Coherent structures and biological invasions

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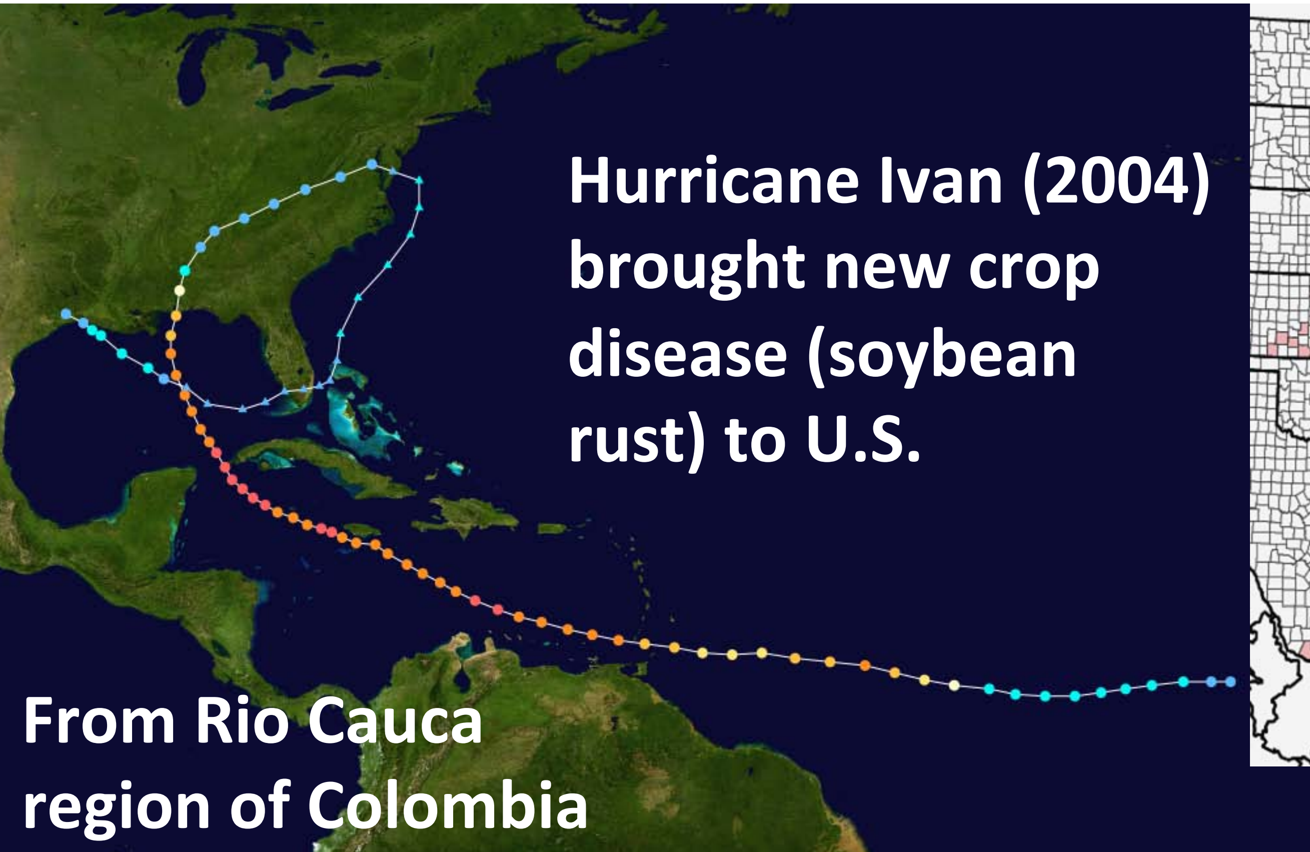
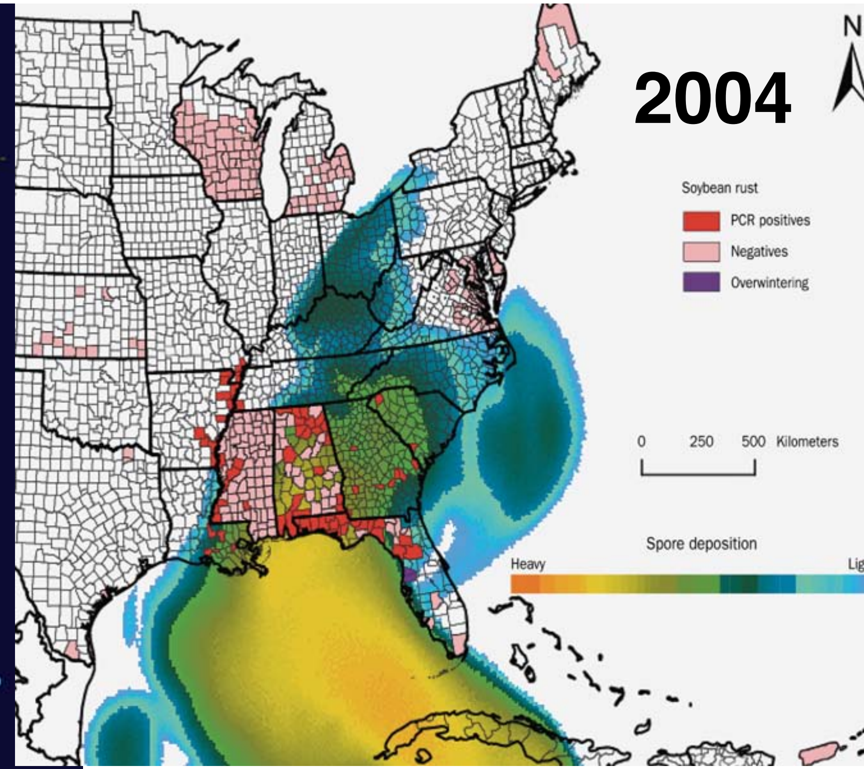
Lorentz Center Workshop, May 2011

Invasive species riding the atmosphere



Hurricane Ivan (2004) brought new crop disease (soybean rust) to U.S.

From Rio Cauca region of Colombia



Disease extent

Invasive species riding the atmosphere

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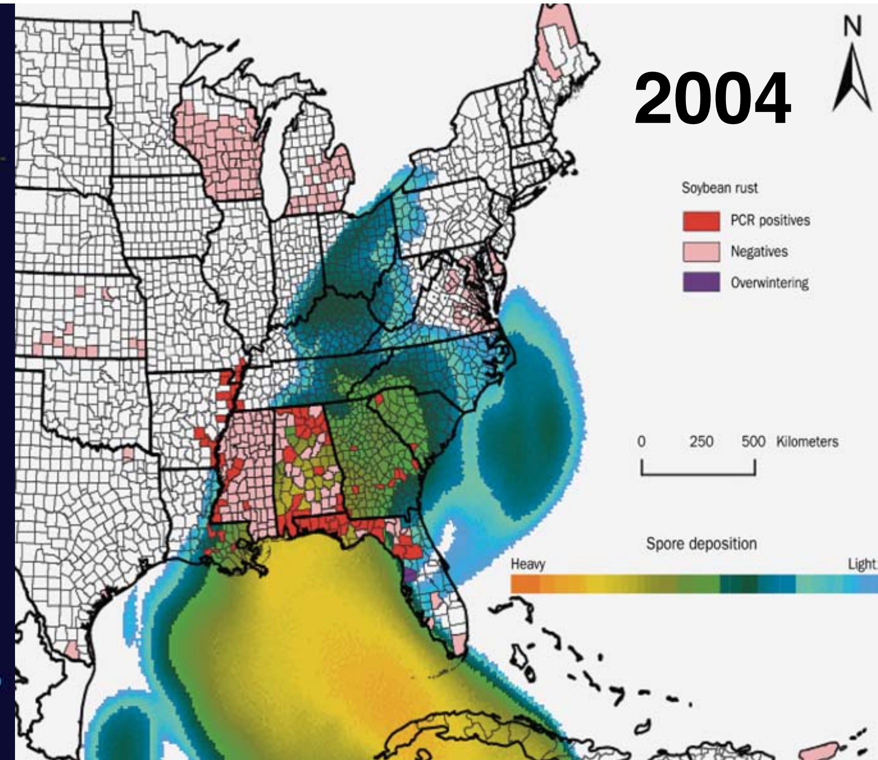


Disease extent

Cost of invasive organisms is ***$137 billion*** **per year in U.S.**

Atmospheric transport network relevant for aeroecology

Skeleton of large-scale horizontal transport

relevant for large-scale

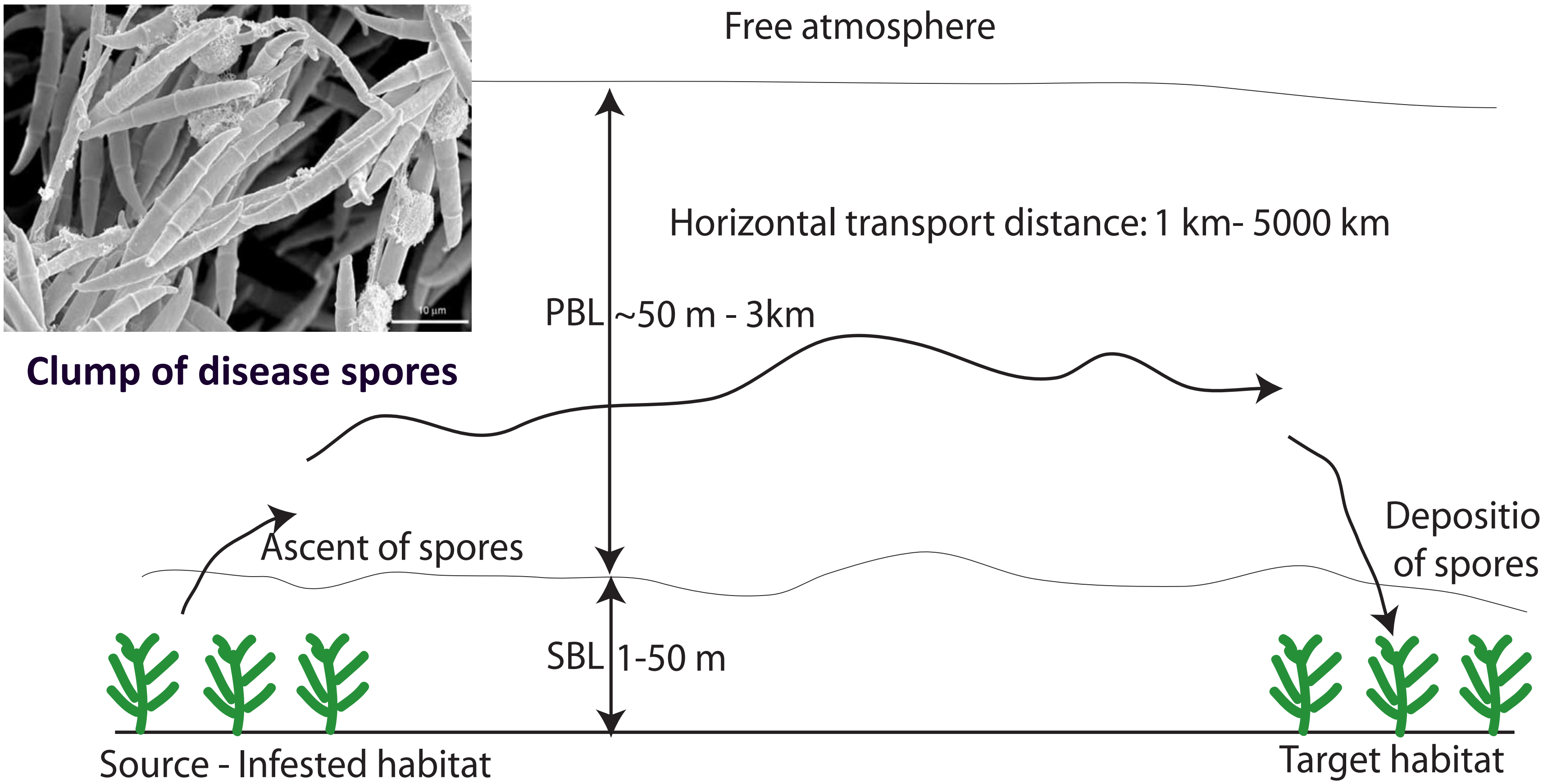
spatiotemporal patterns

of important biota

e.g., plant pathogens

orange = repelling LCSs, blue = attracting LCSs

Atmospheric transport of microorganisms

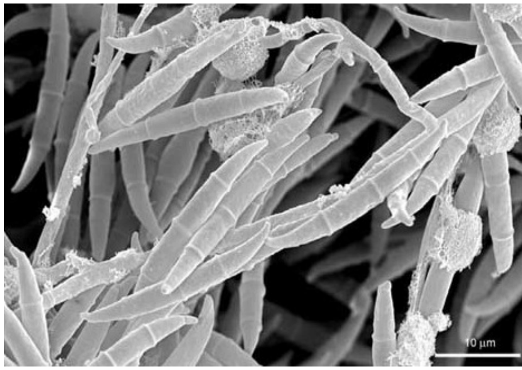


e.g., *Fusarium*

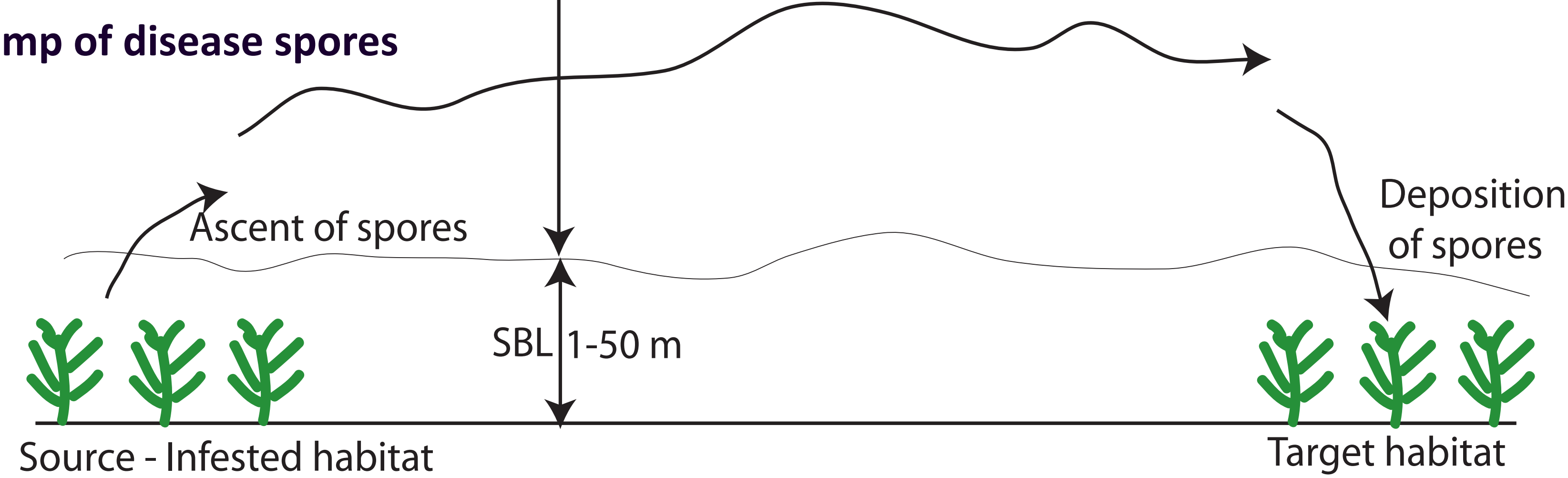
- **Spore production, release, escape from surface**
- **Long-range transport (time-scale hours to days)**
- **Deposition, infection efficiency, host susceptibility**

Isard & Gage [2001]

Atmospheric transport of microorganisms



Clump of disease spores



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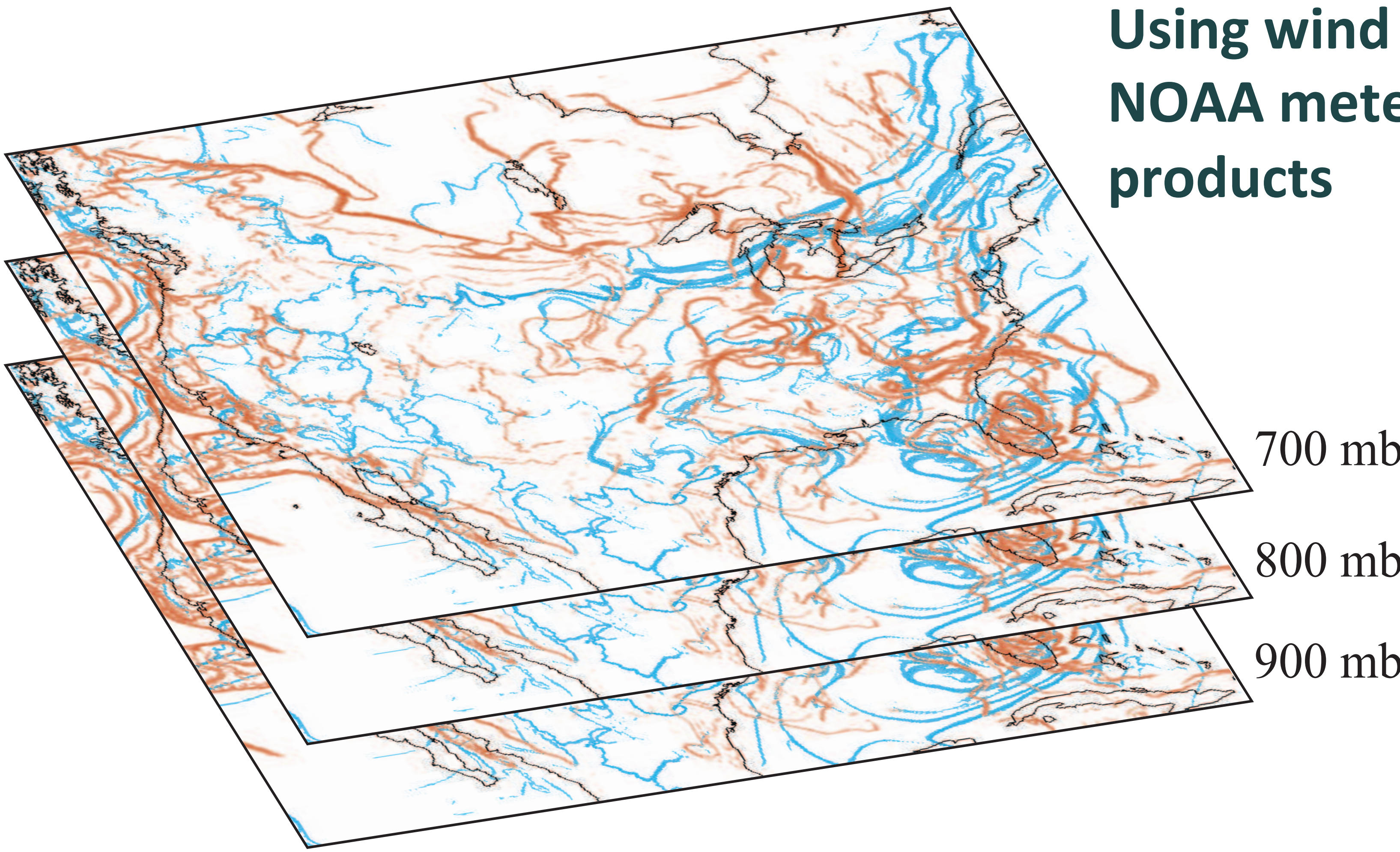
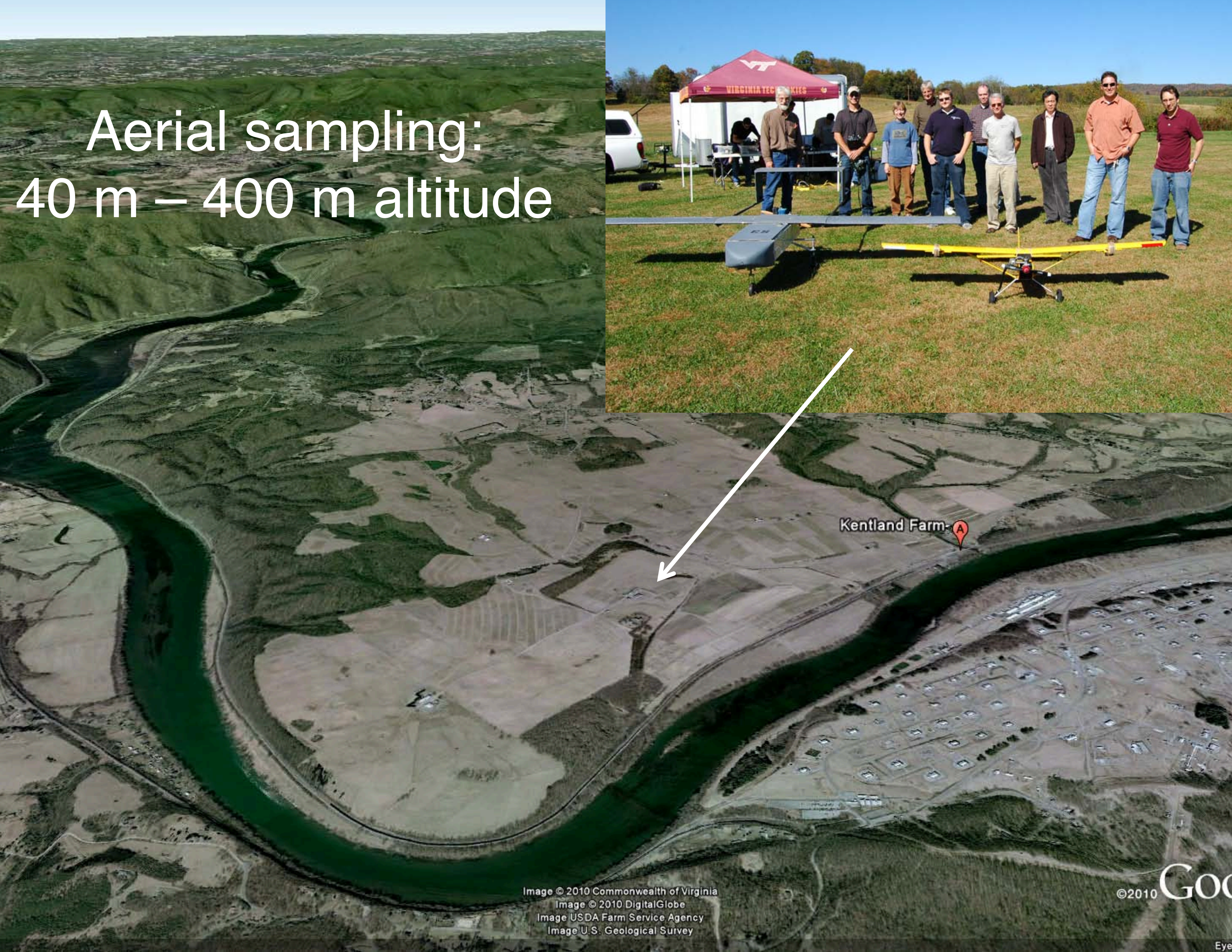
Isard & Gage [2001]

