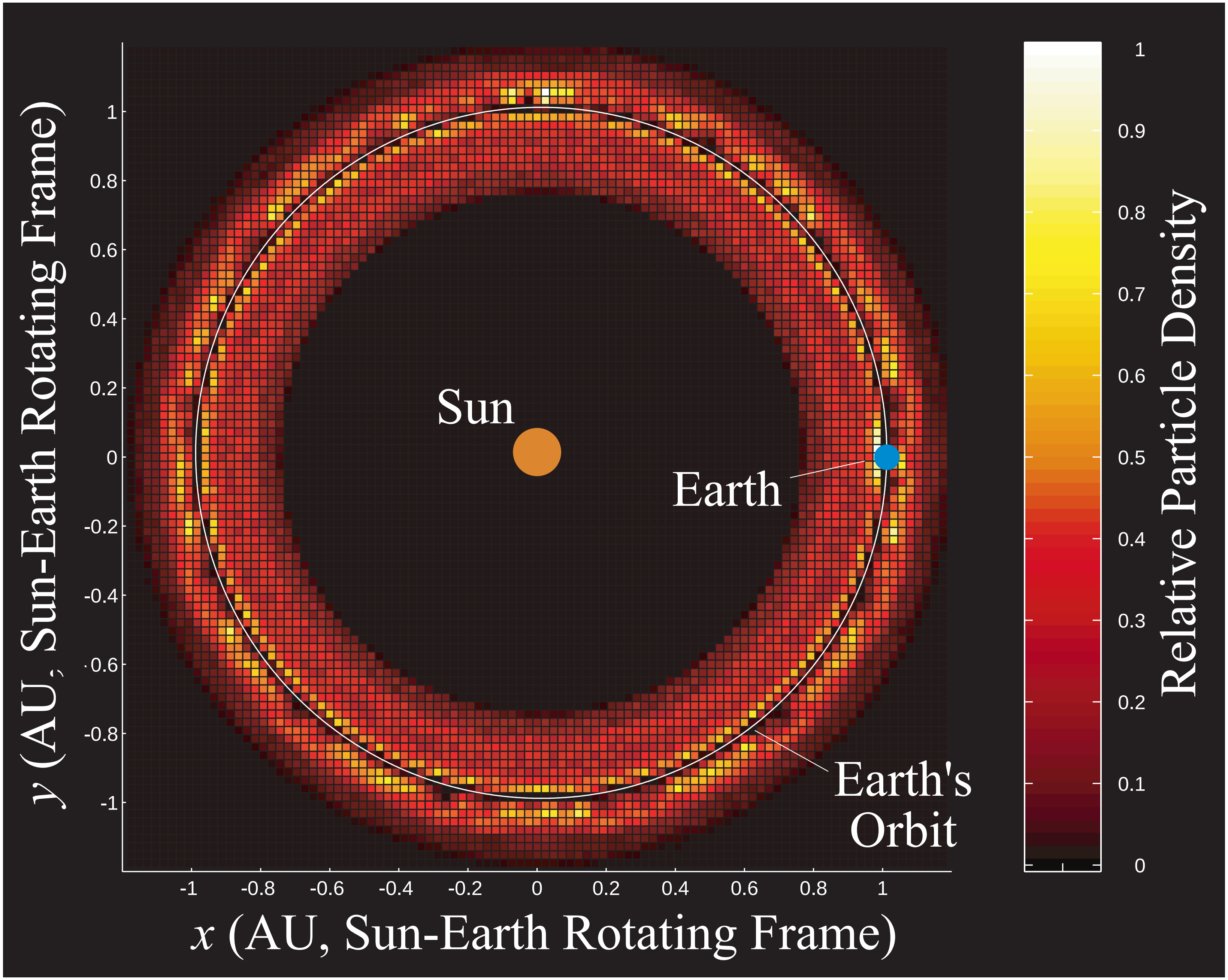
# Impact Trajectories

- Deeper understanding of low velocity impacts
  - eg, *Shoemaker-Levy 9* and Earth crossers

Example Collision Trajectory



# Circumstellar Dust Clouds



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- use graph theoretic methods (Dellnitz, *et al.*)
- use symplectic integrators (Wisdom, Marsden, *et al.*)
  - combine the above methods

# References


- Gómez, G., W.S. Koon, M.W. Lo, J.E. Marsden, J. Masdemont and S.D. Ross [2001] ***Invariant manifolds, the spatial three-body problem and space mission design.*** *AAS/AIAA Astrodynamics Specialist Conference.*
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***The End***