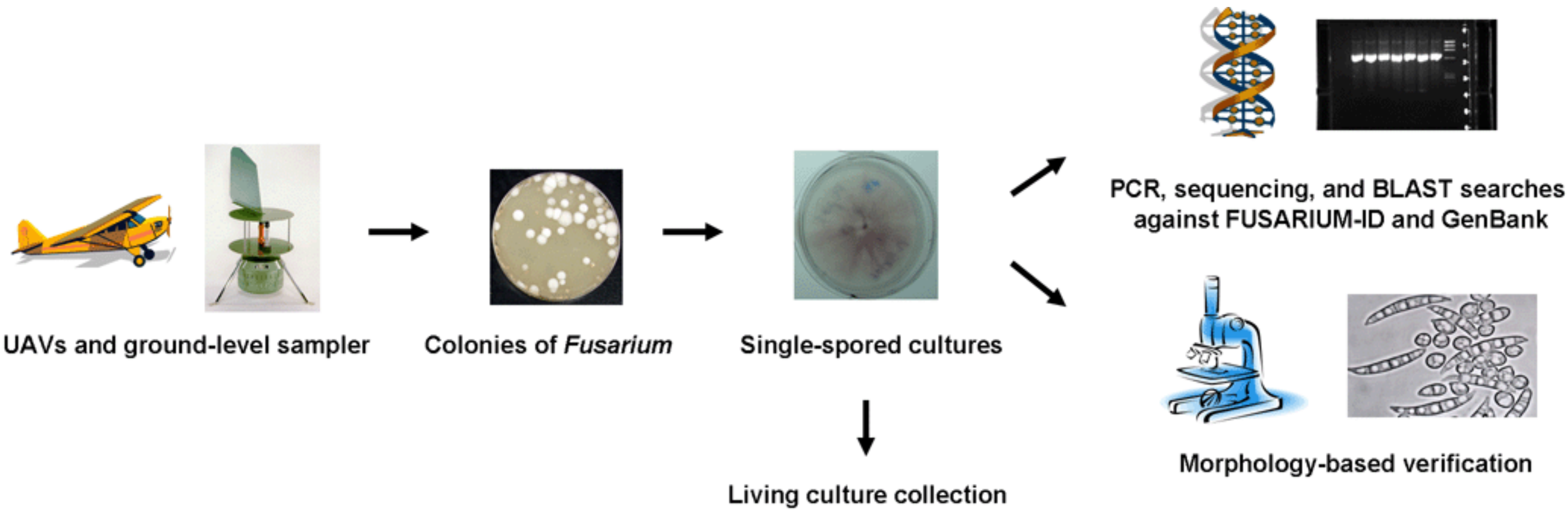
Mesoscale to synoptic scale motion

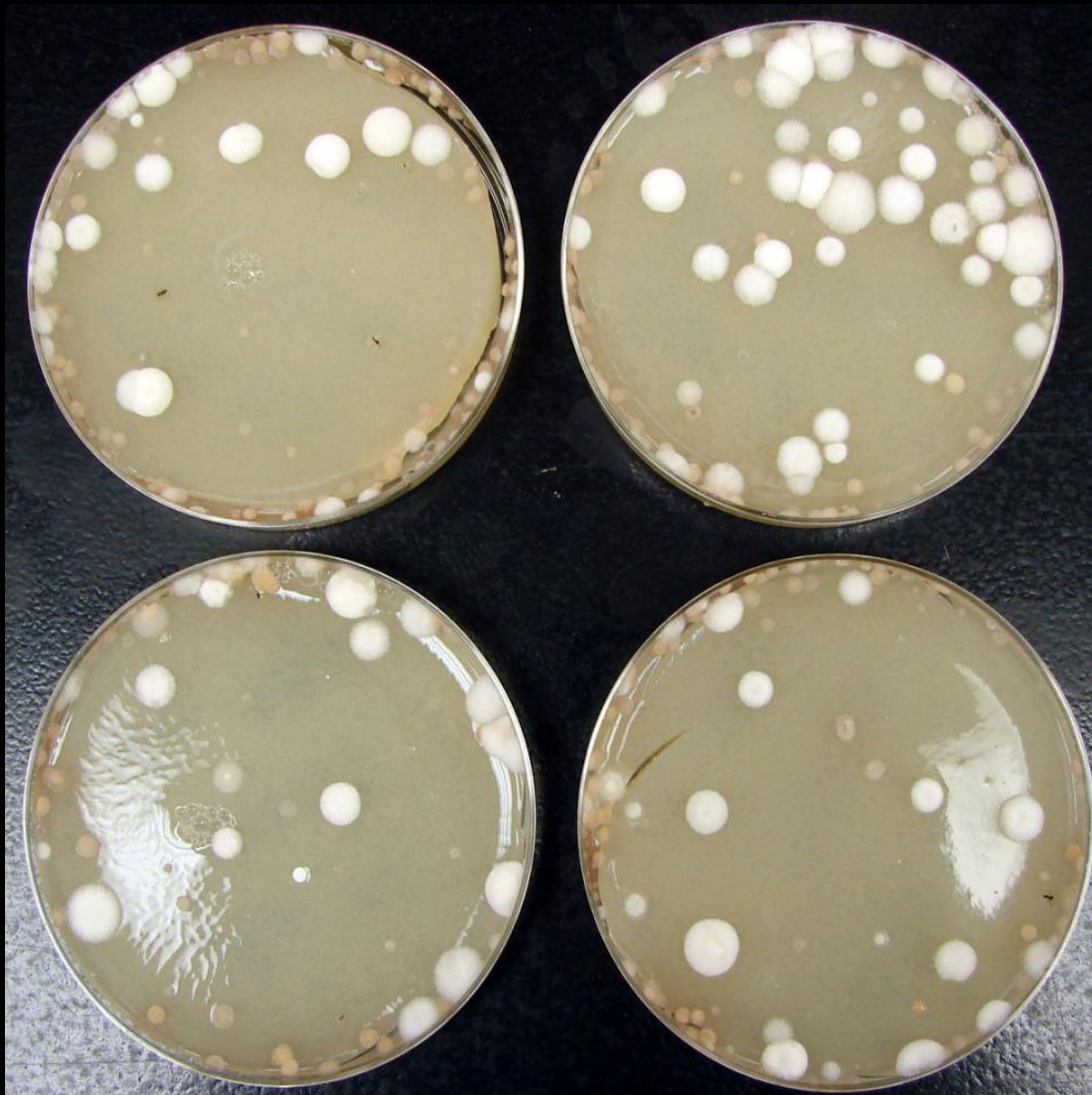
- Consider first 2D motion, then fully 3D
- Quasi-2D motion (isobaric) over timescales of interest, < 12-24 hrs, limited by fungal spore viability

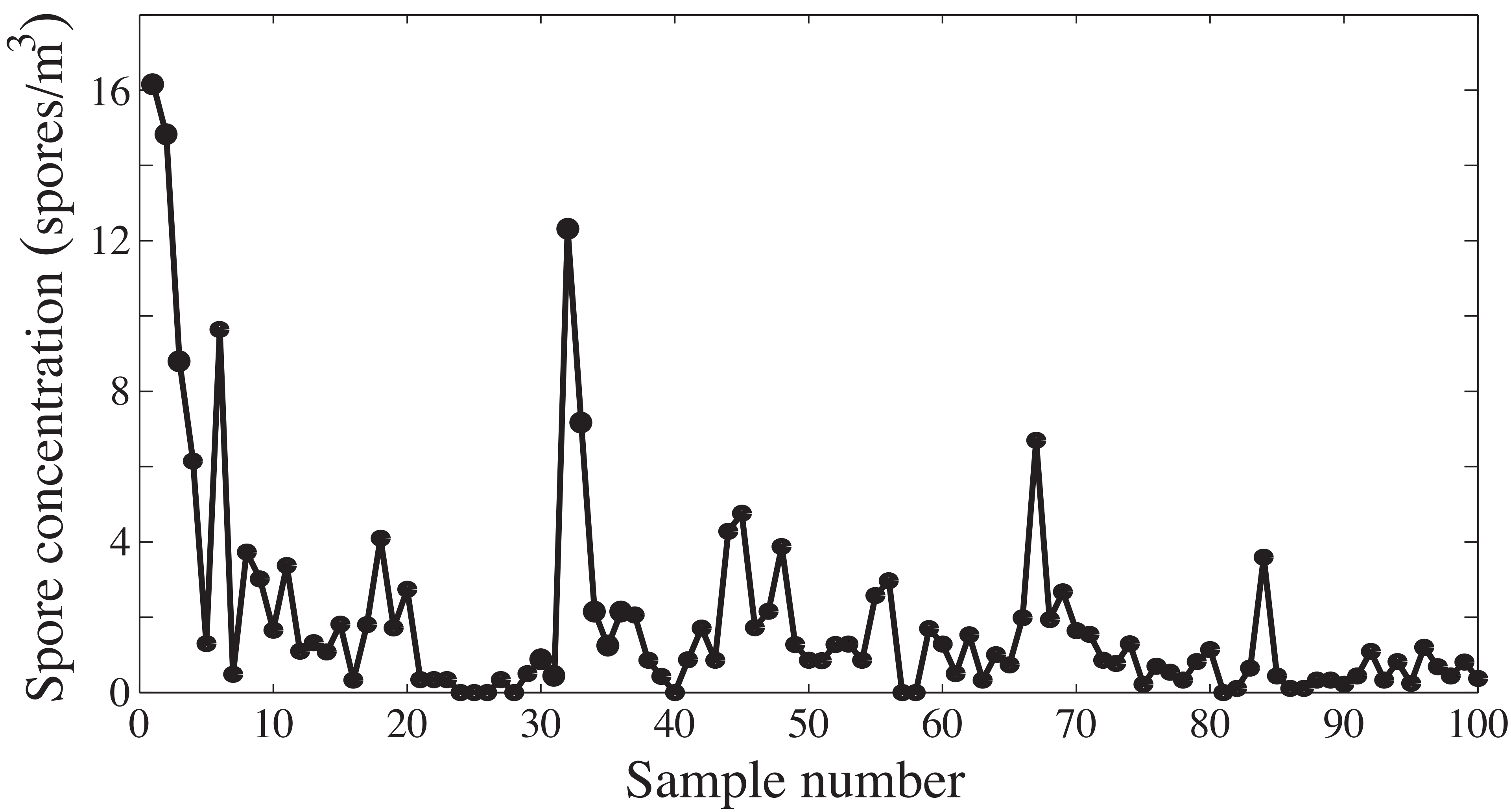
Using wind fields from NOAA meteorological products



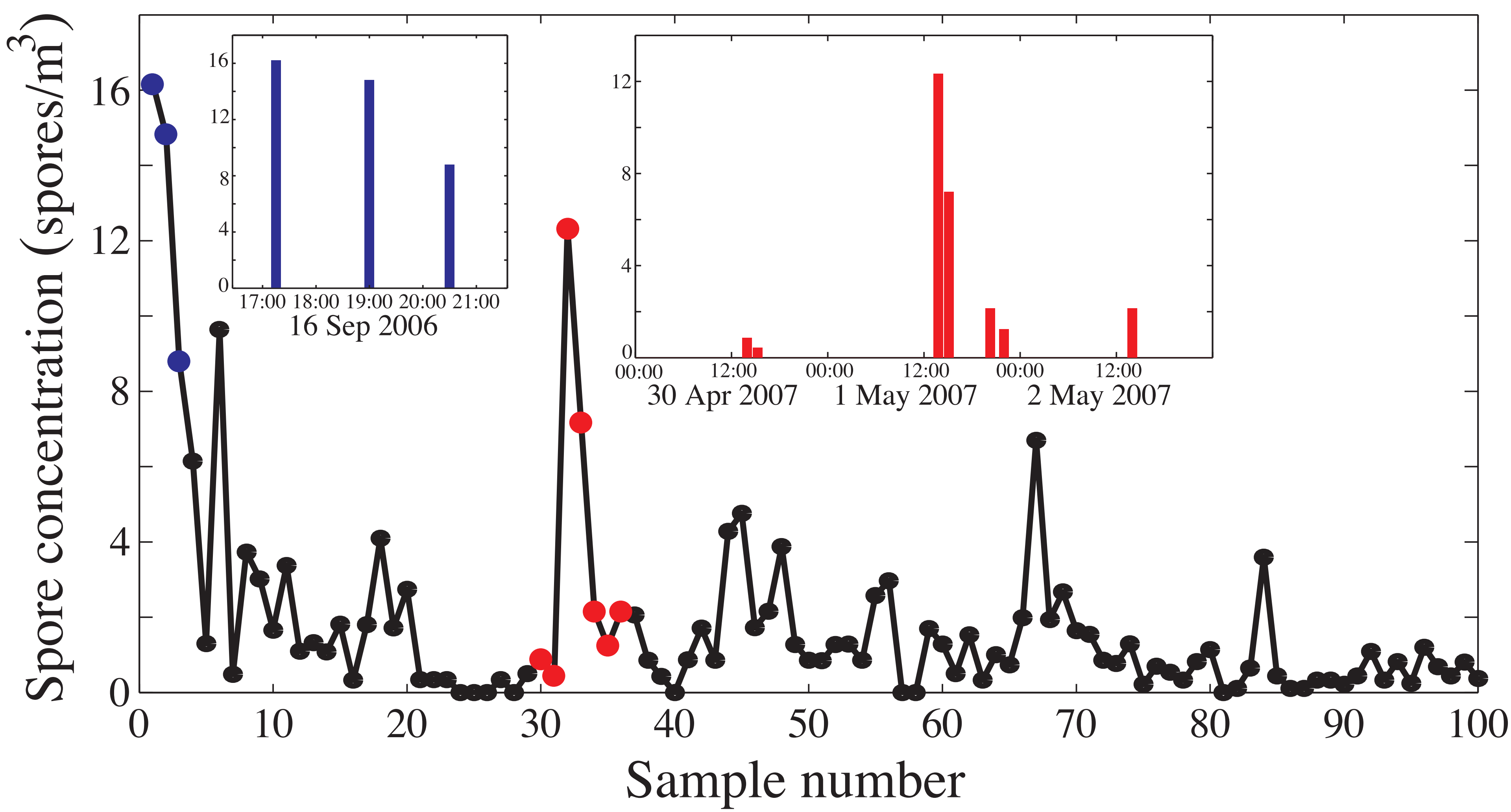

Aerial sampling:
40 m – 400 m altitude
Kentland Farm
A
Image © 2010 Commonwealth of Virginia
Image © 2010 DigitalGlobe
Image USDA Farm Service Agency
Image U.S. Geological Survey


UAVs and ground-level sampler
Colonies of *Fusarium*
Single-spored cultures
PCR, sequencing, and BLAST searches against FUSARIUM-ID and GenBank
Morphology-based verification
Living culture collection





Concentration of *Fusarium* spores (number/m^3) for samples from 100 flights conducted between August 2006 and March 2010.



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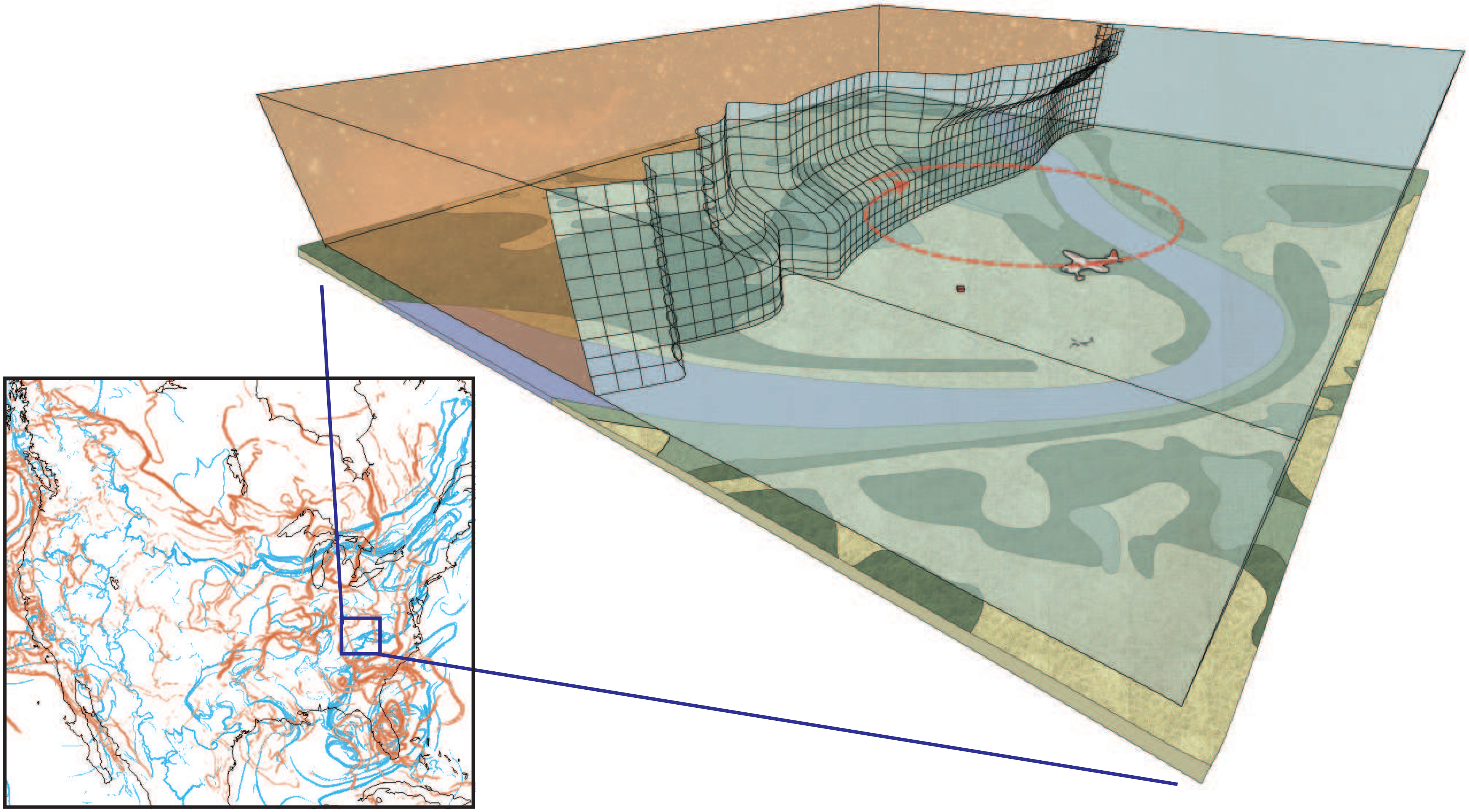
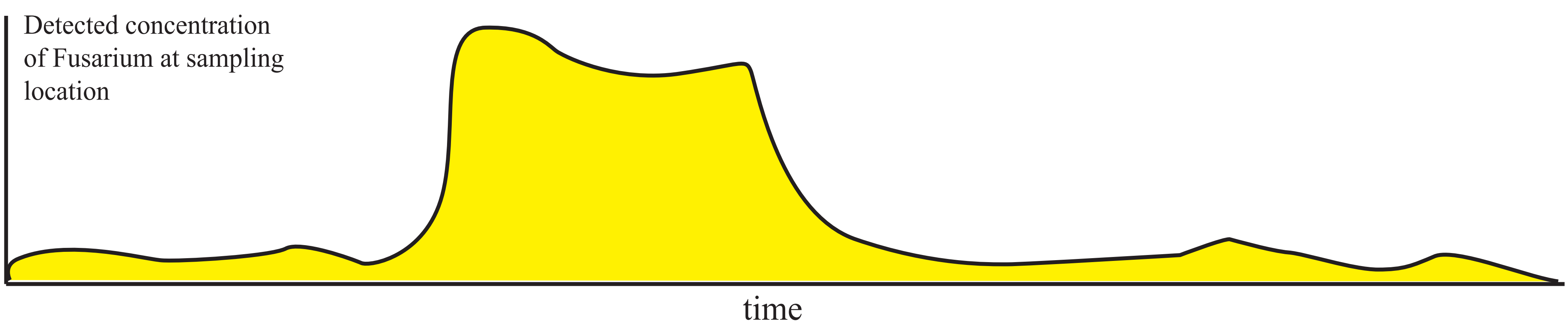


Punctuated changes: correlated to LCS passage?



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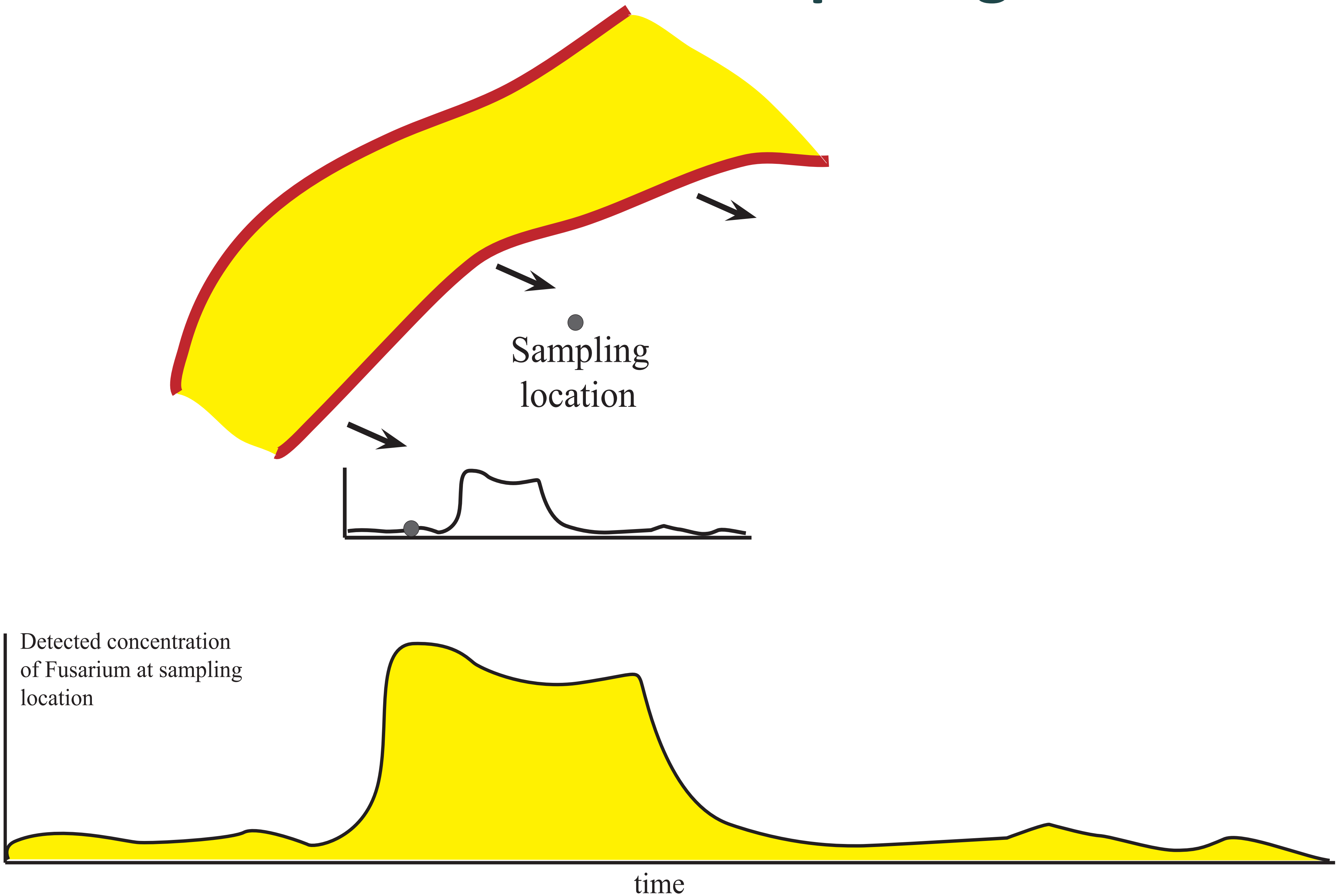

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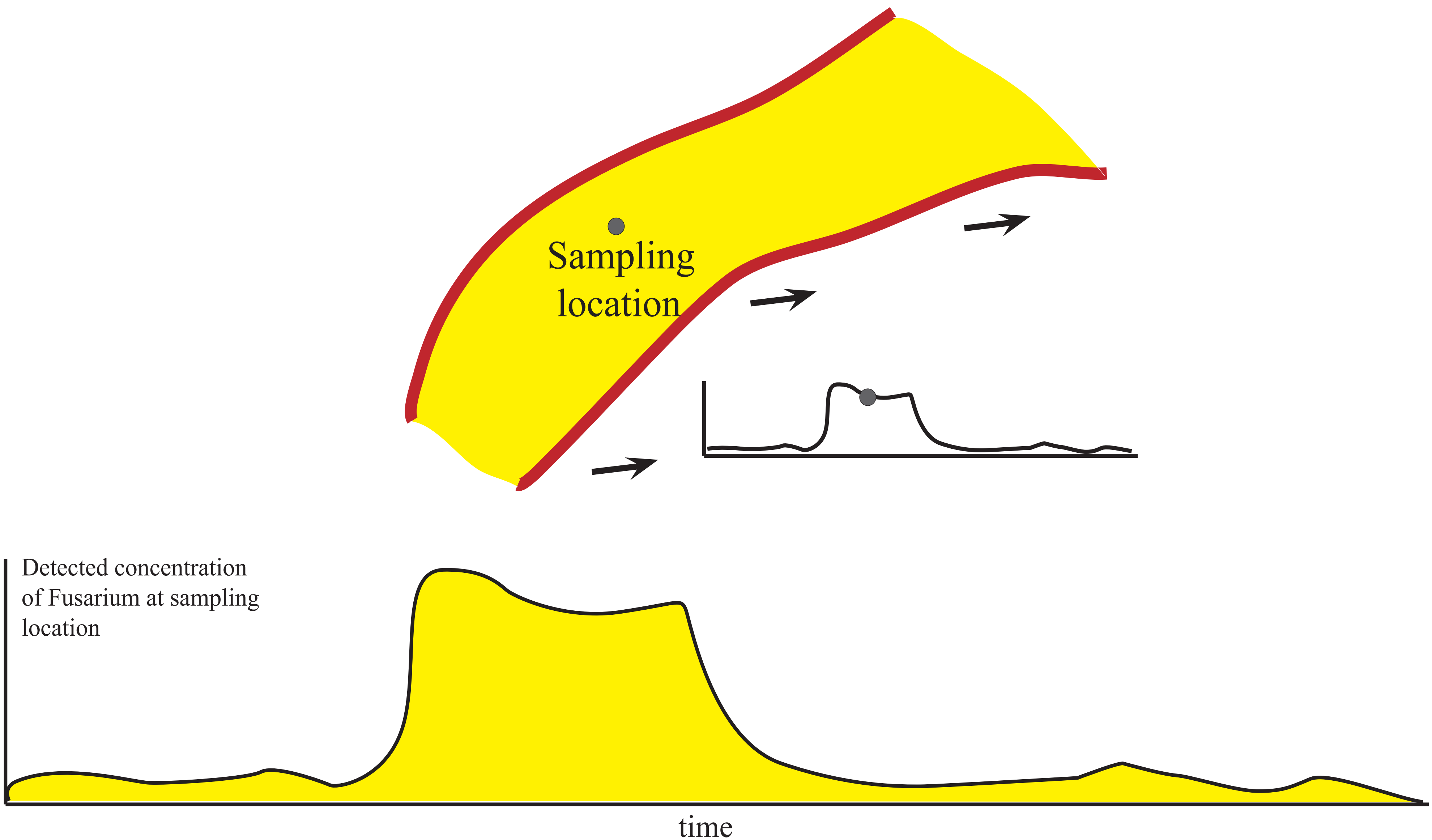

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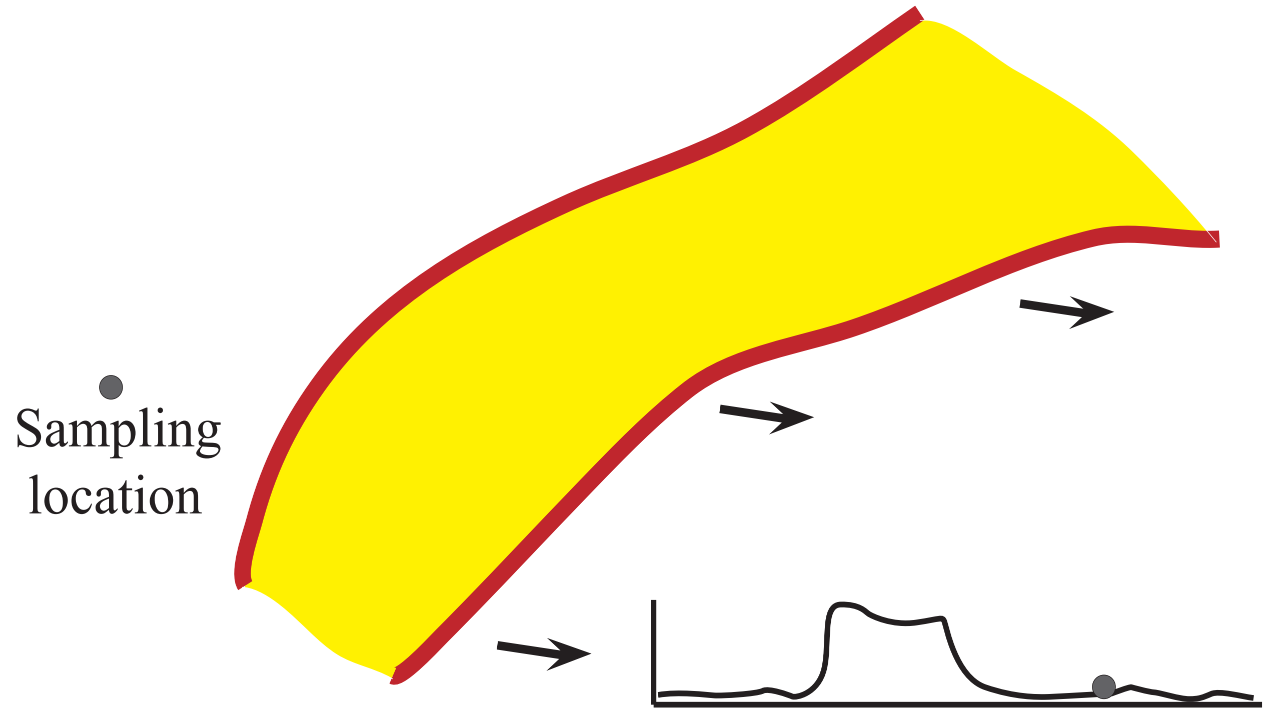
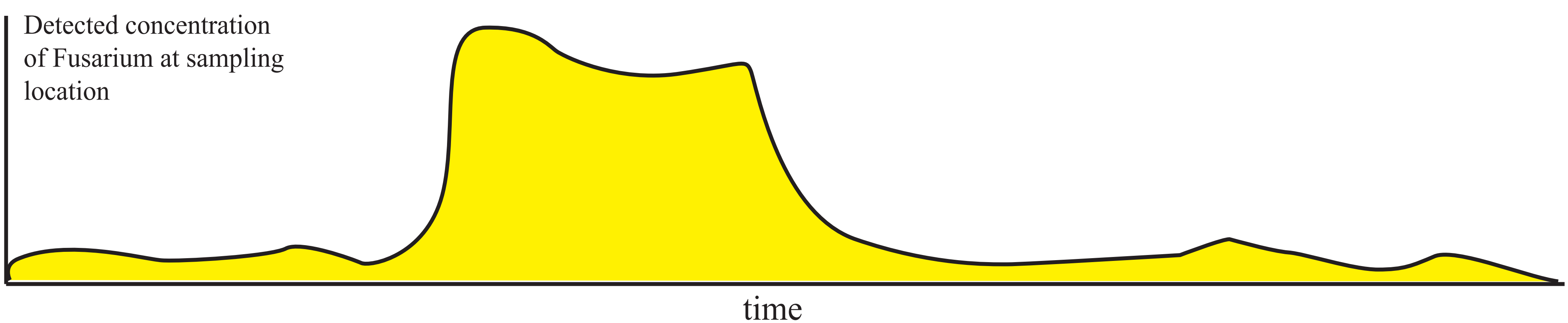
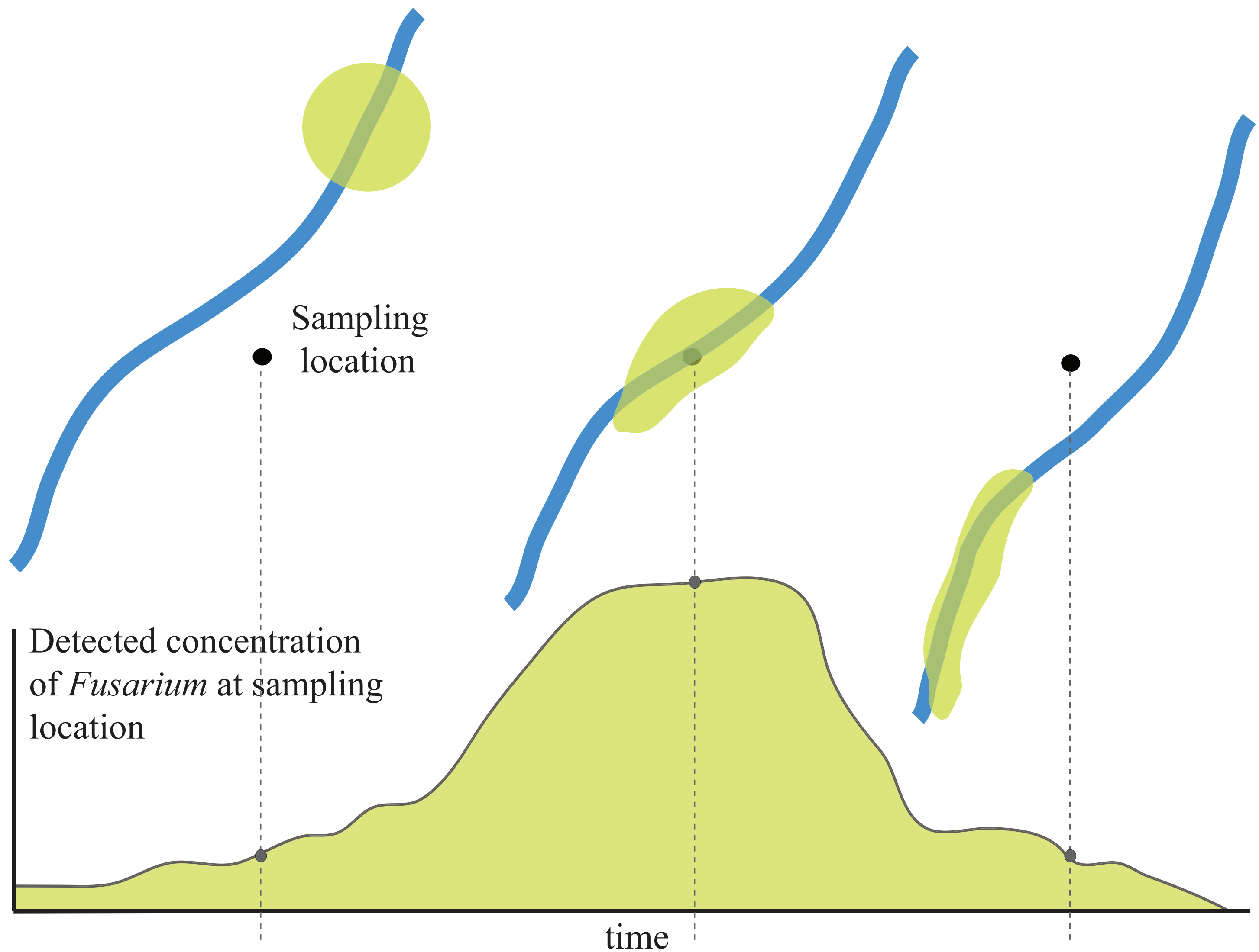
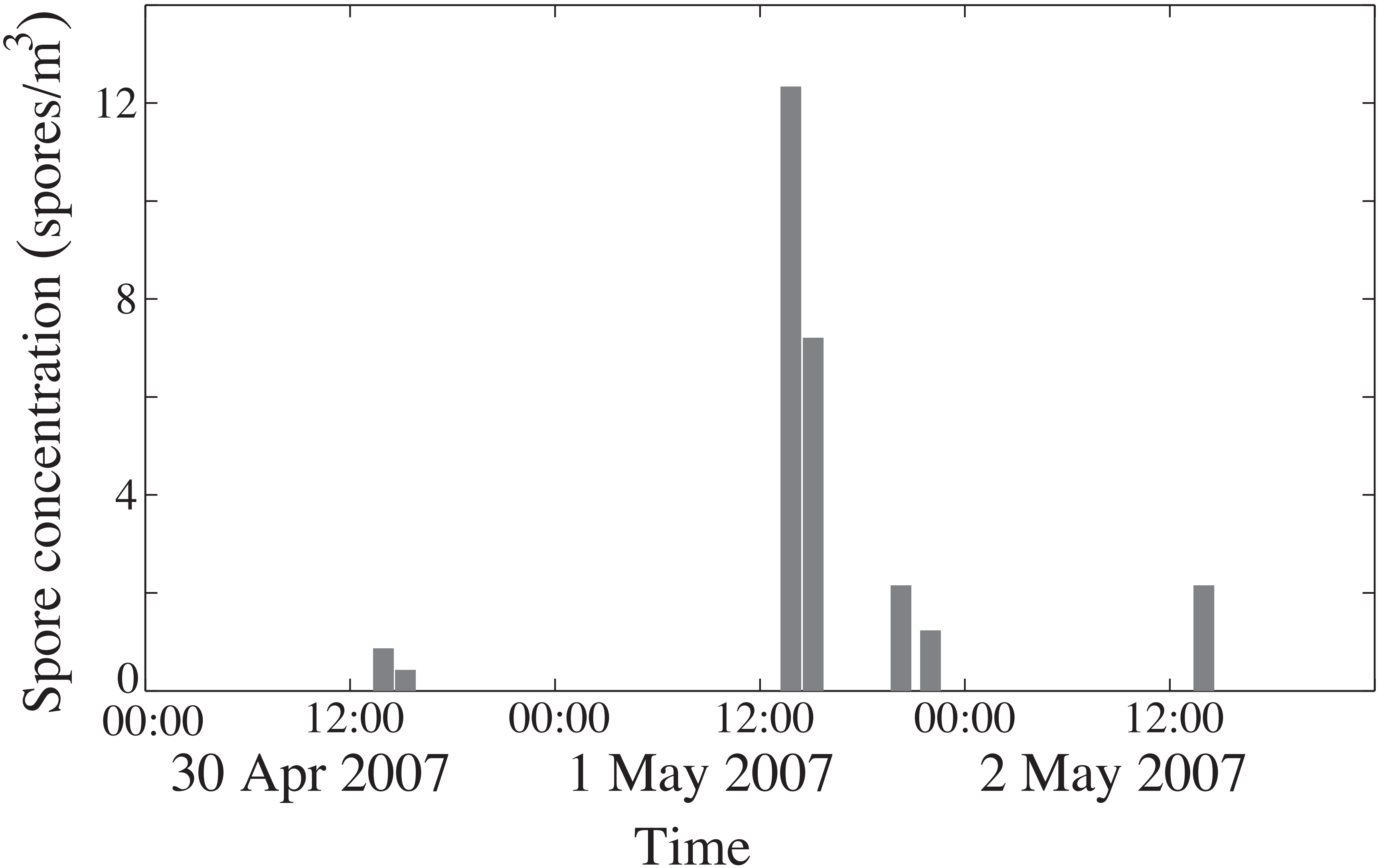


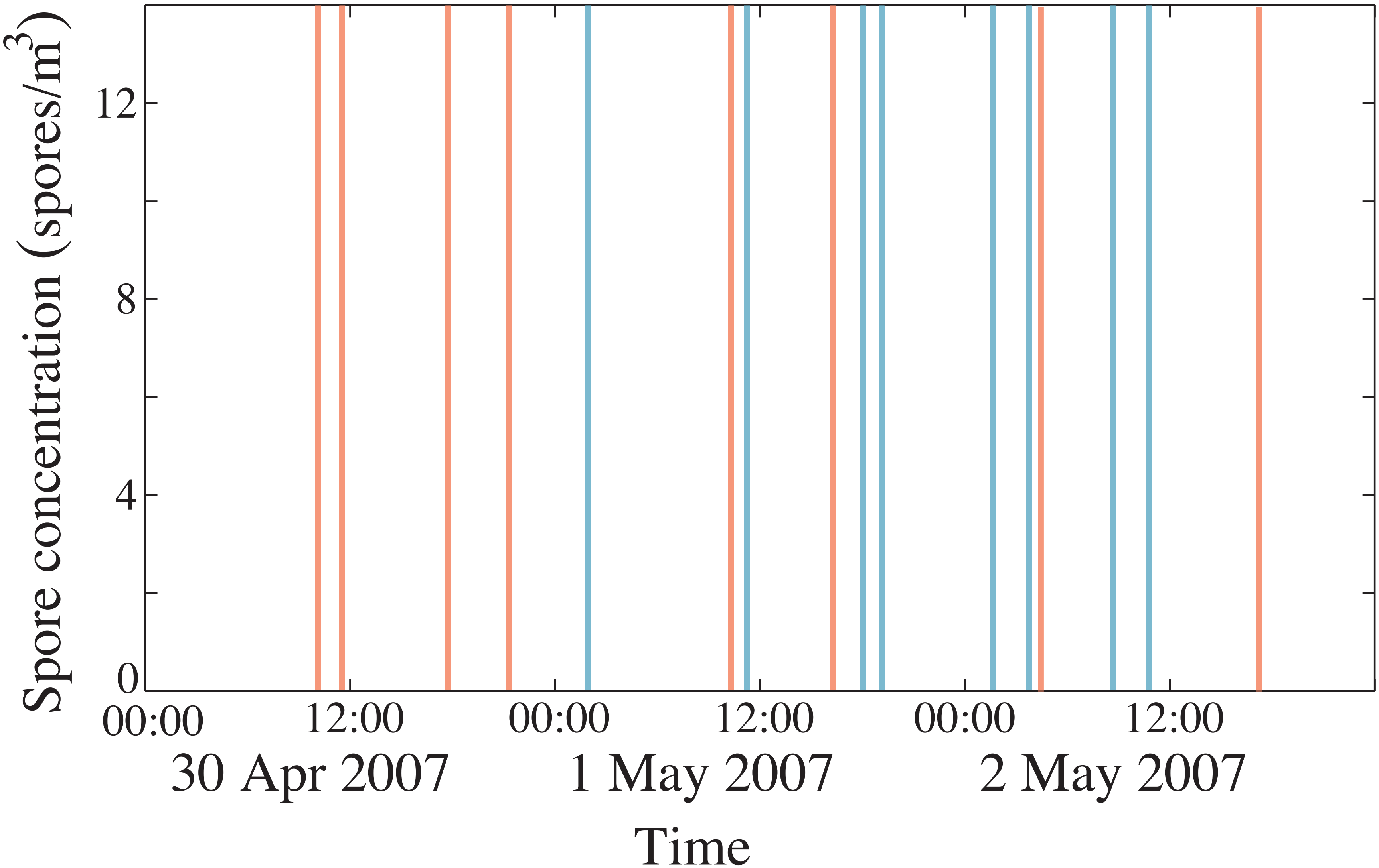




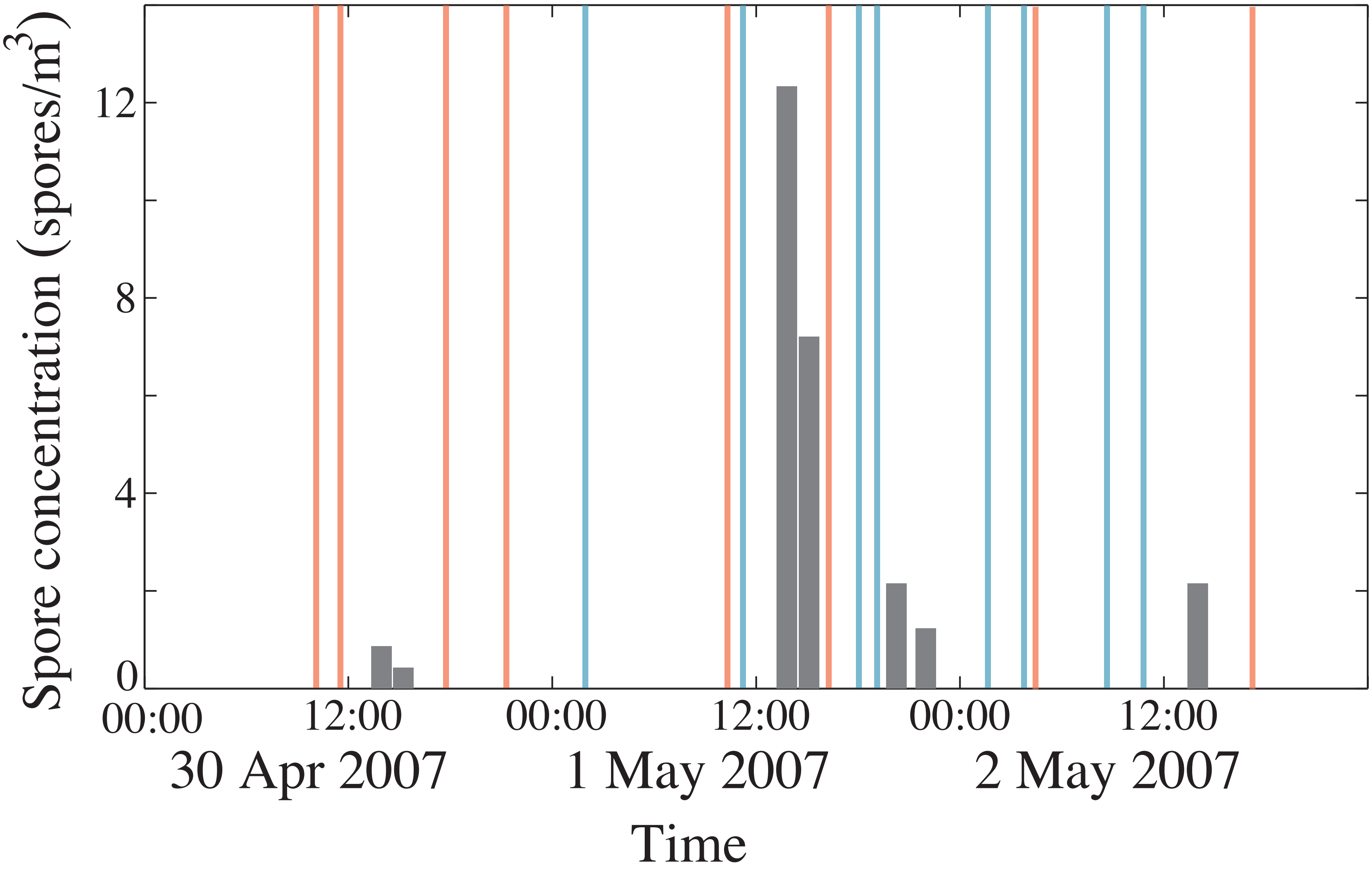
Movement of a 'cloud' of relatively high concentration of *Fusarium* along an attracting LCSs (upper panel, three time snapshots going from left to right) and the corresponding abrupt changes in detected concentrations of *Fusarium* at a geographically fixed sampling location (below).



Time series of concentration $\{(t_0, C_0), \ldots, (t_{N-1}, C_{N-1})\}$



LCS passage times: orange = repelling LCSs, blue = attracting

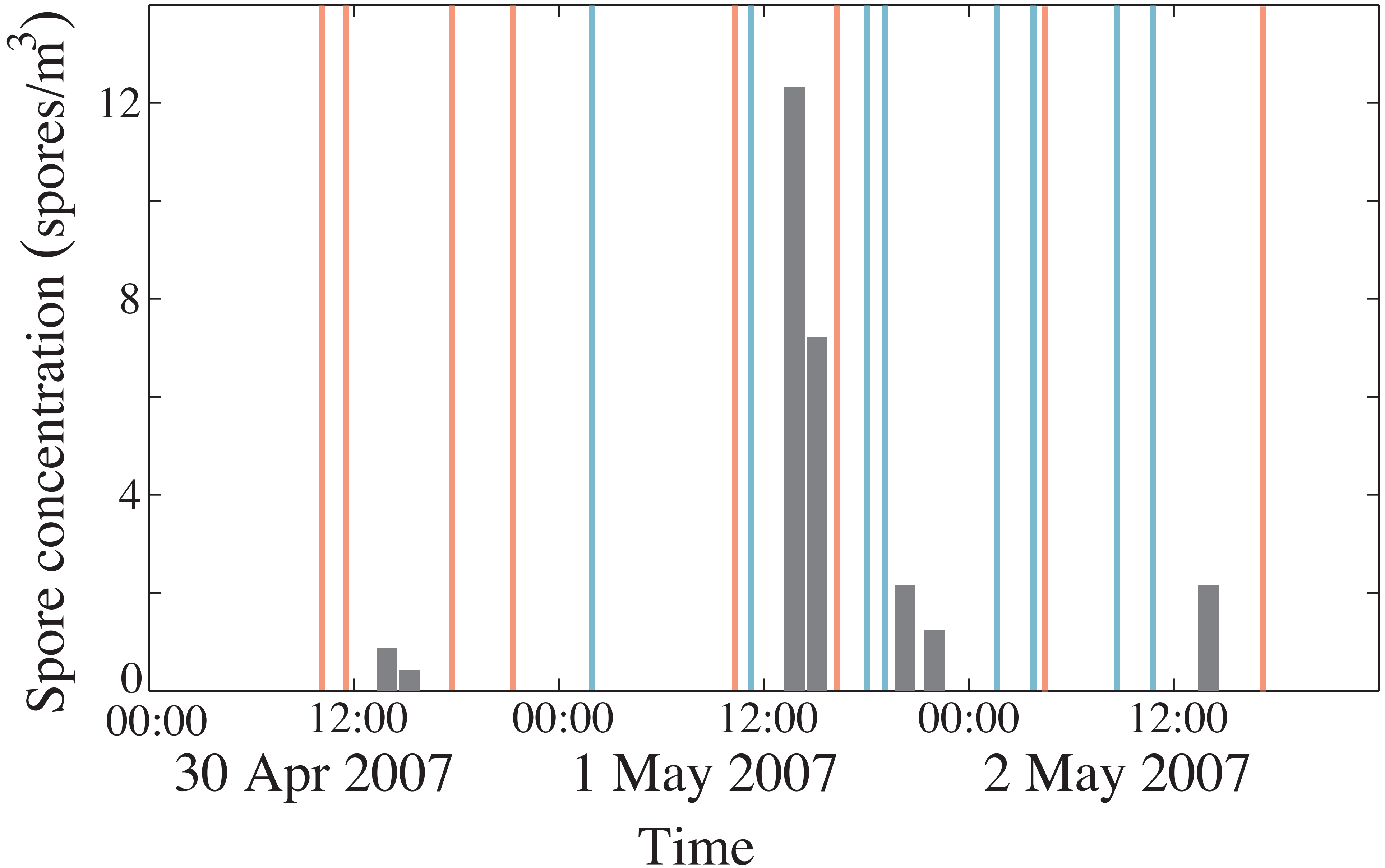


Compare: are there patterns?

Statistical framework for hypotheses testing

- Time series of concentration $\{(t_0, C_0), \ldots, (t_{N-1}, C_{N-1})\}$
- Consider concentration changes $\Delta C_k \equiv C_k - C_{k-1}$
  if $\Delta t_k \equiv t_k - t_{k-1} < T$, where, e.g., $T = 24$ hr or $T = 12$ hr
- Categorize each ΔC_k as (1) a punctuated change or (0) not
- Independently determine if LCS passed over sampling point during $[t_{k-1}, t_k]$, (1) yes or (0) no

Statistical framework for hypotheses testing



Statistical framework for hypotheses testing

- Can test hypotheses using contingency table for categorical variables. Looking for high $sensitivity$ and high $statistical\ significance$.

- Sensitivity of 1 means that the LCS diagnostic test identifies all punctuated changes in the concentration of atmospheric *Fusarium*.

- Statistical significance? $p < 0.05$ suggest rejection of null hypothesis.

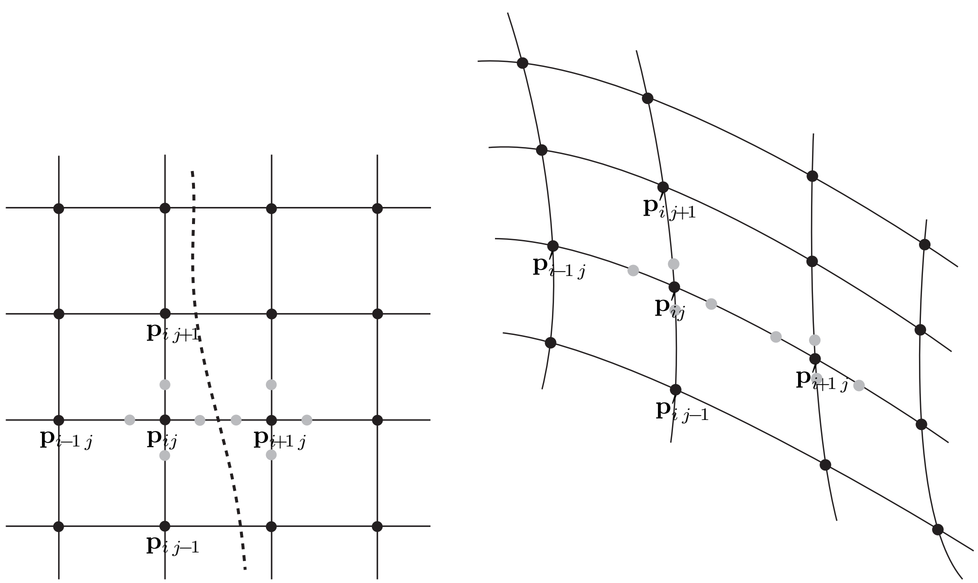
Detecting LCS numerically: FTLE field approach

- The finite-time Lyapunov exponent (FTLE),

$$\sigma_t^T(x) = \frac{1}{|T|} \ln \left\| \frac{d\phi_t^{t+T}(x)}{dx} \right\|$$

measures the maximum stretching rate over the interval T of trajectories starting near the point x at time t

cf. Bowman, 1999; Haller & Yuan, 2000; Haller, 2001; Shadden, Lekien, Marsden, 2005

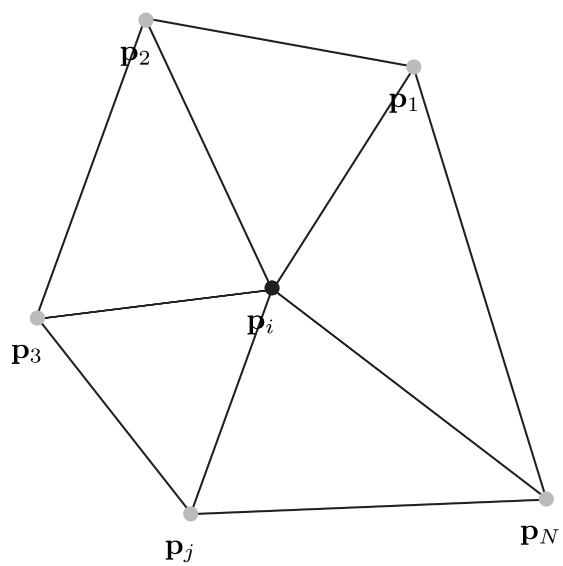


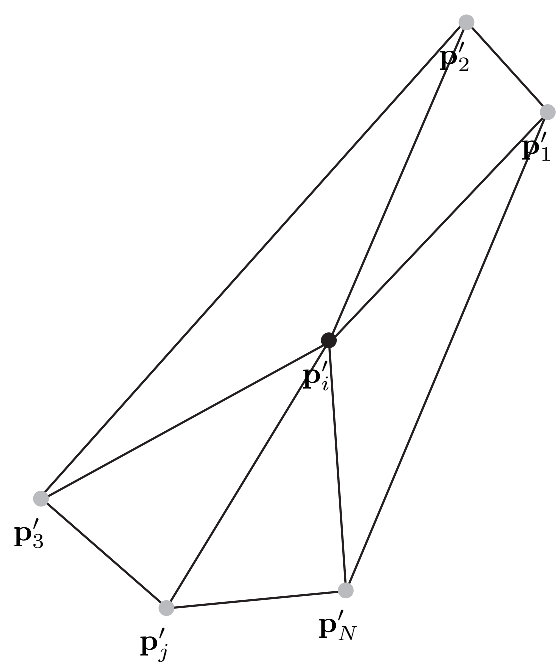
Detecting LCS numerically: FTLE field approach

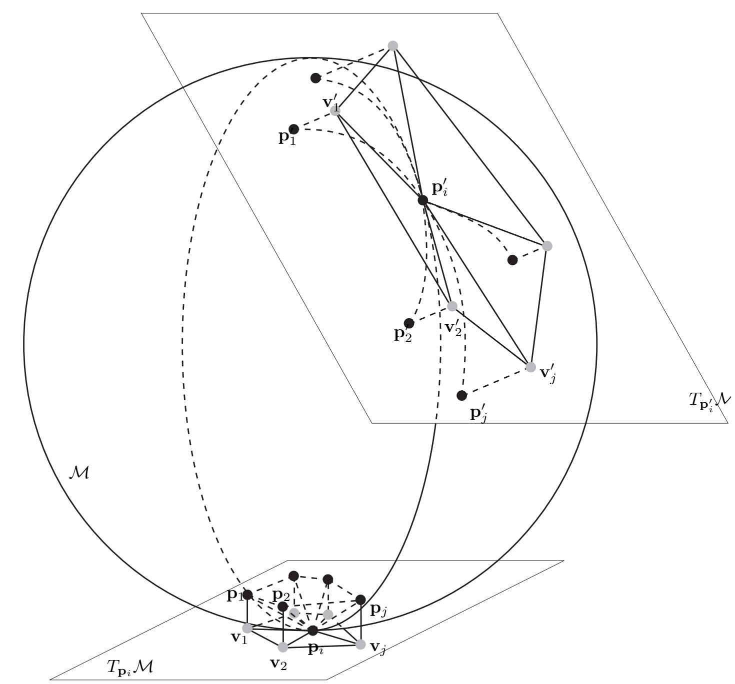
- We can define the FTLE for Riemannian manifolds[1]

$$\sigma_t^T(x) = \frac{1}{|T|} \ln \left\| \mathrm{D}\phi_t^{t+T} \right\| \doteq \frac{1}{|T|} \ln \left(\max_{\mathrm{y} \neq 0} \frac{\left\| \mathrm{D}\phi_t^{t+T}(\mathrm{y}) \right\|}{\|\mathrm{y}\|} \right)$$

with y a small perturbation in the tangent space at x.







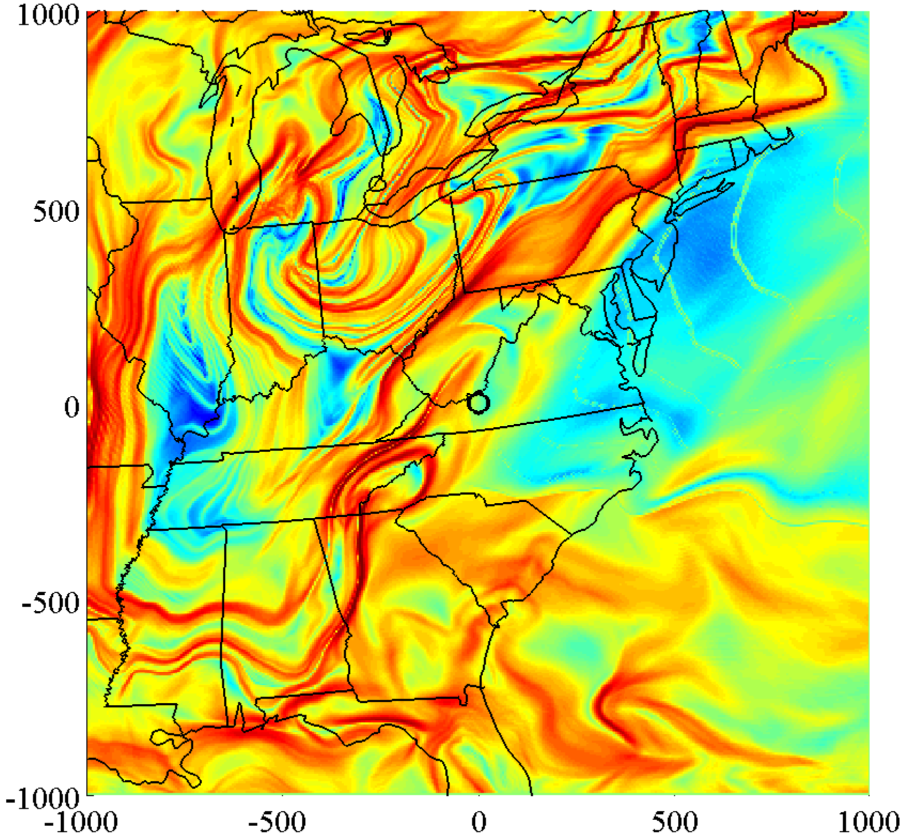
[1] Lekien & Ross [2010] Chaos

Atmosphere: Antarctic polar vortex

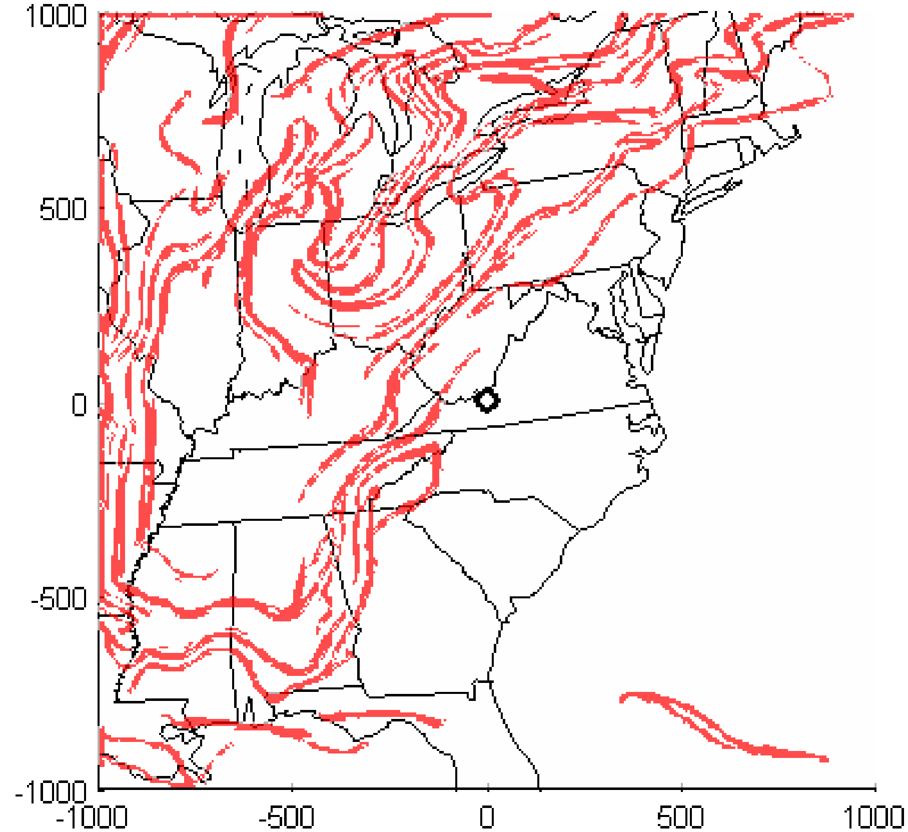
Atmosphere: Antarctic polar vortex

Atmosphere: continental U.S.

Computation of LCS



(a)



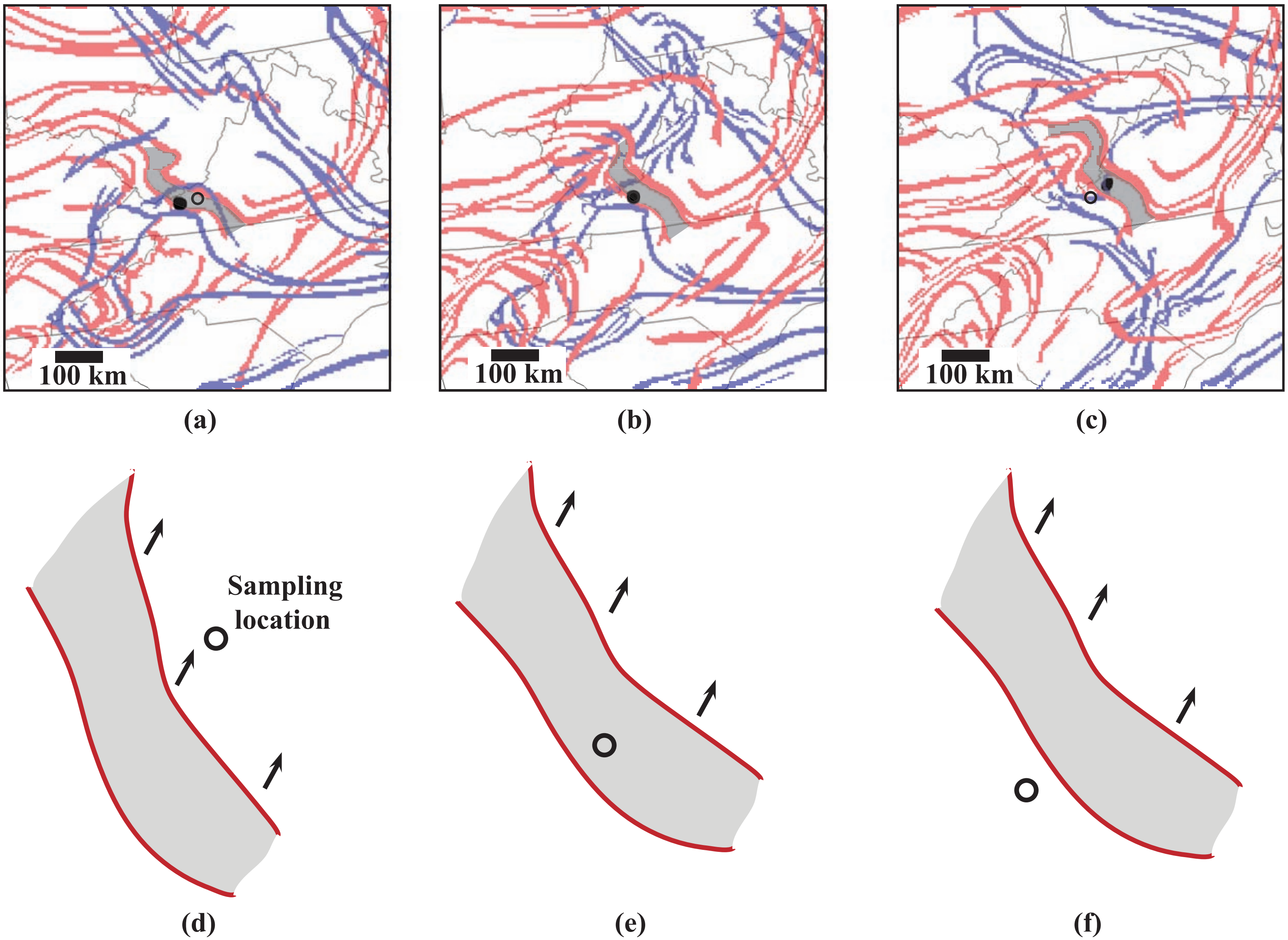
(b)

(a) Sample FTLE field over the eastern United State at 21:00 UTC 15 May 2007, using WRF NAM-218 velocity data set provided by NOAA. (b) Ridges extracted from the FTLE field in (a).

Summary of Hypothesis Testing

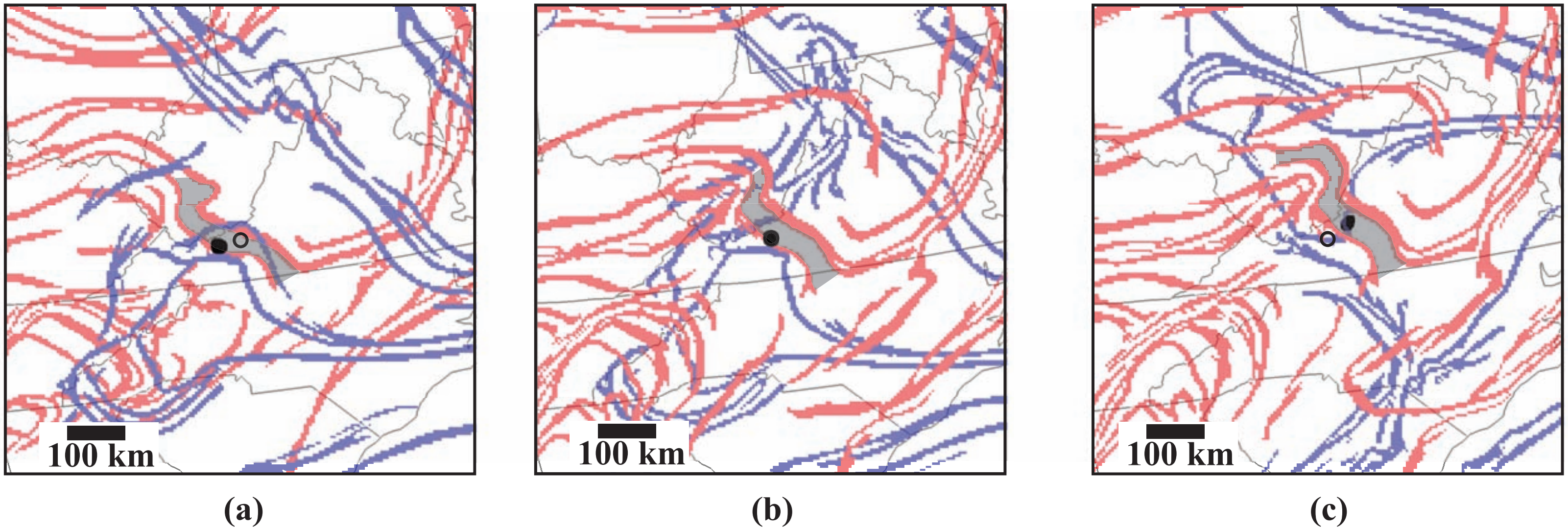
- Of 100 samples, only 73 sample pairs within 24 hours
- Of those, 16 show punctuated changes in the concentration of *Fusarium*
- Punctuated change $\Rightarrow$ repelling LCS passage **70% of the time** ($p = 0.0017$)
- **Punctuated changes were significantly associated with the movement of a repelling LCS**
- Correlation poor for attracting LCS: punctuated change $\Rightarrow$ attracting LCS passage 37% of the time ($p = 0.33$)

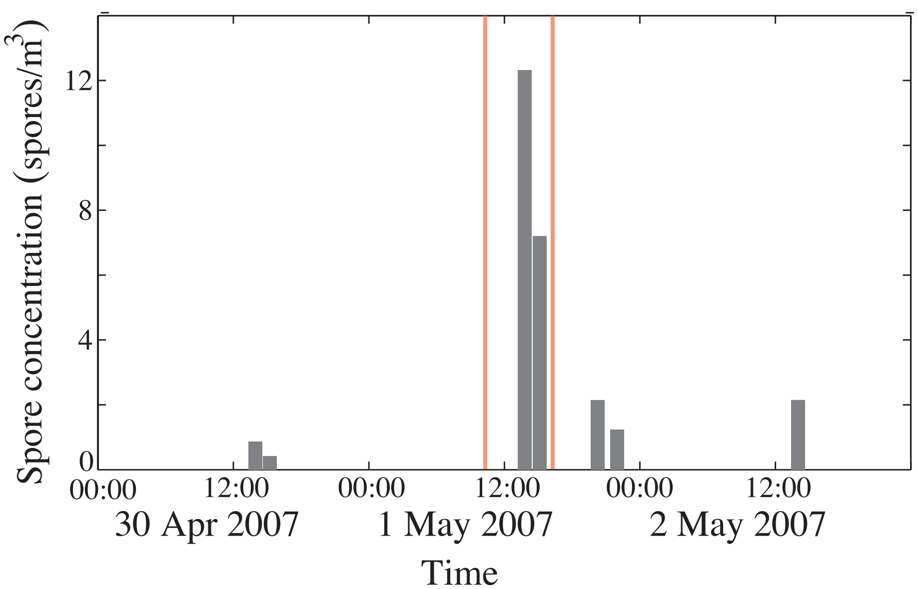
Example: Filament bounded by repelling LCS



12:00 UTC 1 May 2007 | 15:00 UTC 1 May 2007 | 18:00 UTC 1 May 2007

Example: Filament bounded by repelling LCS





12:00 UTC 1 May 2007 15:00 UTC 1 May 2007 18:00 UTC 1 May 2007

Relationship to lobe dynamics and almost-invariant sets?

- As our dynamical system, we consider a discrete map[2]

$$f : \mathcal{M} \longrightarrow \mathcal{M},$$

e.g., $f = \phi_{t_0}^{t_0+T}$, where $\mathcal{M}$ is a differentiable, orientable, two-dimensional manifold e.g., $\mathbb{R}^2$, S^2

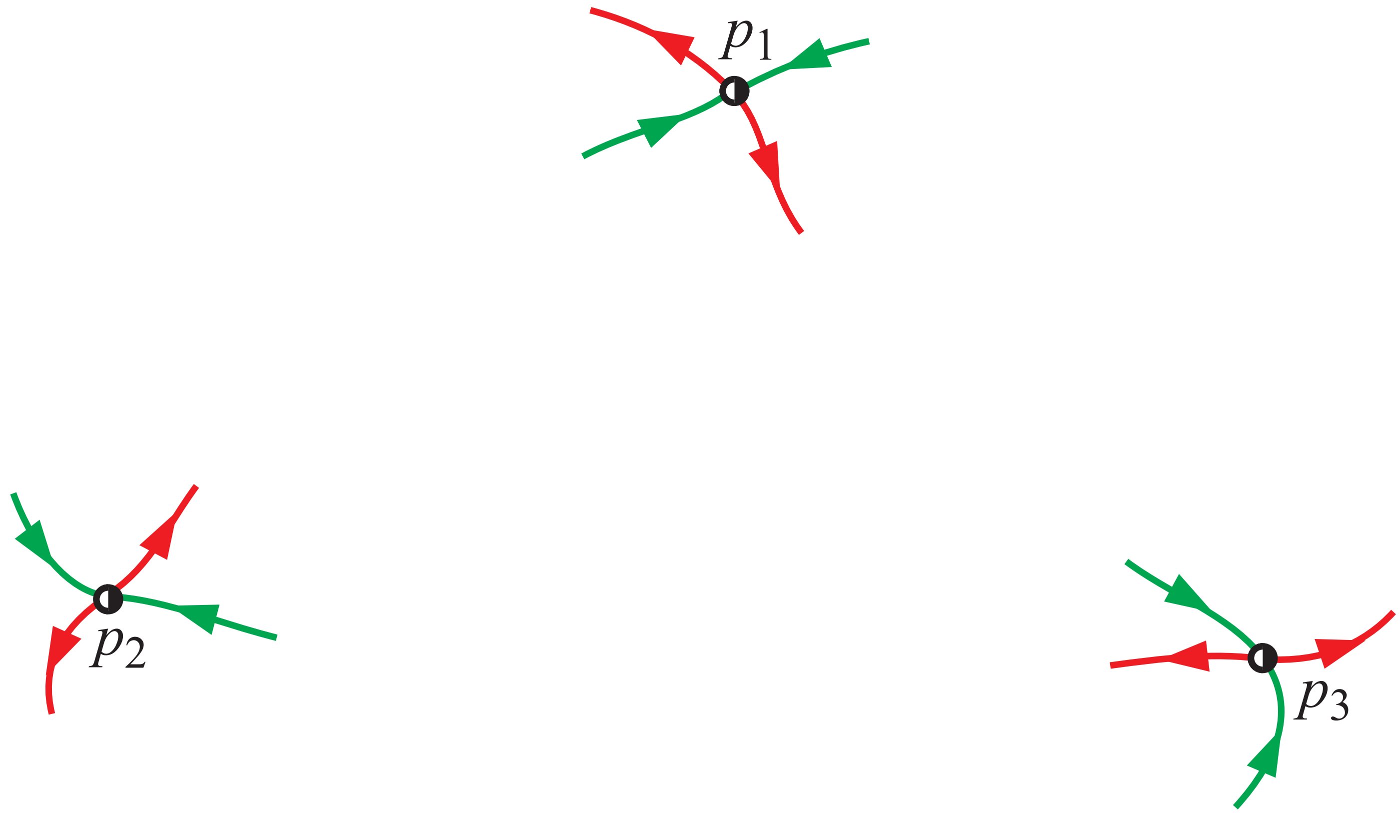
- To understand the transport of points under the map f, we consider the **invariant manifolds of unstable fixed points**

- Let $p_i, i = 1, ..., N_p$, denote a collection of saddle-type hyperbolic fixed points for f.

[2]Following Rom-Kedar and Wiggins [1990]

Partition phase space into regions

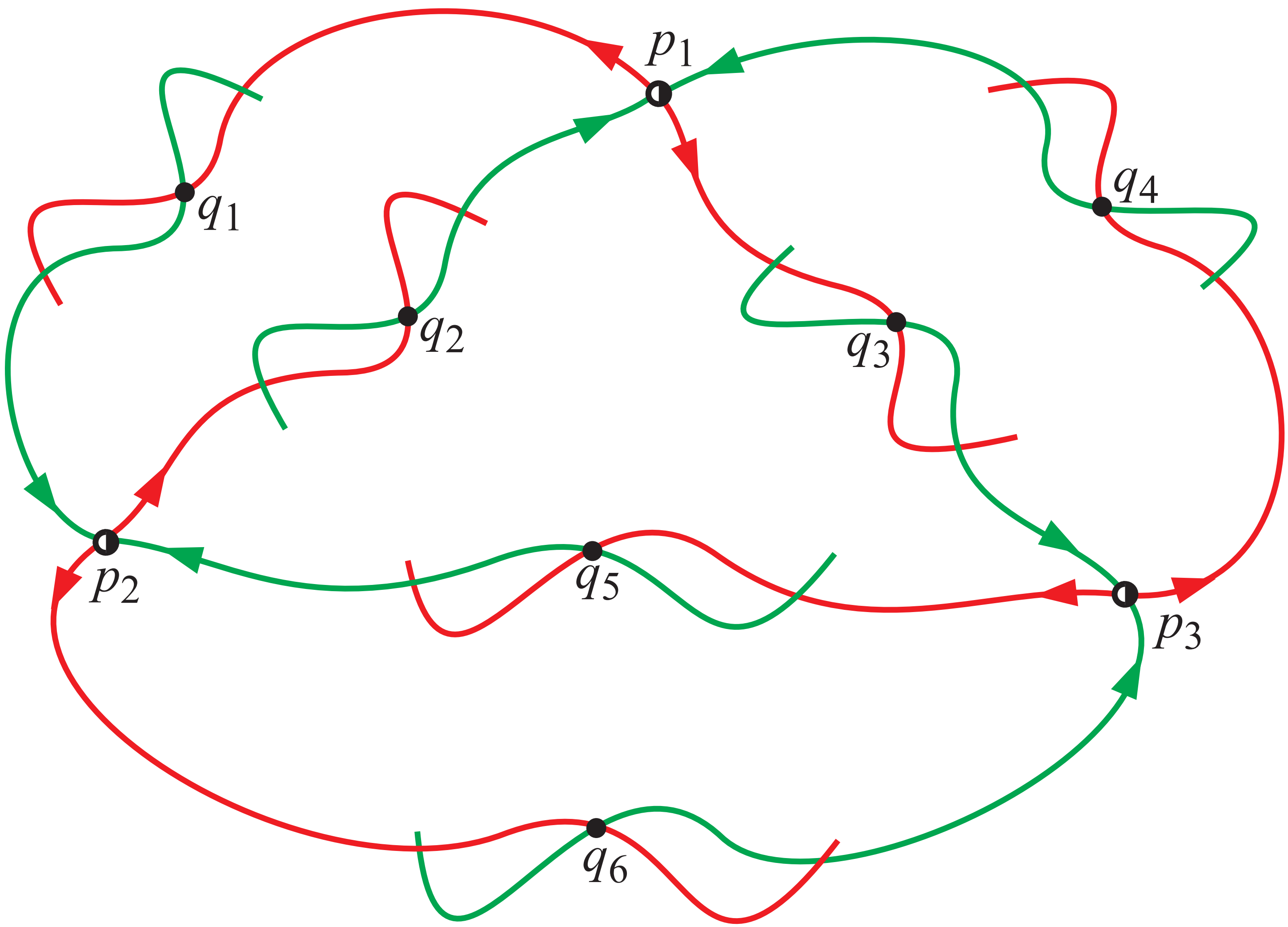
- Natural way to partition phase space
  - Pieces of $W^u(p_i)$ and $W^s(p_i)$ partition $\mathcal{M}$.



Unstable and stable manifolds in **red** and **green**, resp.

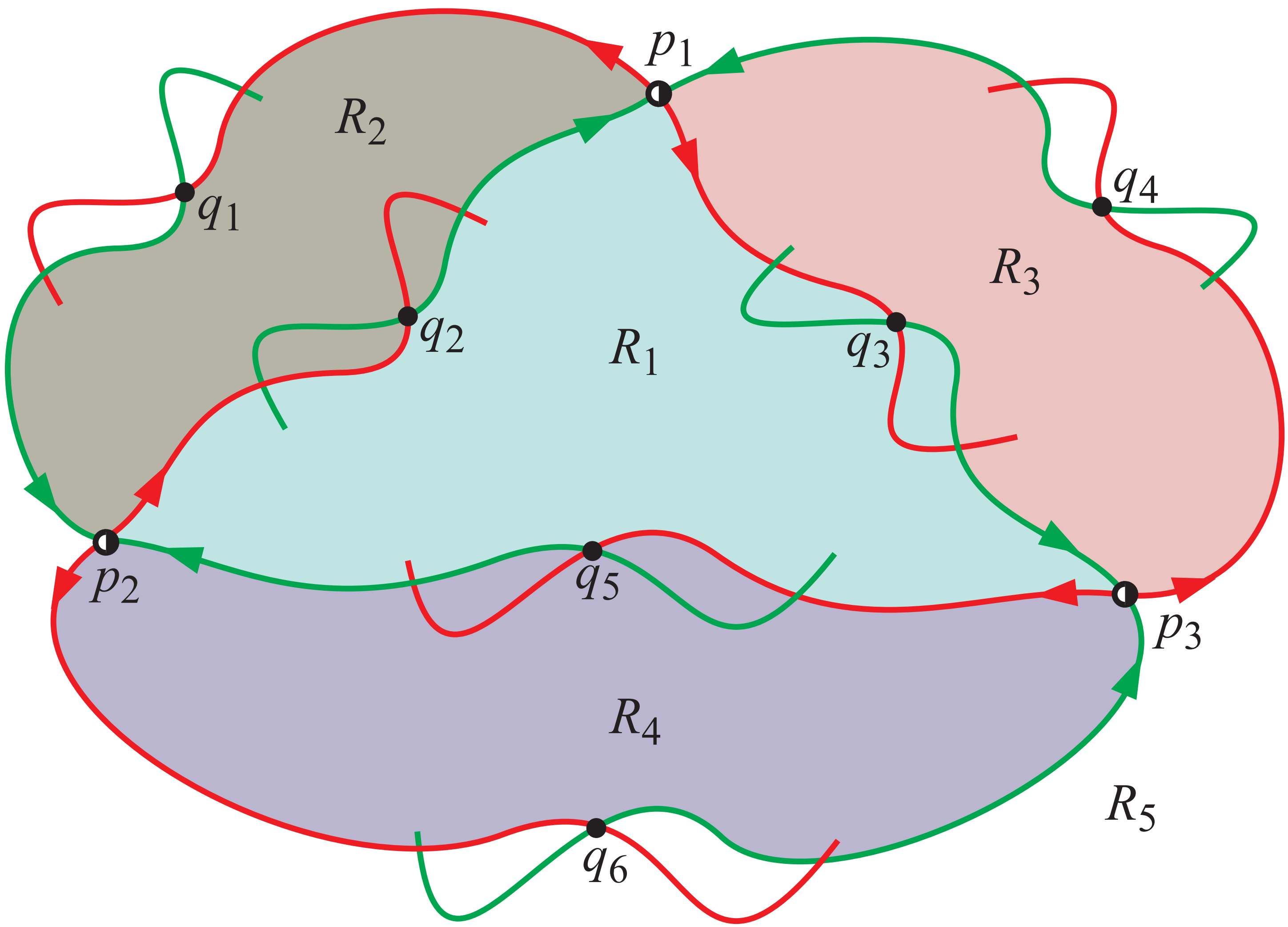
Partition phase space into regions

- Intersection of unstable and stable manifolds define **boundaries**.



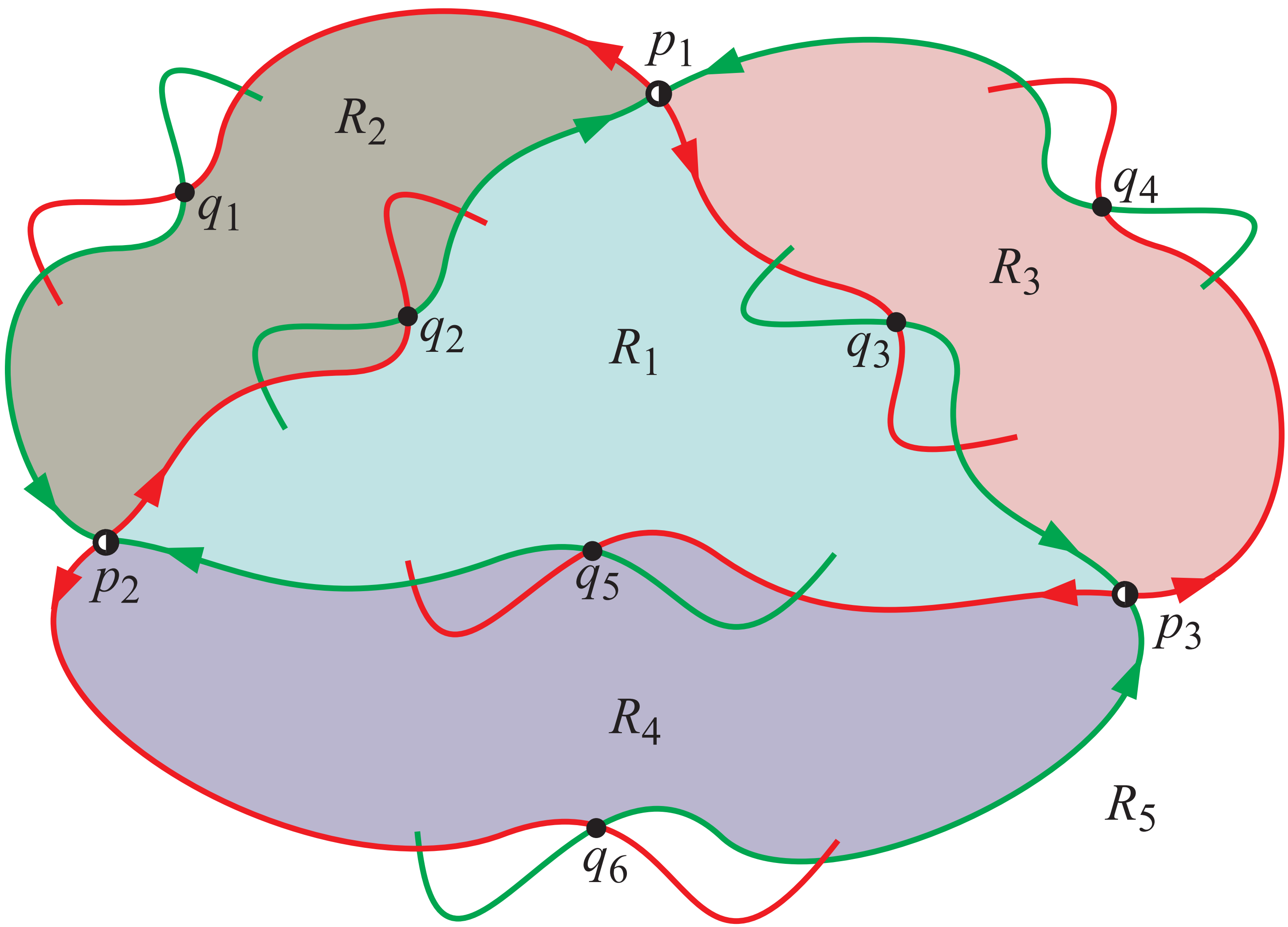
Partition phase space into regions

- These boundaries divide the phase space into **regions**.



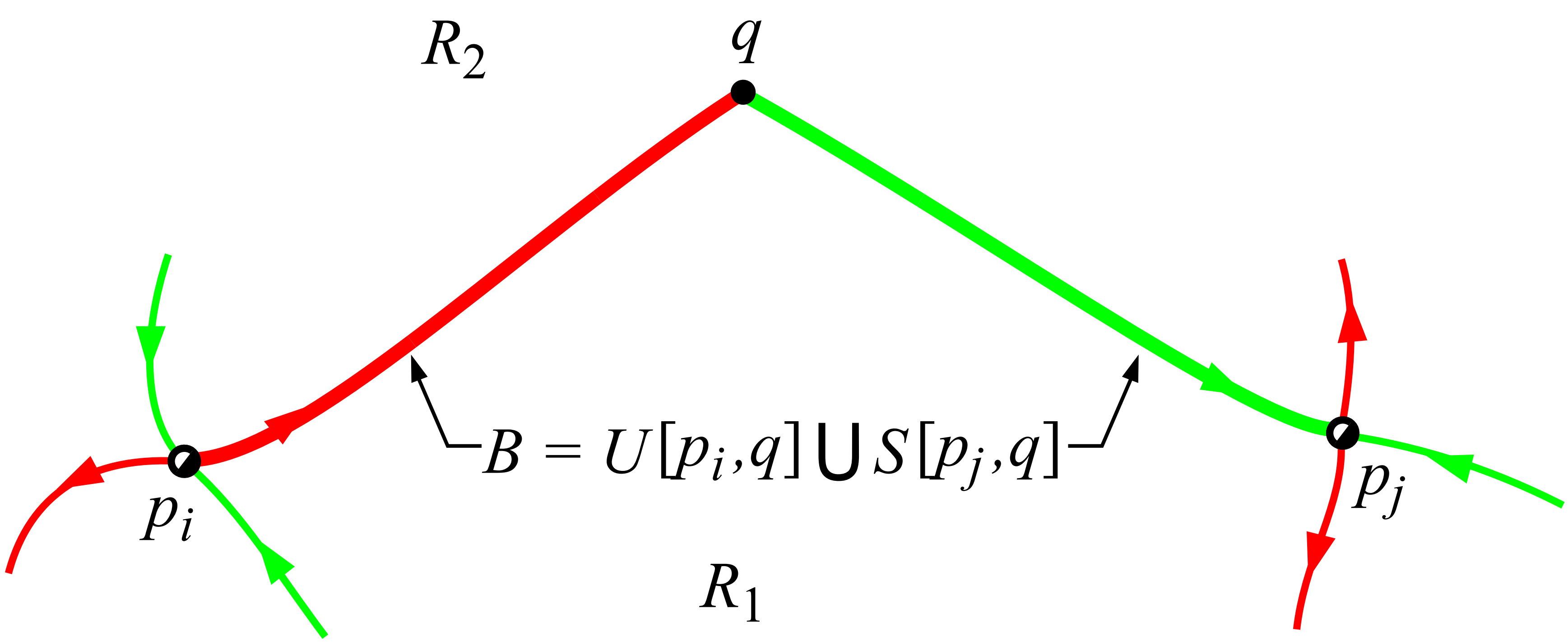
Label mobile subregions: 'atoms' of transport

- Can label mobile subregions based on their past and future whereabouts under one iterate of the map, e.g., $(\ldots, R_3, R_3, [R_1], R_1, R_2, \ldots)$



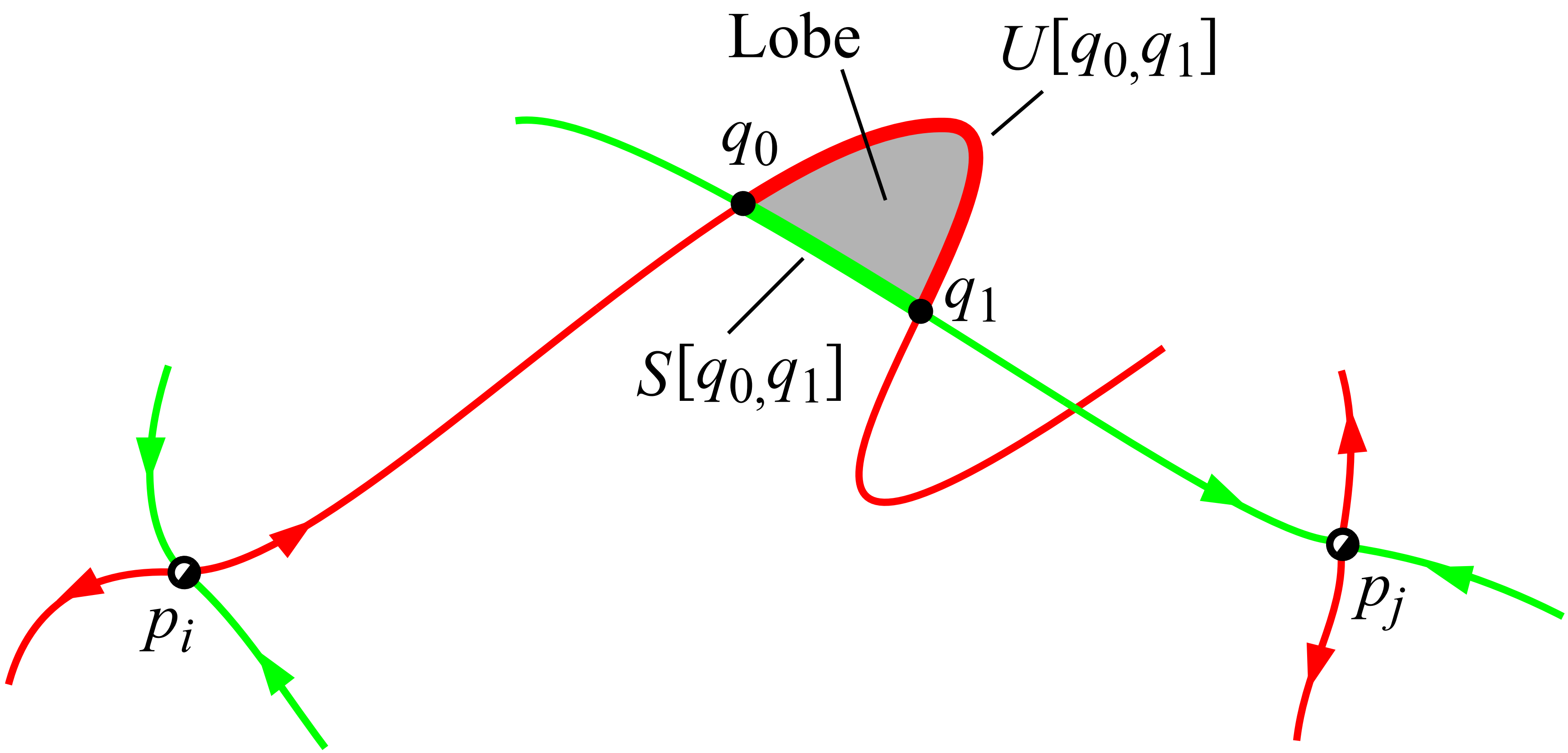
Primary intersection points (pips) and boundaries

- Suppose $W^u(p_i)$ and $W^s(p_j)$ intersect in the pip q. Define $\mathcal{B} \equiv U[p_i, q] \bigcup S[p_j, q]$ as a **boundary** between "two sides," R_1 and R_2.



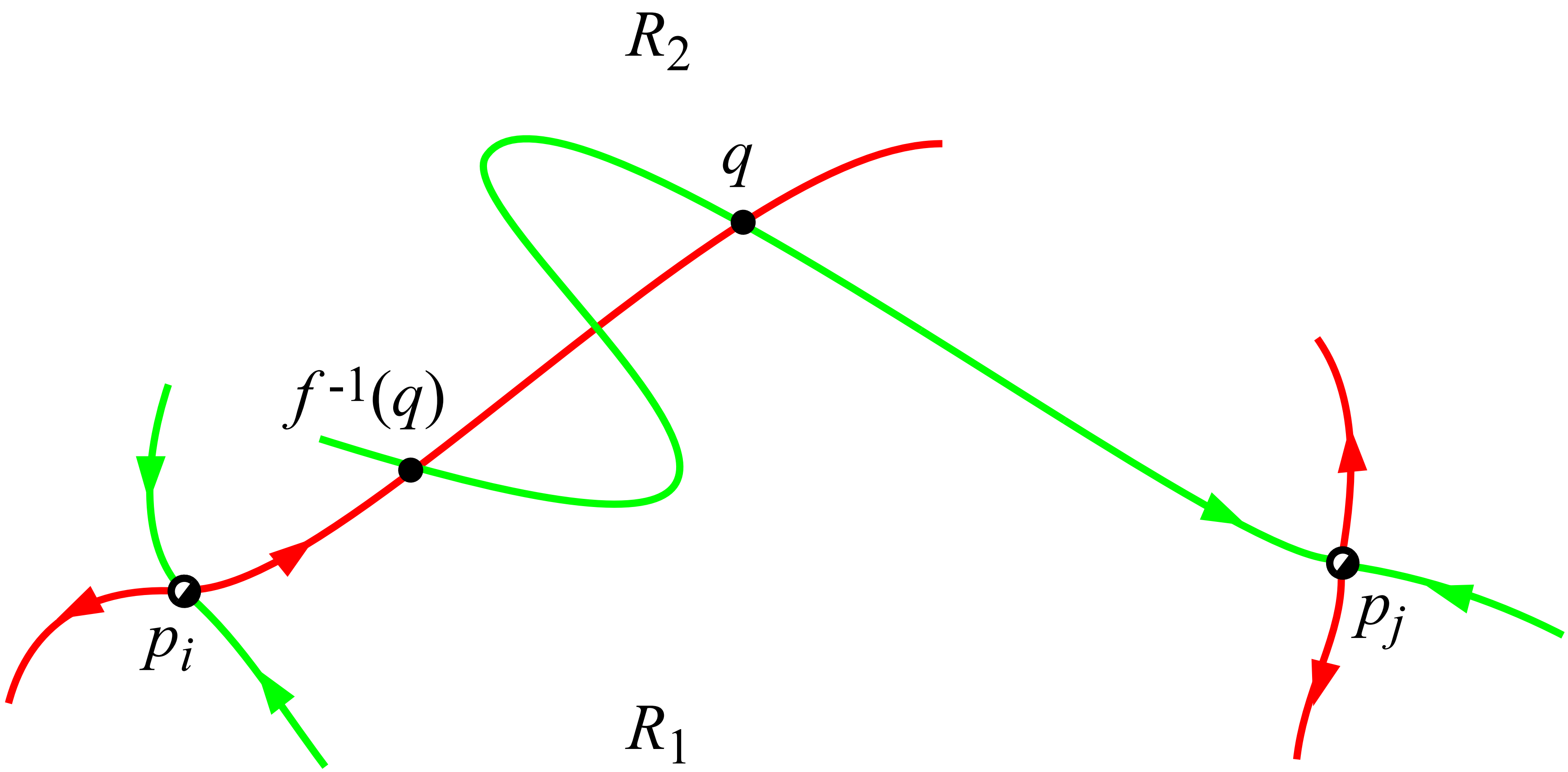
Lobes: the mobile subregions

- Let $q_0, q_1 \in W^u(p_i) \bigcap W^s(p_j)$ be two adjacent pips, i.e., there are no other pips on $U[q_0, q_1]$ and $S[q_0, q_1]$. The region interior to $U[q_0, q_1] \bigcup S[q_0, q_1]$ is a **lobe**.



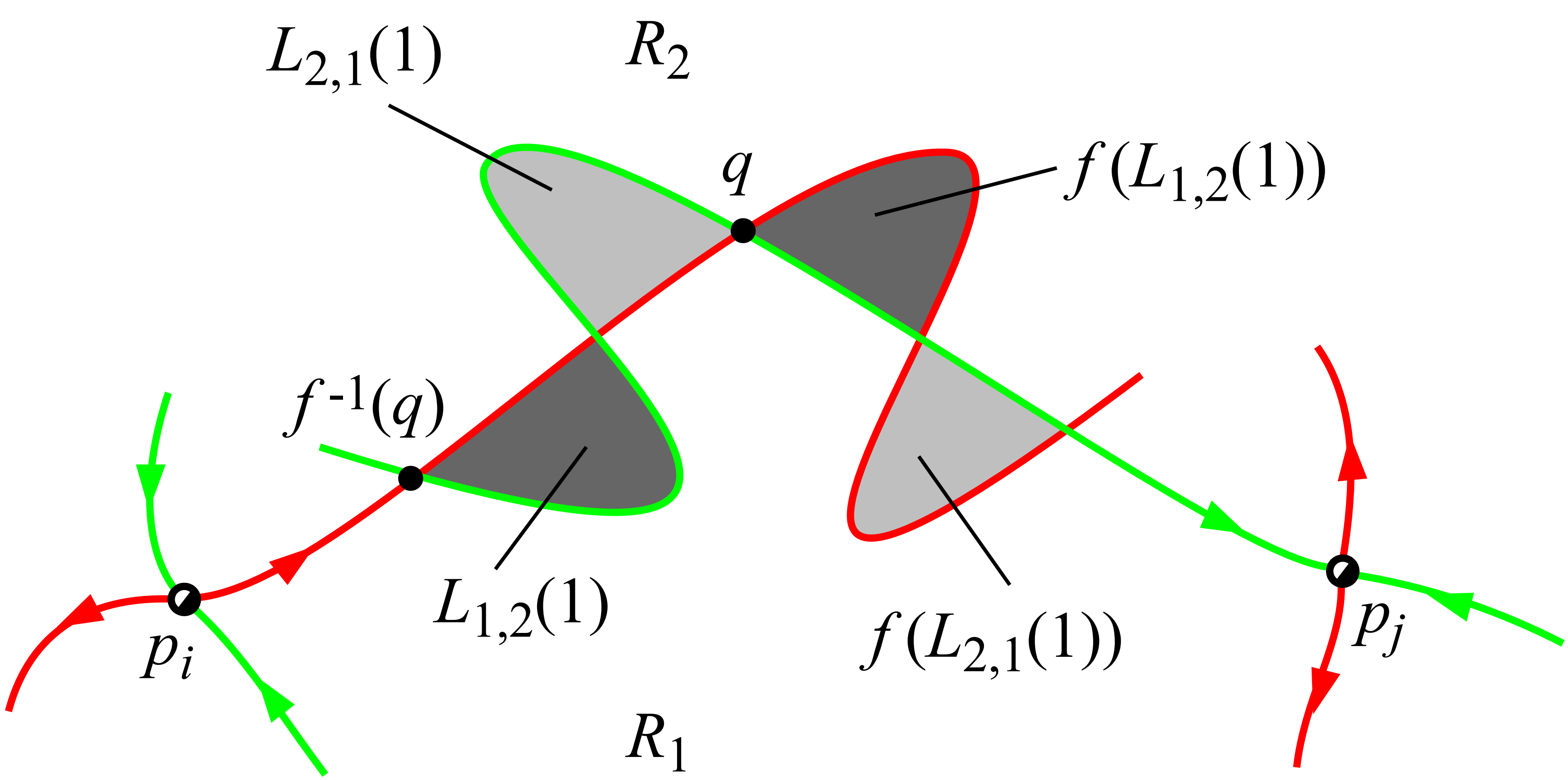
Lobe dynamics: transport across a boundary

- $f^{-1}(q)$ is a pip (by lemma). f is orientation-preserving $\Rightarrow$ there's *at least one* pip on $U[f^{-1}(q), q]$ where the $W^u(p_i), W^s(p_j)$ intersection is topologically transverse.



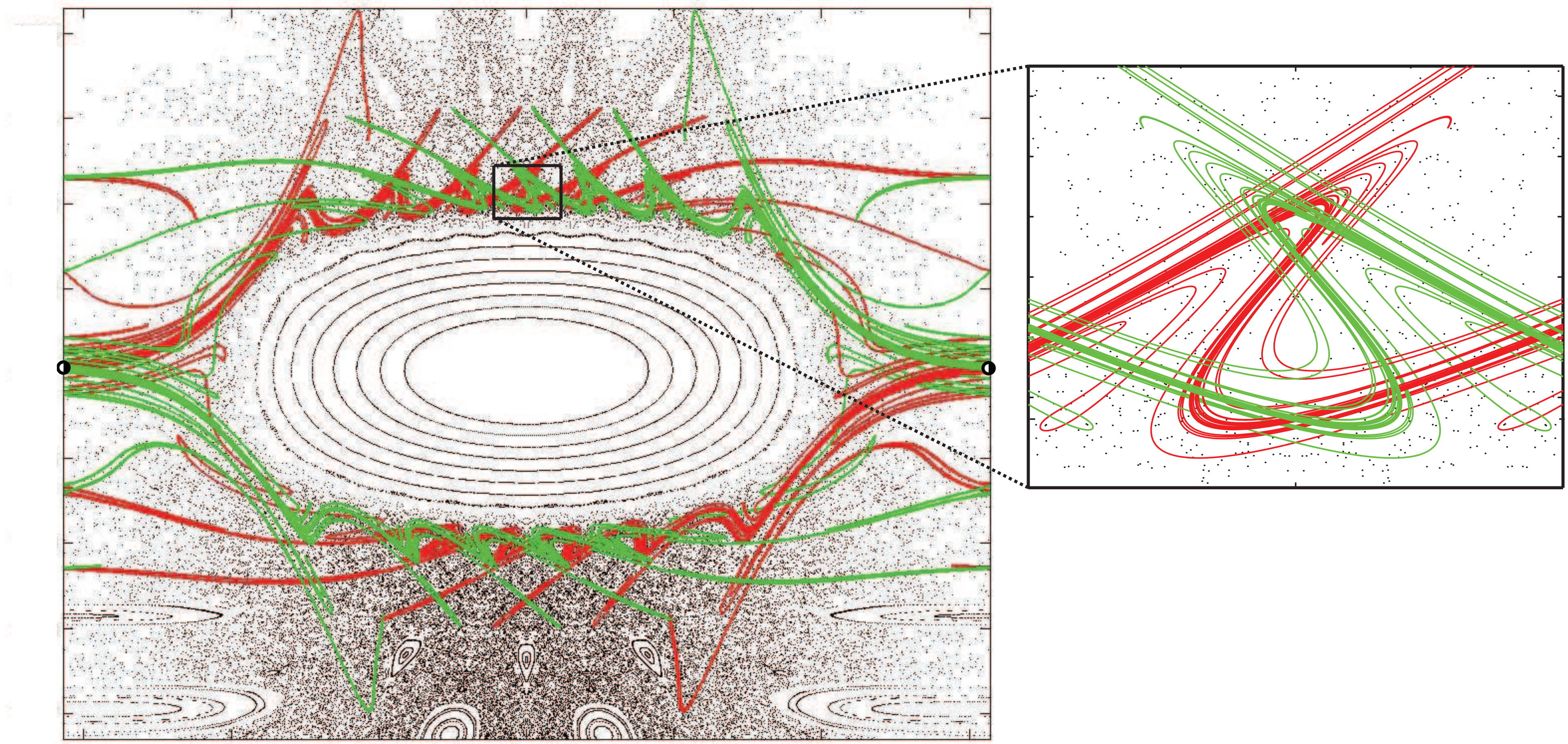
Lobe dynamics: transport across a boundary

- Under one iteration of f, *only points in* $L_{1,2}(1)$ can move from R_1 into R_2 by crossing $\mathcal{B}$, etc.
- The two lobes $L_{1,2}(1)$ and $L_{2,1}(1)$ are called a **turnstile**.



Identifying atoms of transport by itinerary

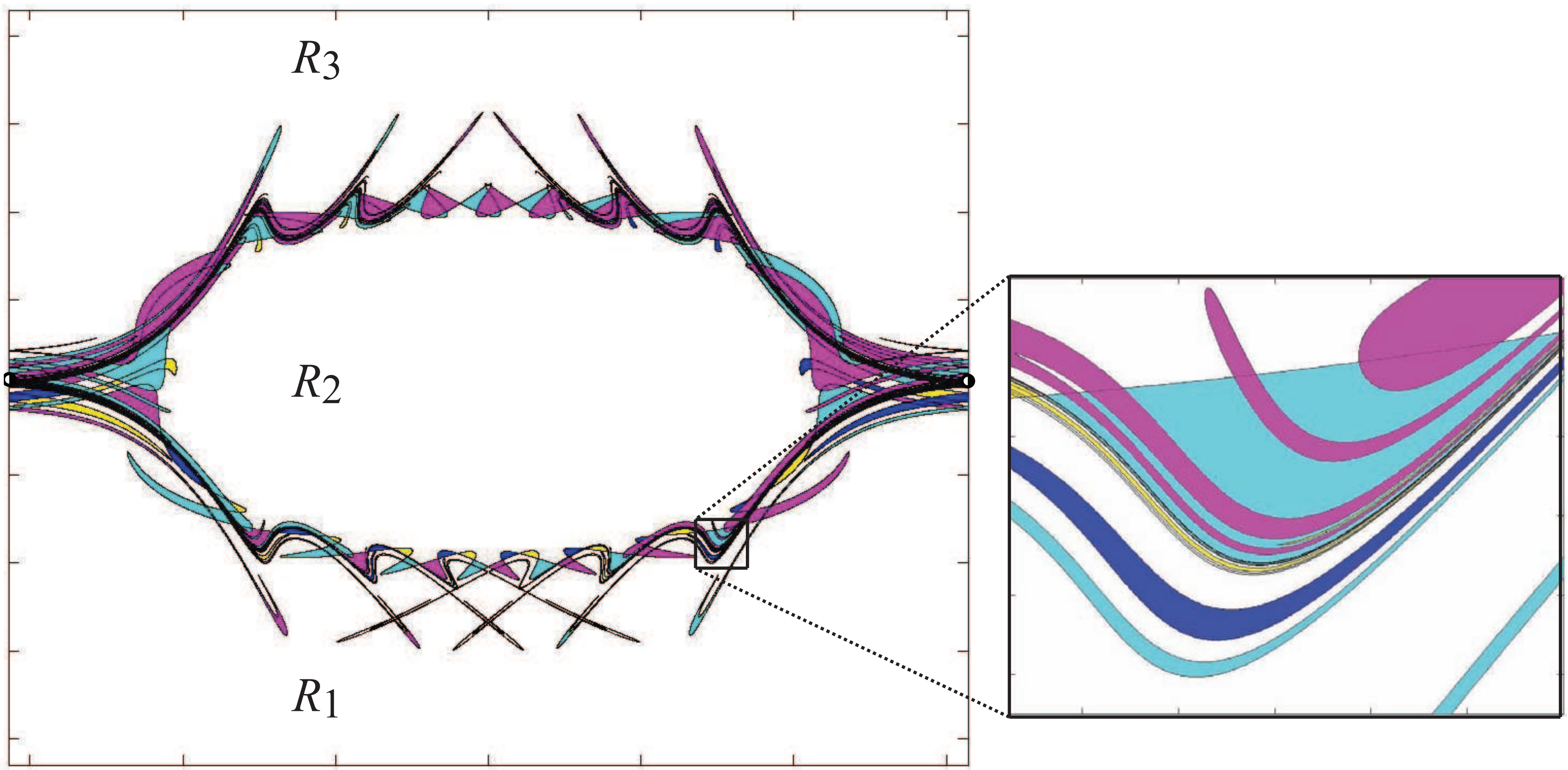
- In a complicated flow, can still identify manifolds ...



Unstable and stable manifolds in **red** and **green**, resp.

Identifying atoms of transport by itinerary

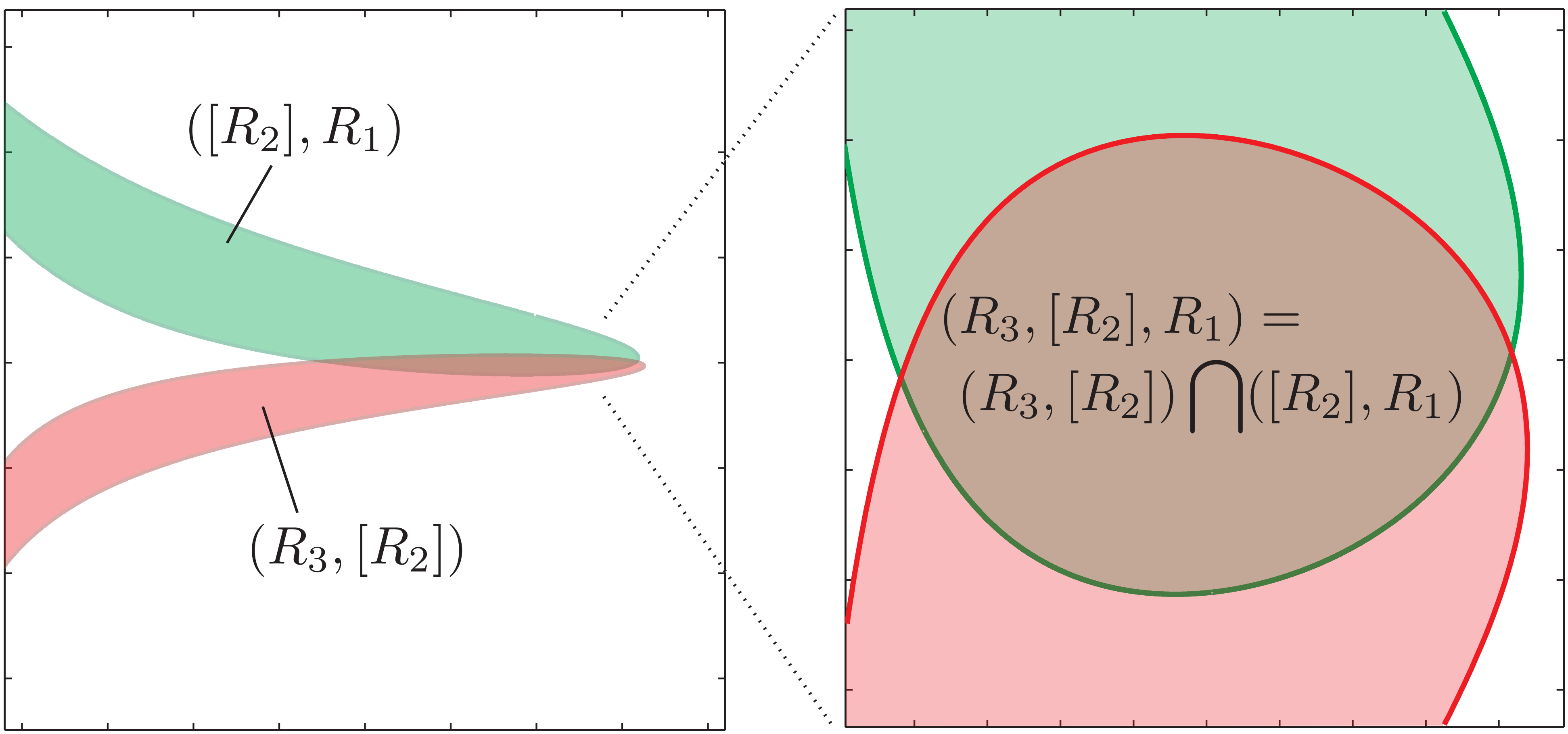
- ... and lobes



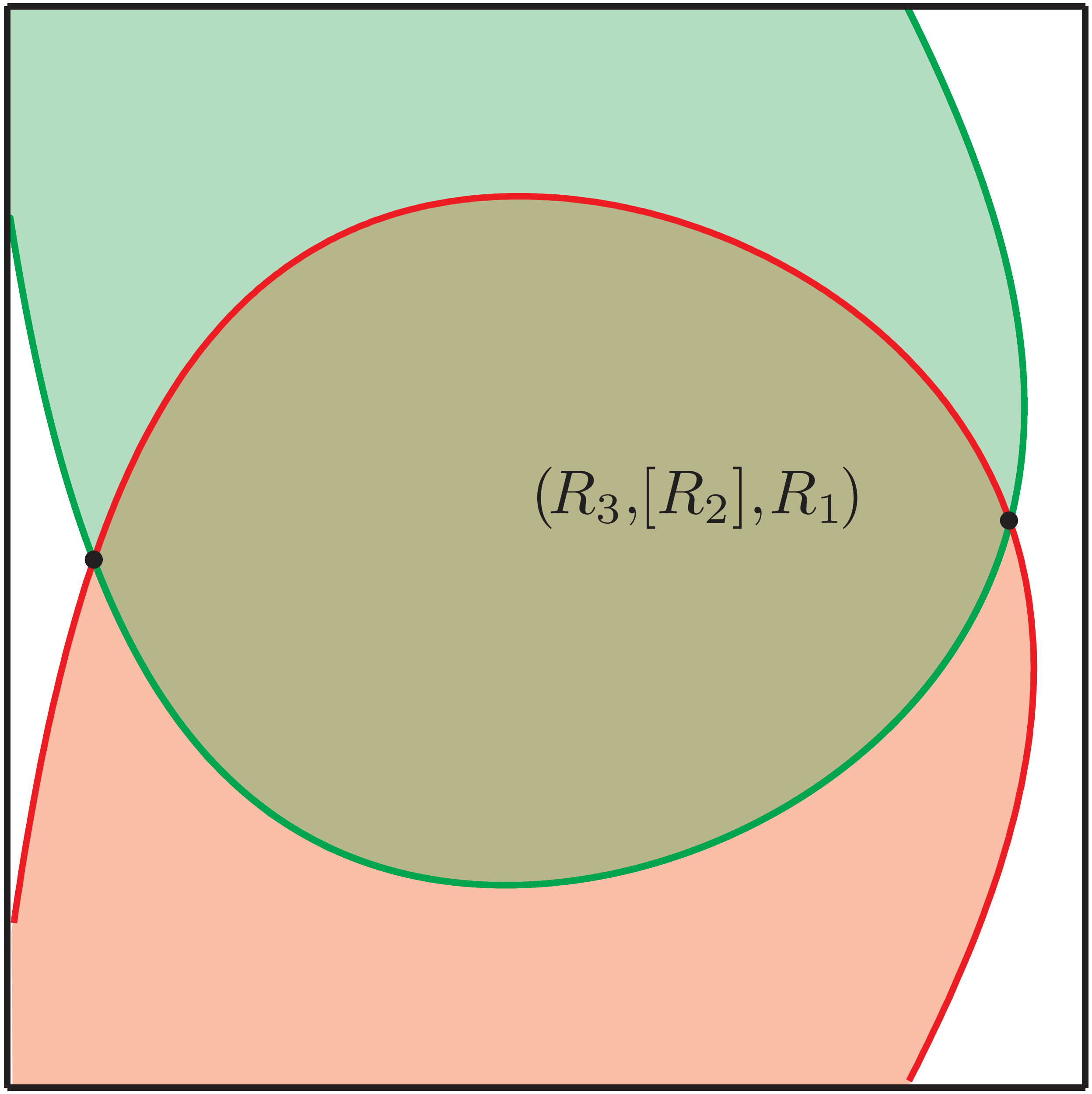
Significant amount of fine, filamentary structure.

Identifying atoms of transport by itinerary

- e.g., with three regions $\{R_1, R_2, R_3\}$, label lobe intersections accordingly.
  - Denote the intersection $(R_3, [R_2]) \bigcap ([R_2], R_1)$ by $(R_3, [R_2], R_1)$

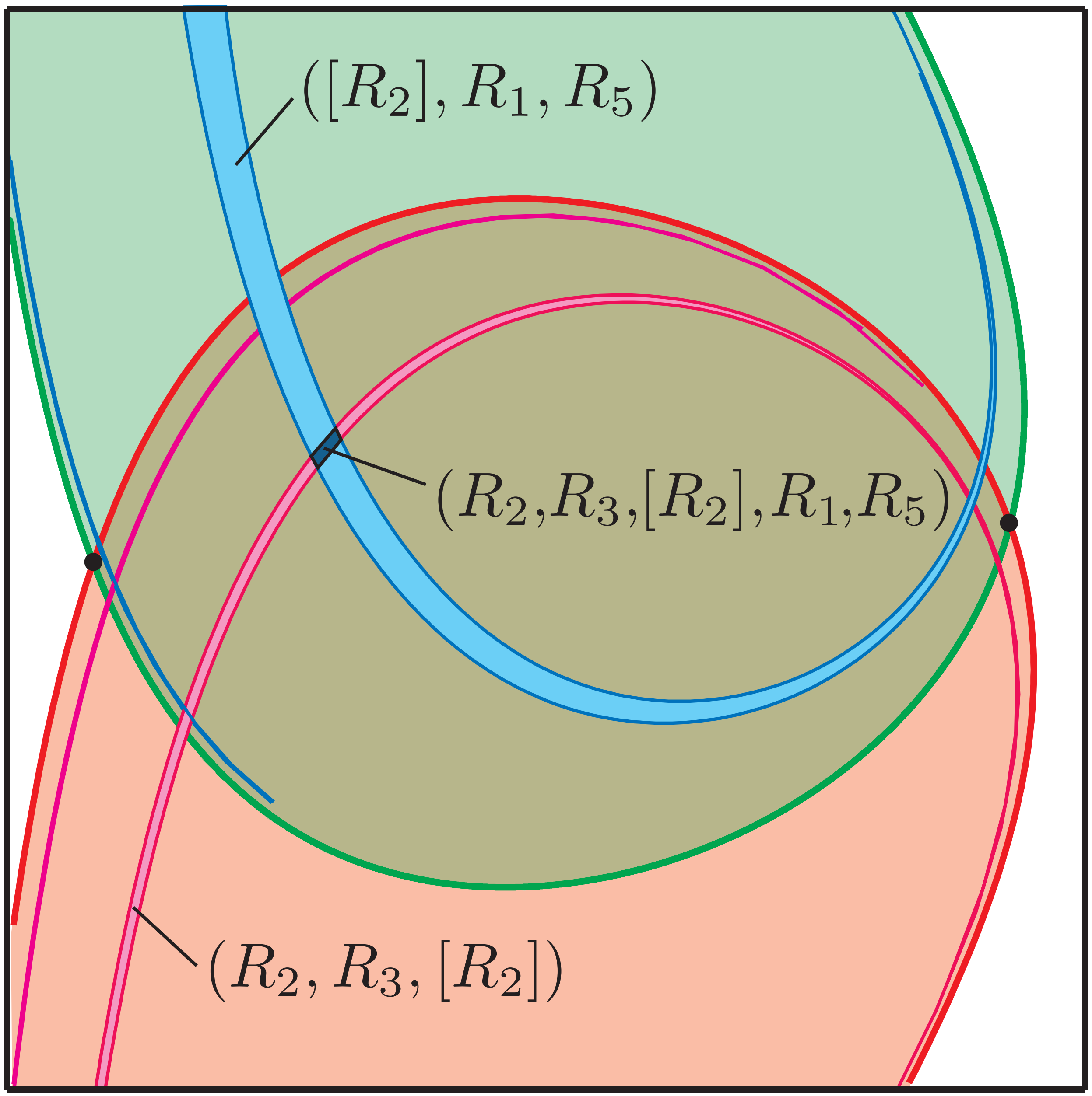


Identifying atoms of transport by itinerary



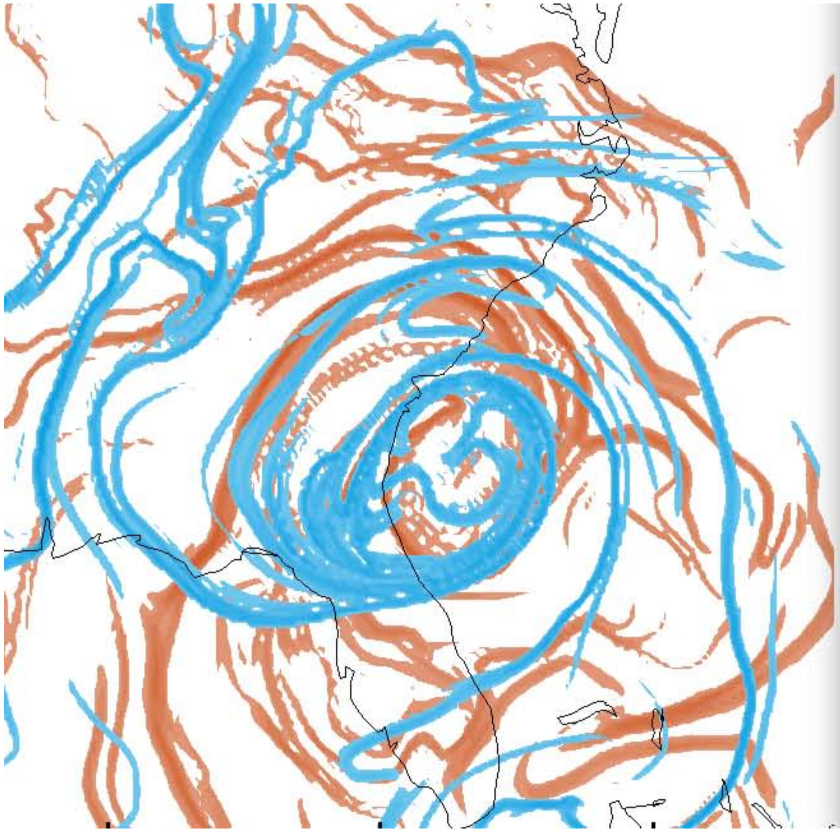
Longer itineraries...

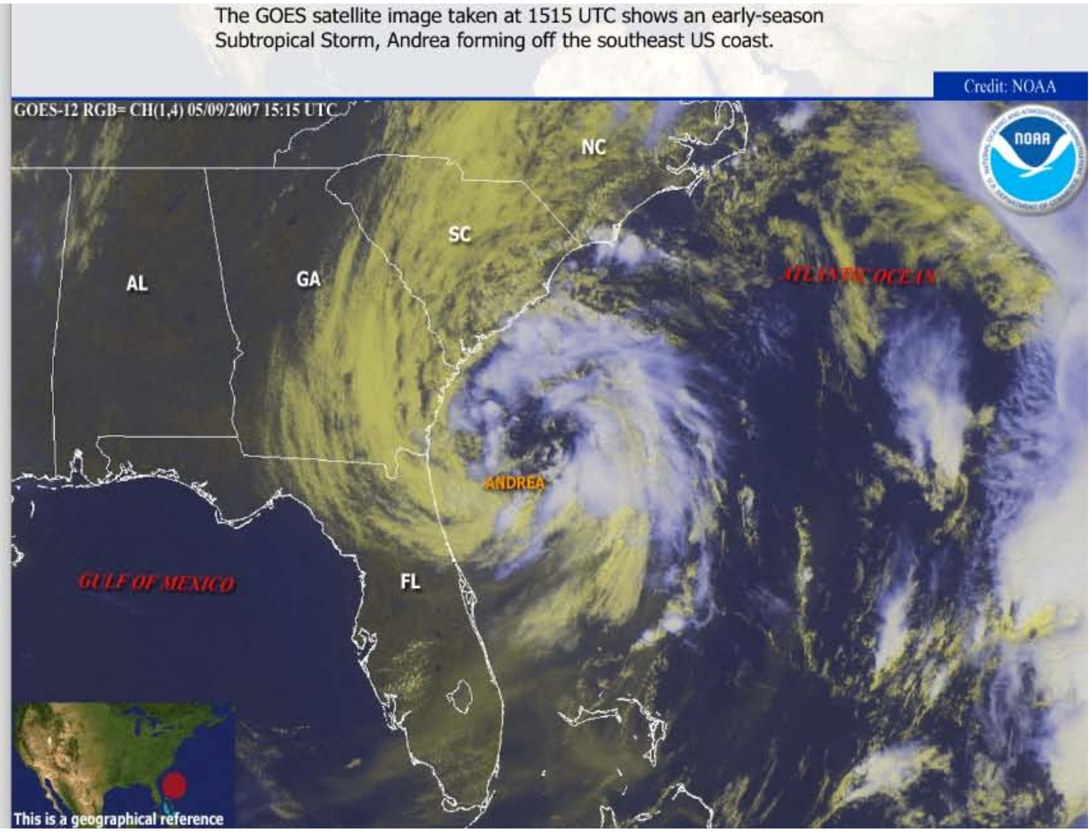
Identifying atoms of transport by itinerary



... correspond to smaller pieces of phase space

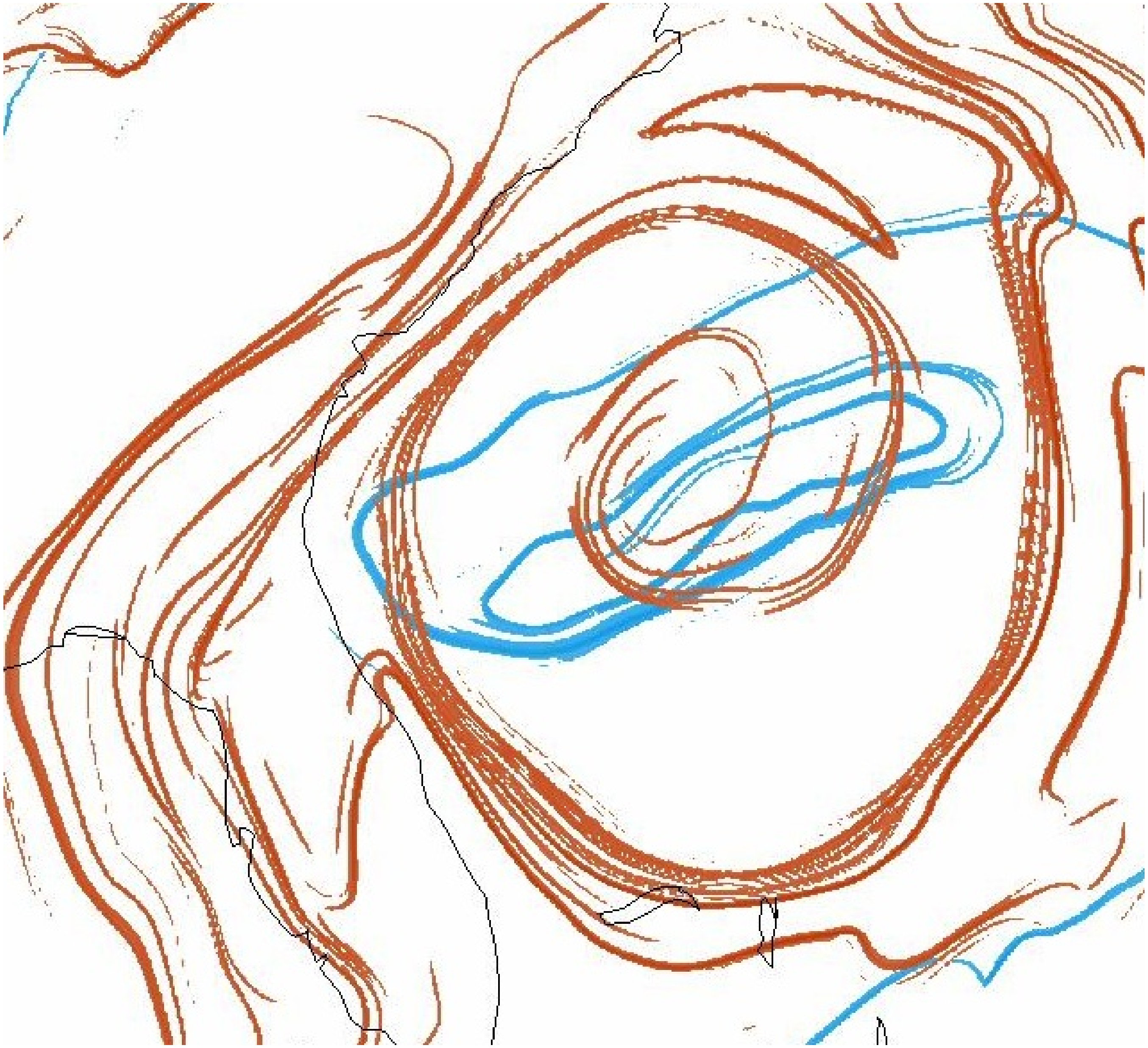
Hurricanes and lobe dynamics



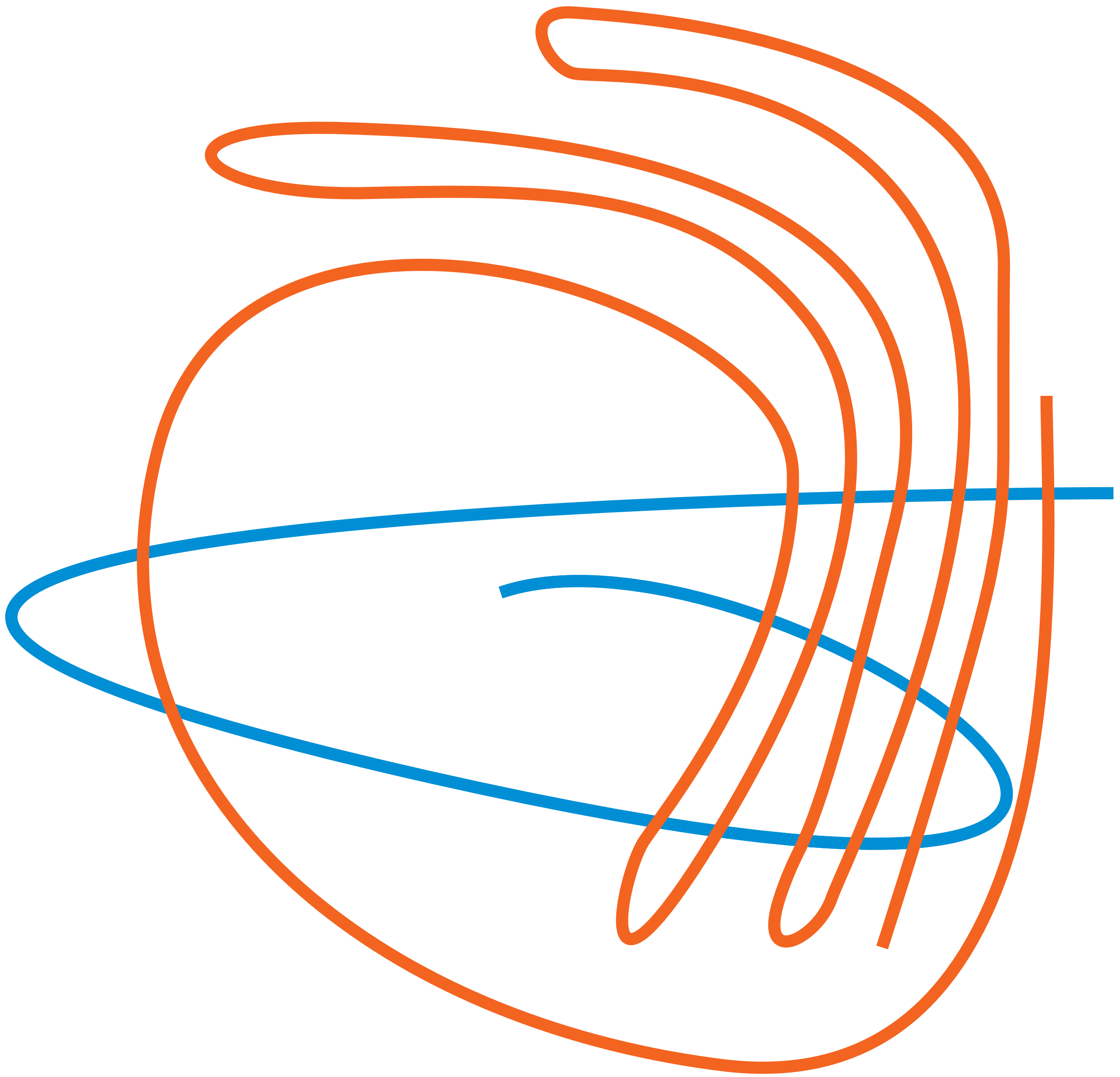


cf. Sapsis & Haller [2009], Lekien & Ross [2010], Du Toit & Marsden [2010]

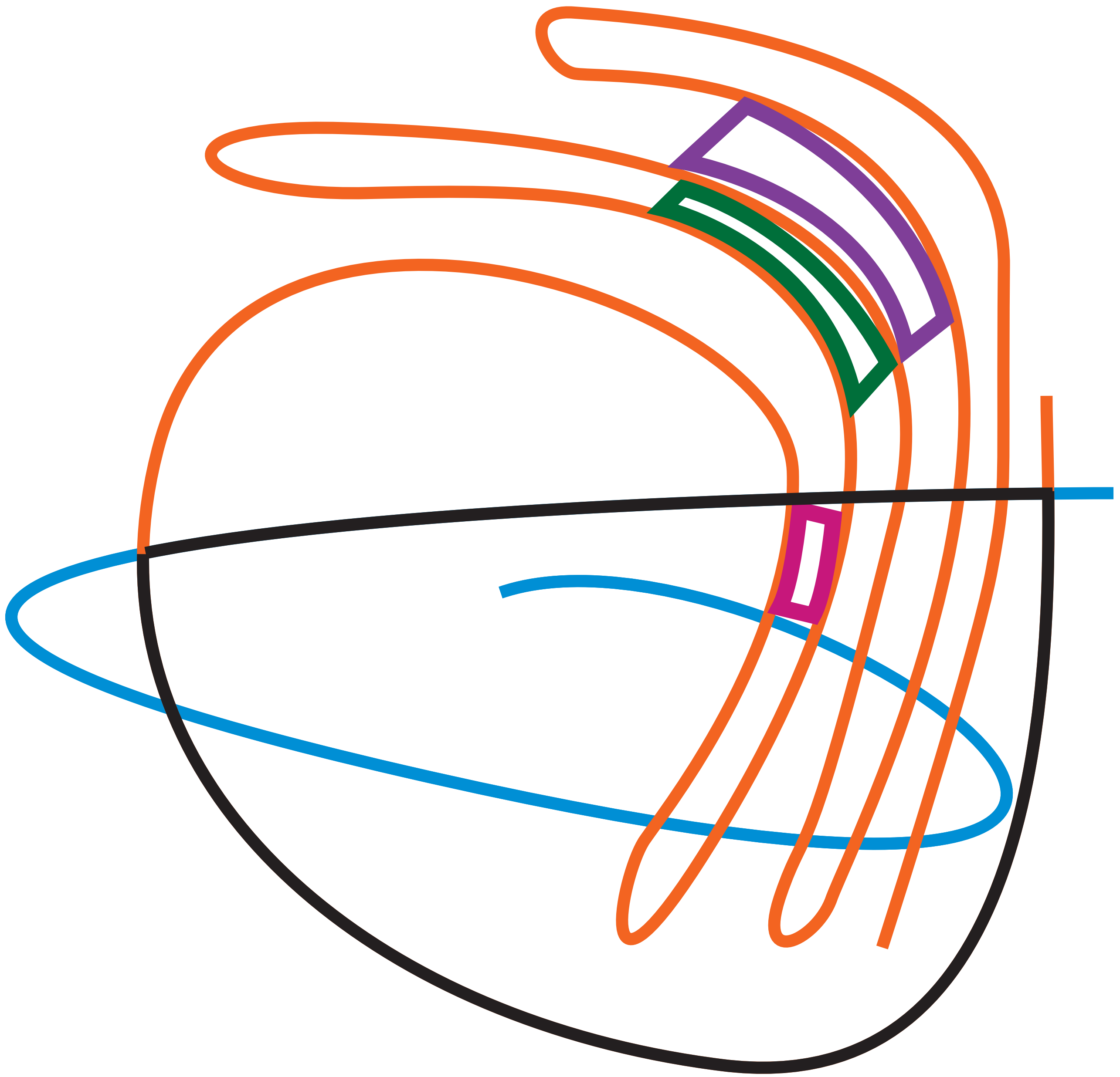
Hurricanes and lobe dynamics



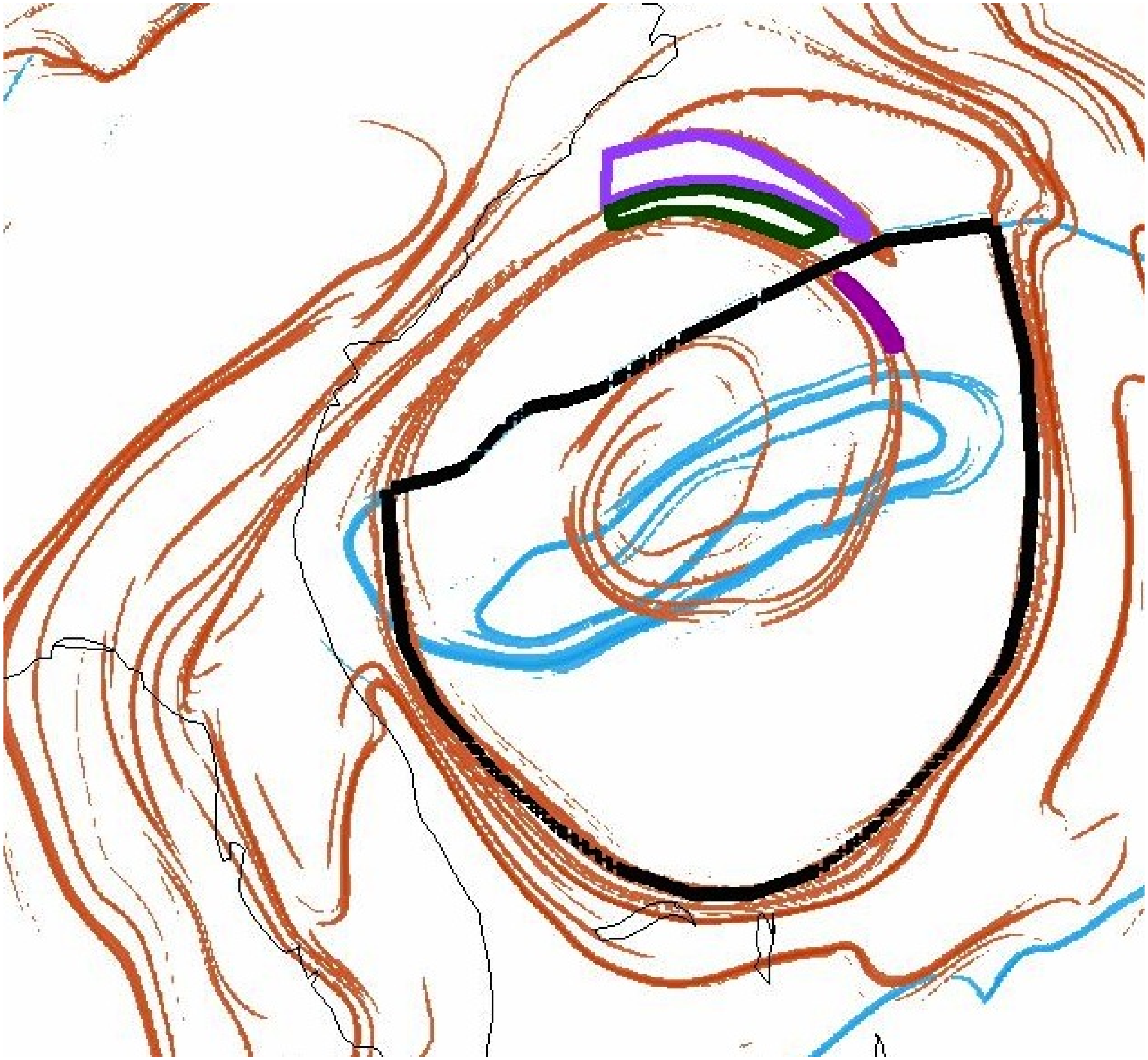
Hurricanes and lobe dynamics



Hurricanes and lobe dynamics



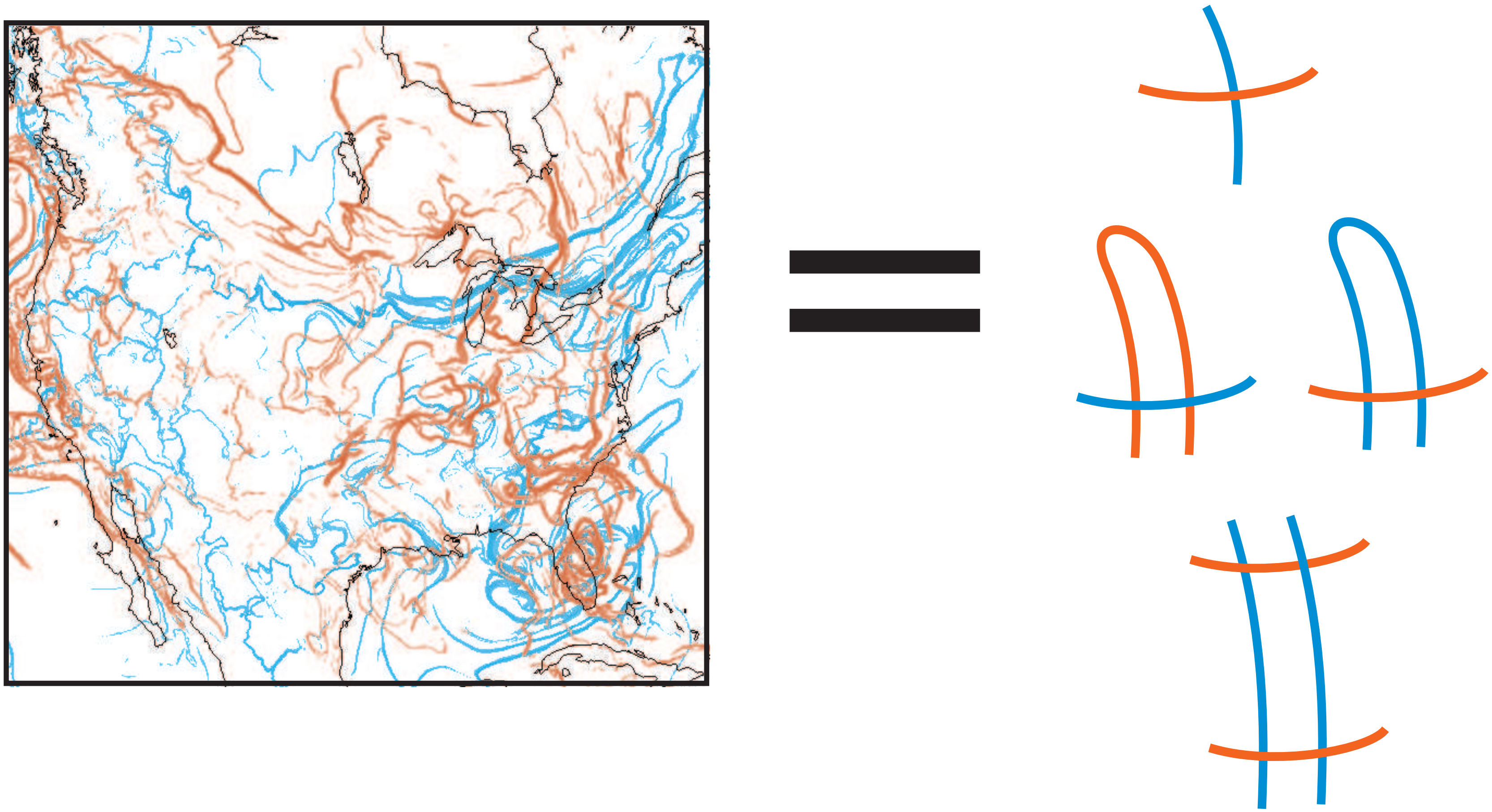
Hurricanes and lobe dynamics



Hurricanes and lobe dynamics

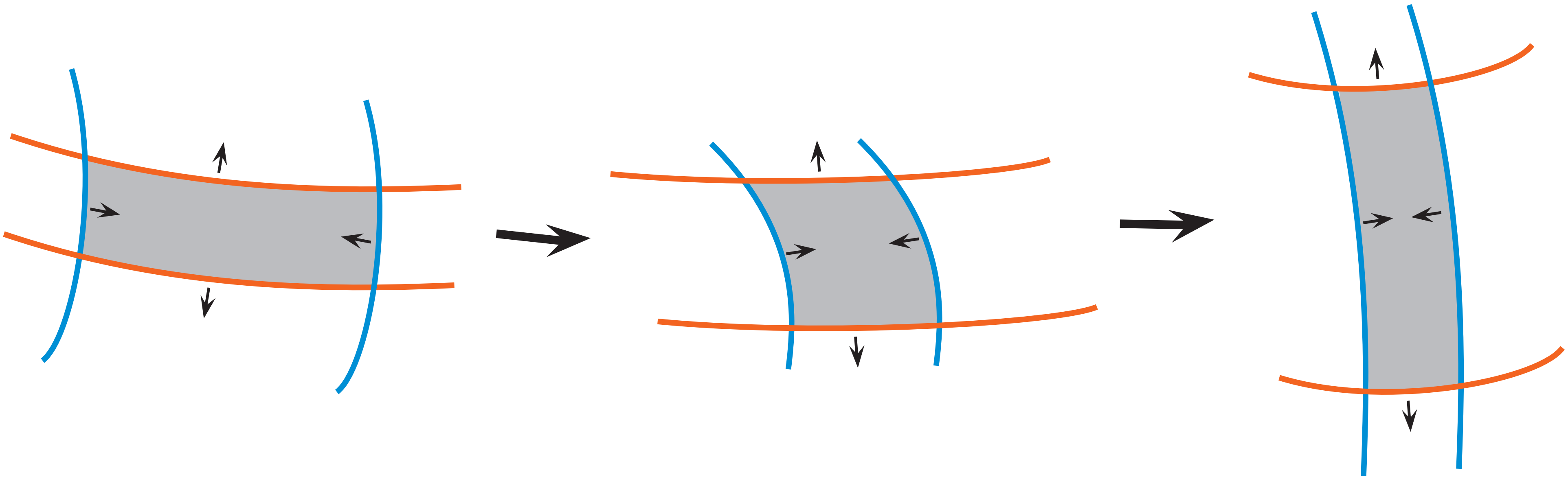
Sets behave as lobe dynamics dictates

Classification of motifs



- **Regions bounded by attracting and repelling curves**
- **Atmosphere is naturally parsed into discrete 'cells'**

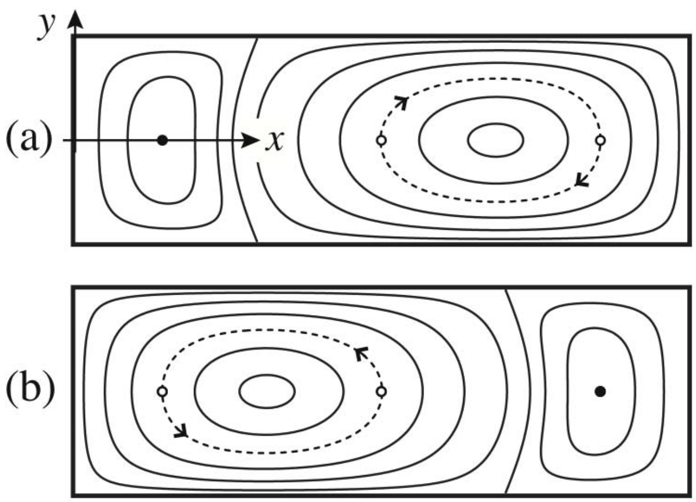
Motion of ‘cells’



- **Packets have their own dynamics as consequence of repelling and attracting natures of boundaries**

Lobe dynamics: another fluid example

- Fluid example: time-periodic Stokes flow[2]



streamlines

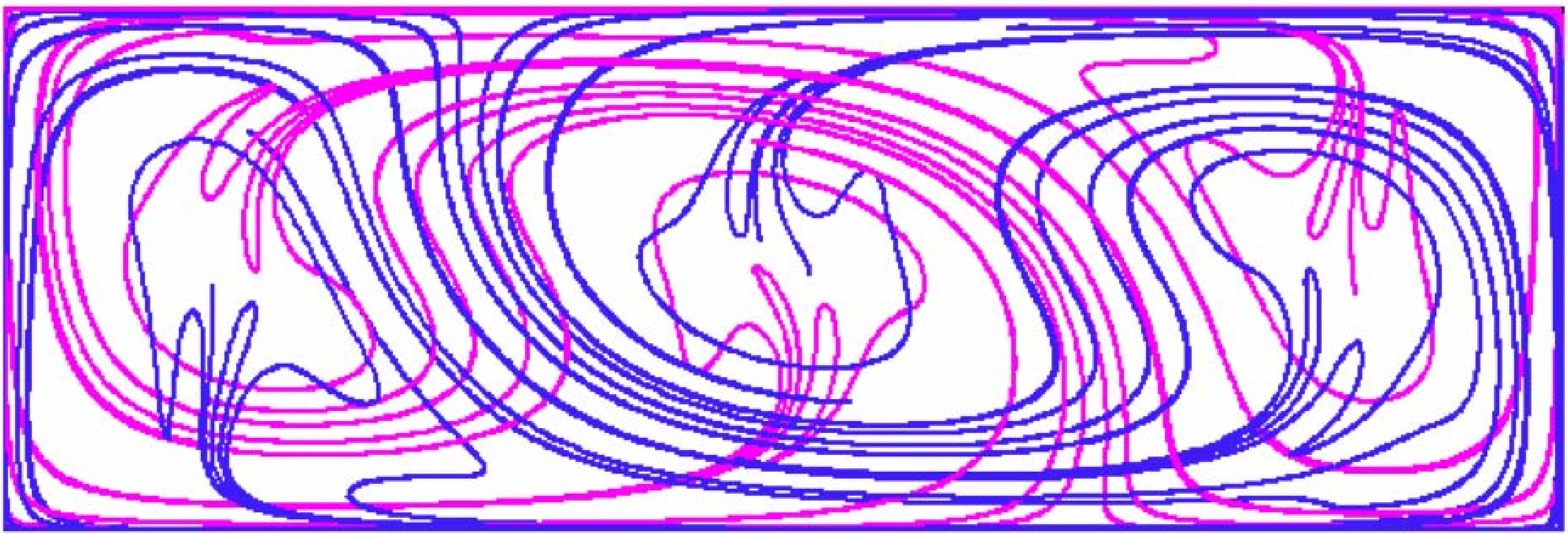
tracer blob

Lid-driven cavity flow

[2] Computations of Mohsen Gheisarieha and Mark Stremler (Virginia Tech)

Lobe dynamics: another fluid example

- Fluid example: time-periodic Stokes flow[2]



some invariant manifolds of saddles

Lid-driven cavity flow

[2] Computations of Mohsen Gheisarieha and Mark Stremler (Virginia Tech)

Lobe dynamics: another fluid example

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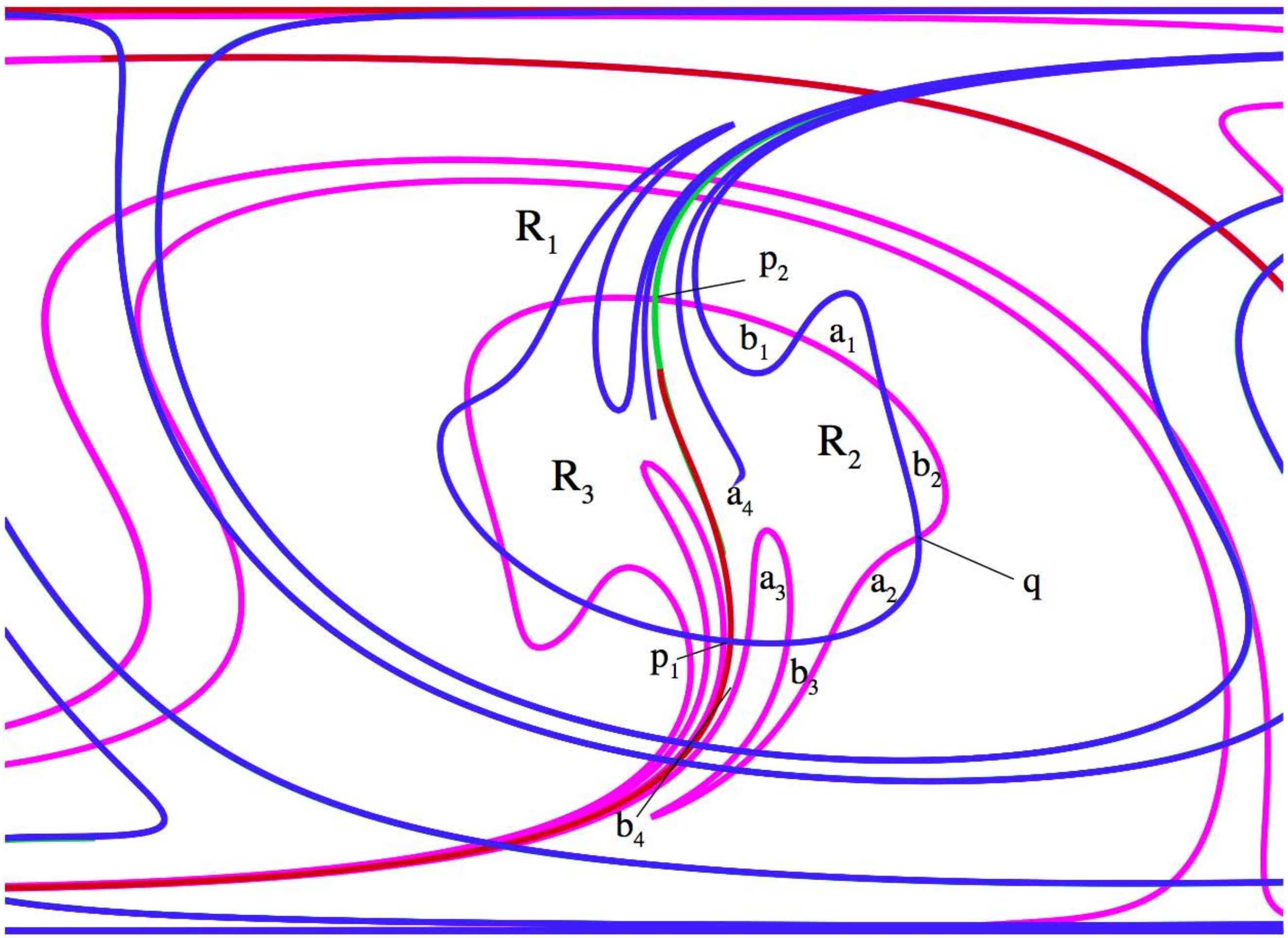
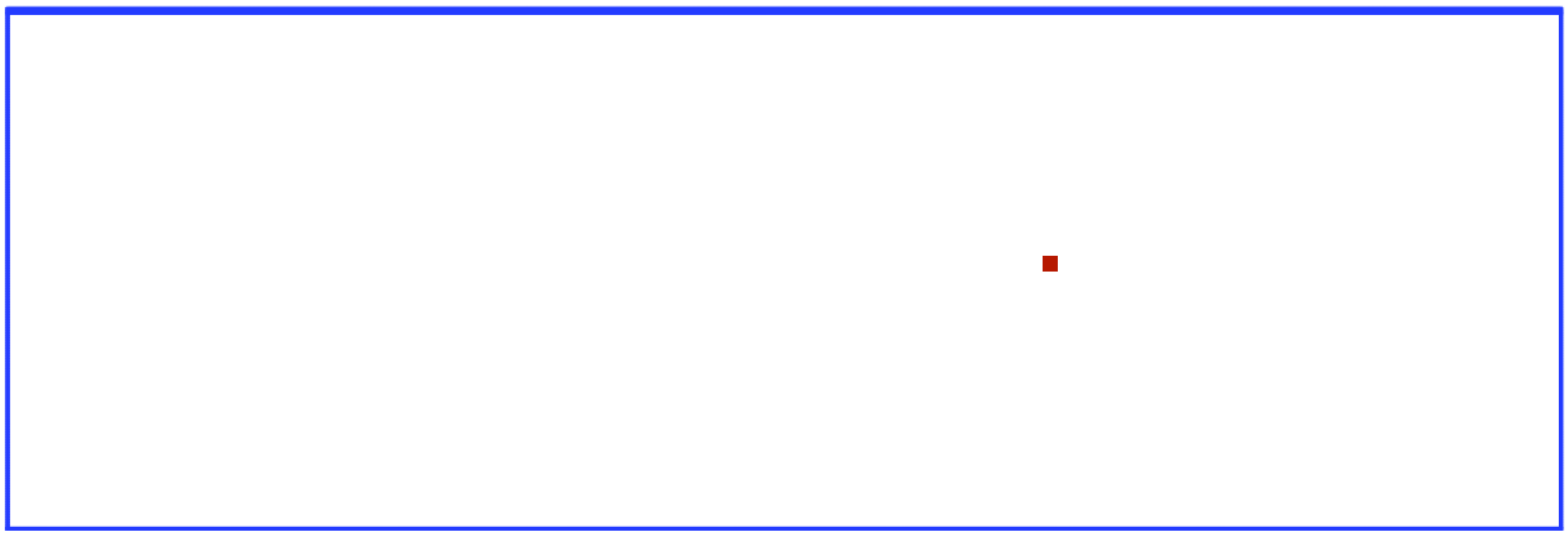


regions and lobes labeled

[2]Computations of Mohsen Gheisarieha and Mark Stremler (Virginia Tech)

Stable/unstable manifolds and lobes in fluids

- Fluid example: time-periodic Stokes flow[2]

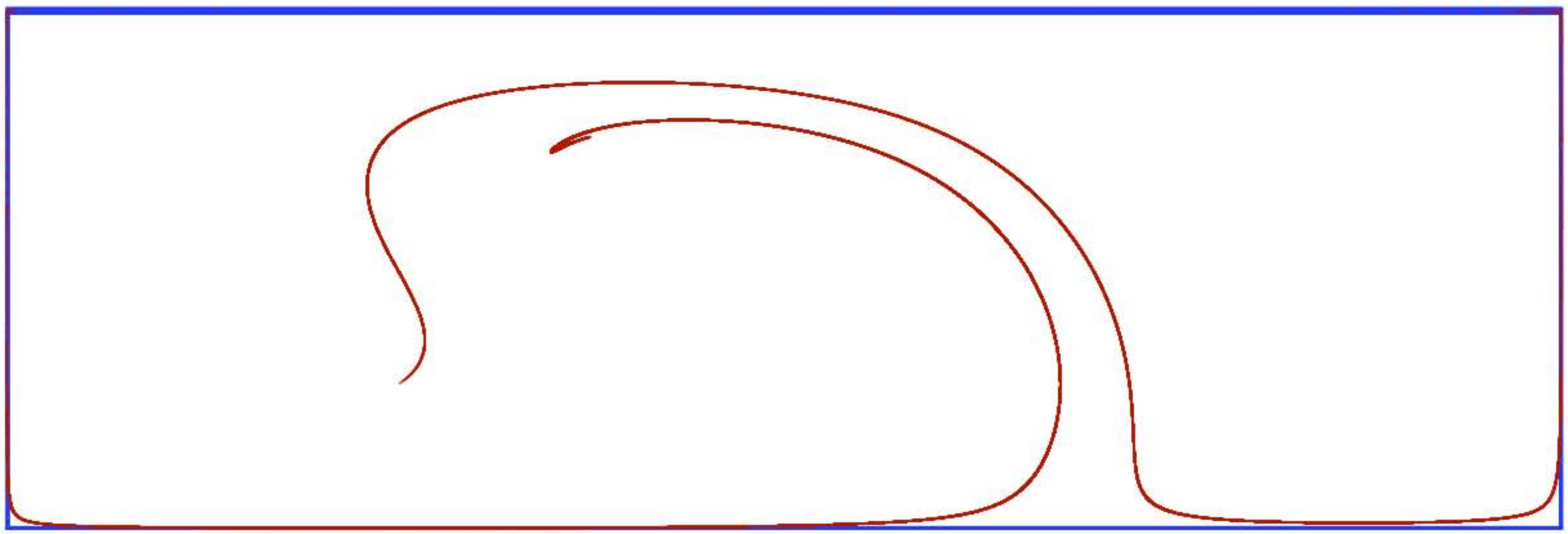


material blob at $t = 0$

[2] Computations of Mohsen Gheisarieha and Mark Stremler (Virginia Tech)

Stable/unstable manifolds and lobes in fluids

- Fluid example: time-periodic Stokes flow[2]

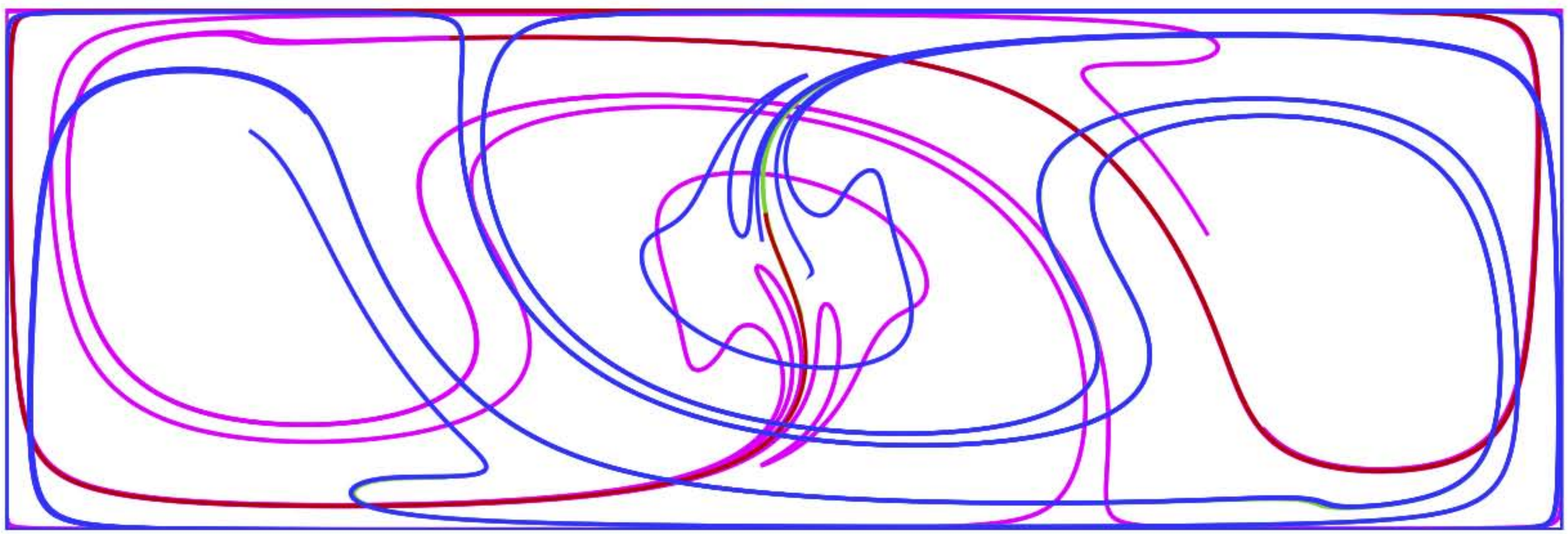


material blob at $t = 5$

[2]Computations of Mohsen Gheisarieha and Mark Stremler (Virginia Tech)

Stable/unstable manifolds and lobes in fluids

- Fluid example: time-periodic Stokes flow[2]

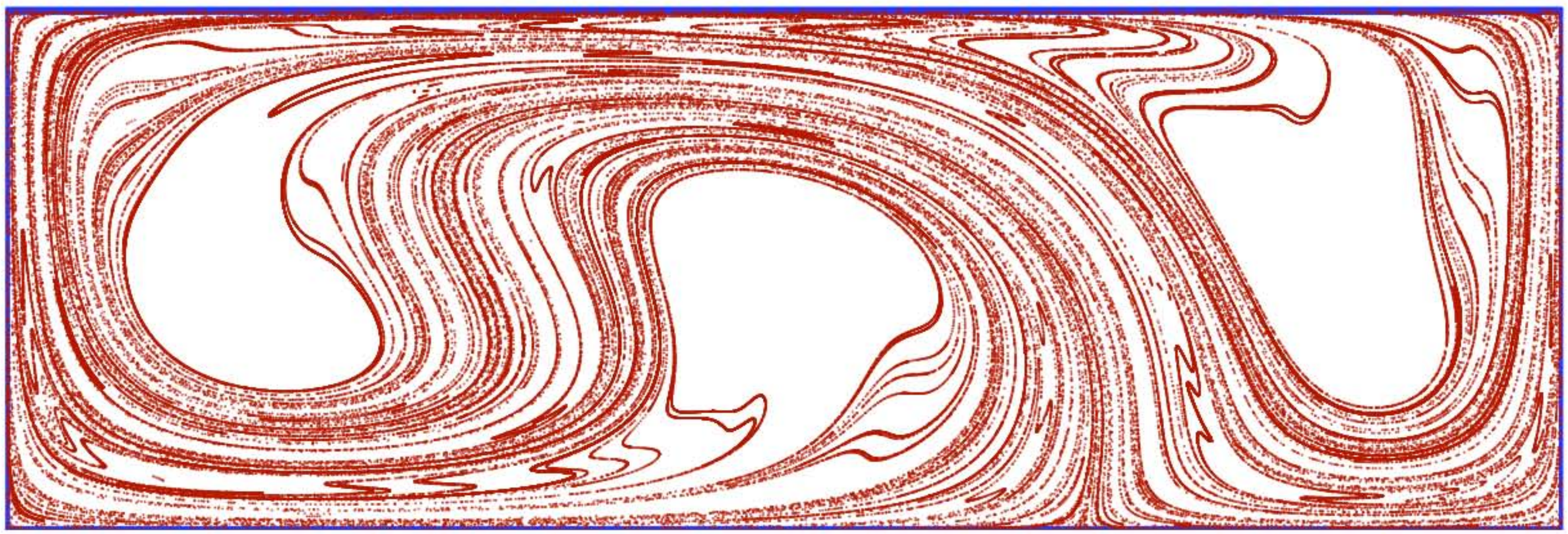


some invariant manifolds of saddles

[2] Computations of Mohsen Gheisarieha and Mark Stremler (Virginia Tech)

Stable/unstable manifolds and lobes in fluids

- Fluid example: time-periodic Stokes flow[2]

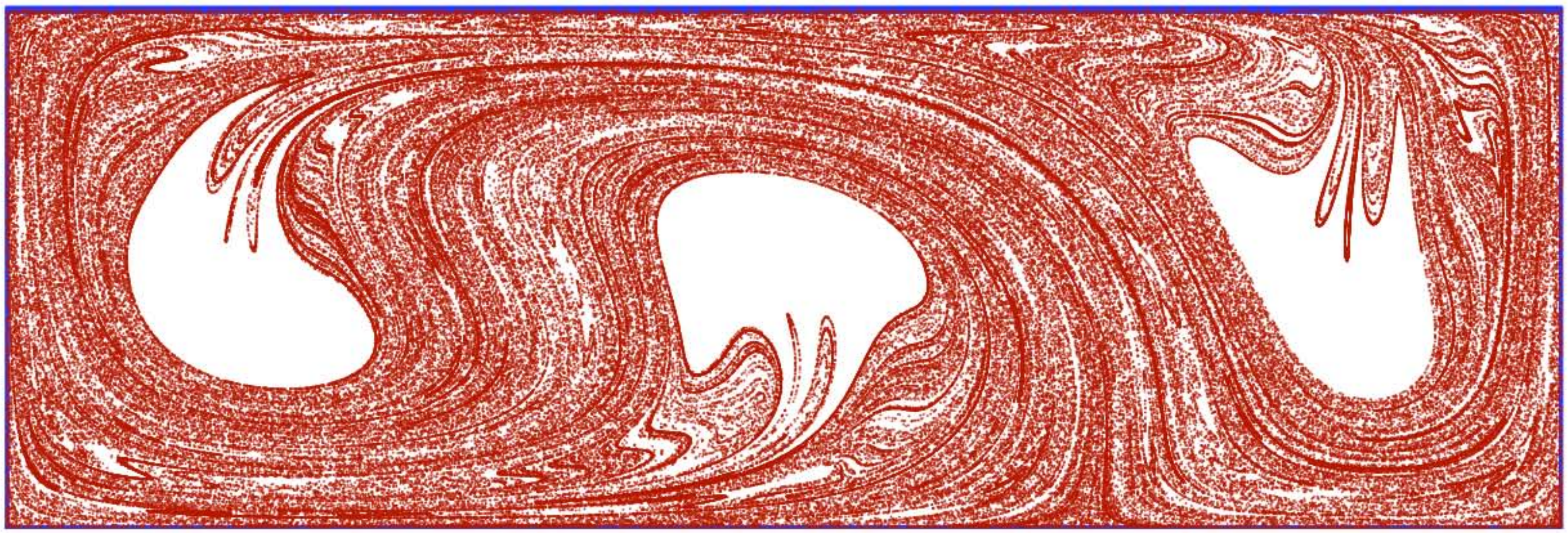


material blob at $t = 10$

[2]Computations of Mohsen Gheisarieha and Mark Stremler (Virginia Tech)

Stable/unstable manifolds and lobes in fluids

- Fluid example: time-periodic Stokes flow[2]

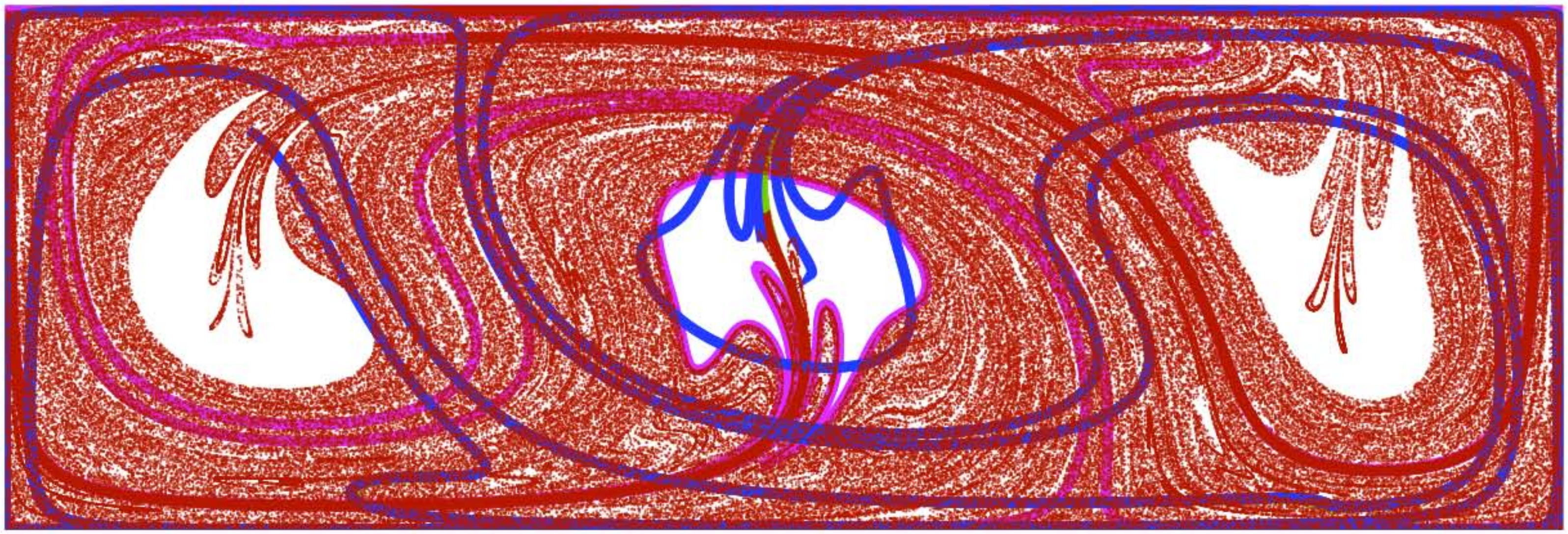


material blob at $t = 15$

[2] Computations of Mohsen Gheisarieha and Mark Stremler (Virginia Tech)

Stable/unstable manifolds and lobes in fluids

- Fluid example: time-periodic Stokes flow[2]

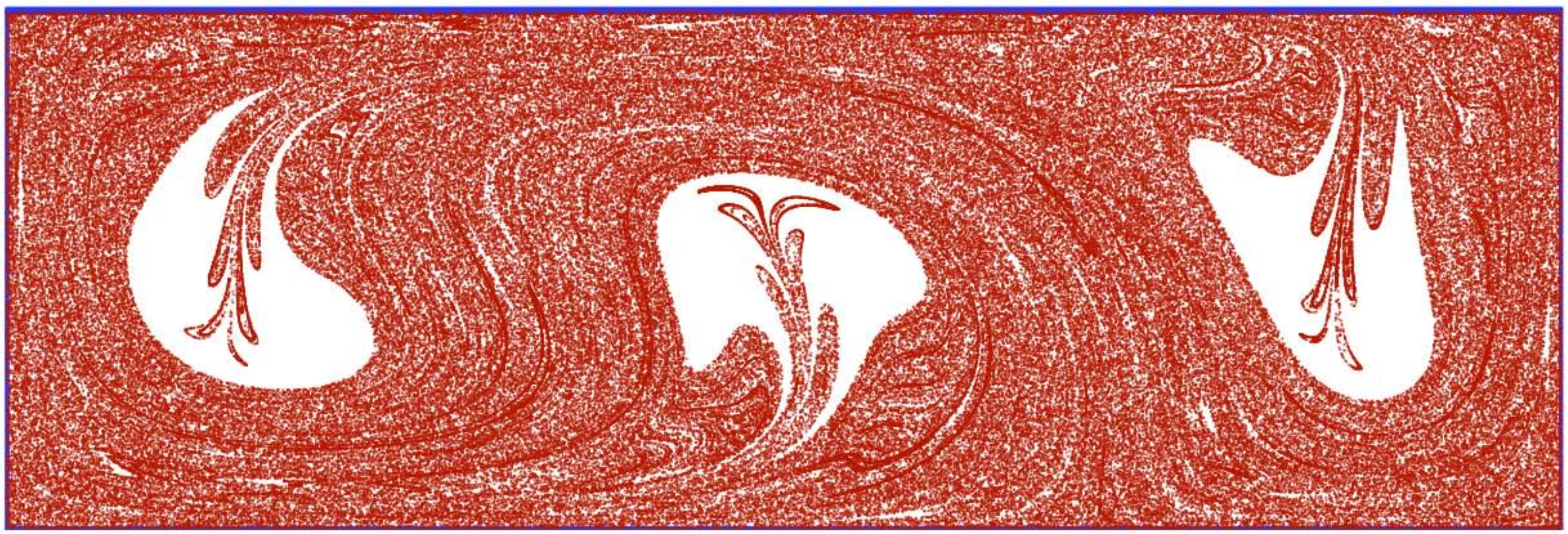


material blob and manifolds

[2]Computations of Mohsen Gheisarieha and Mark Stremler (Virginia Tech)

Stable/unstable manifolds and lobes in fluids

- Fluid example: time-periodic Stokes flow[2]

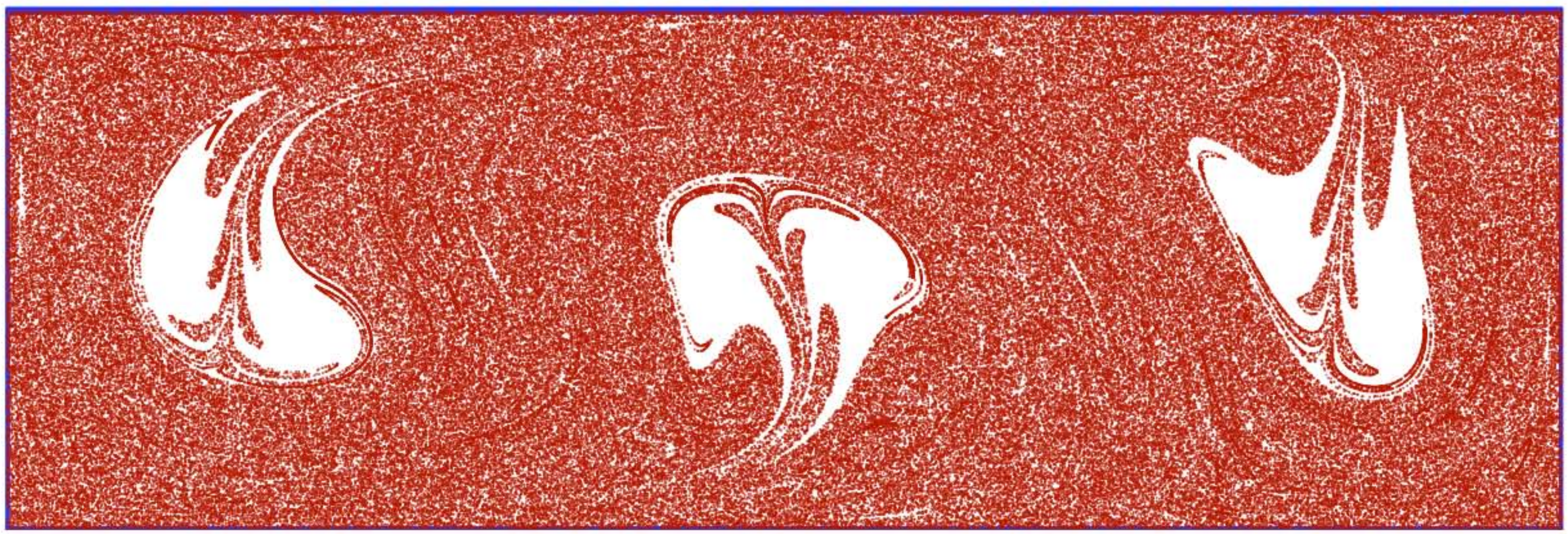


material blob at $t = 20$

[2] Computations of Mohsen Gheisarieha and Mark Stremler (Virginia Tech)

Stable/unstable manifolds and lobes in fluids

- Fluid example: time-periodic Stokes flow[2]

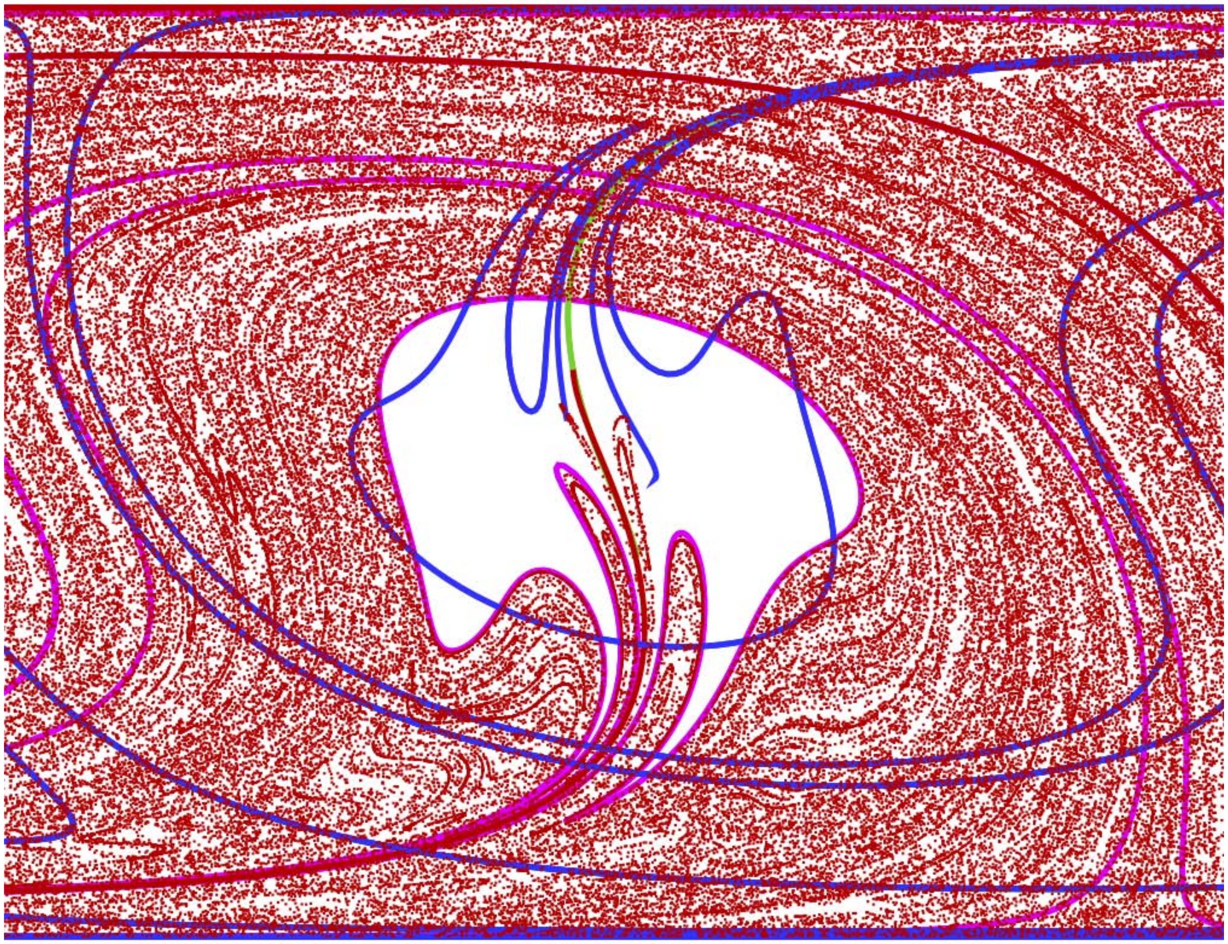


material blob at $t = 25$

[2] Computations of Mohsen Gheisarieha and Mark Stremler (Virginia Tech)

Stable/unstable manifolds and lobes in fluids

- Fluid example: time-periodic Stokes flow[2]

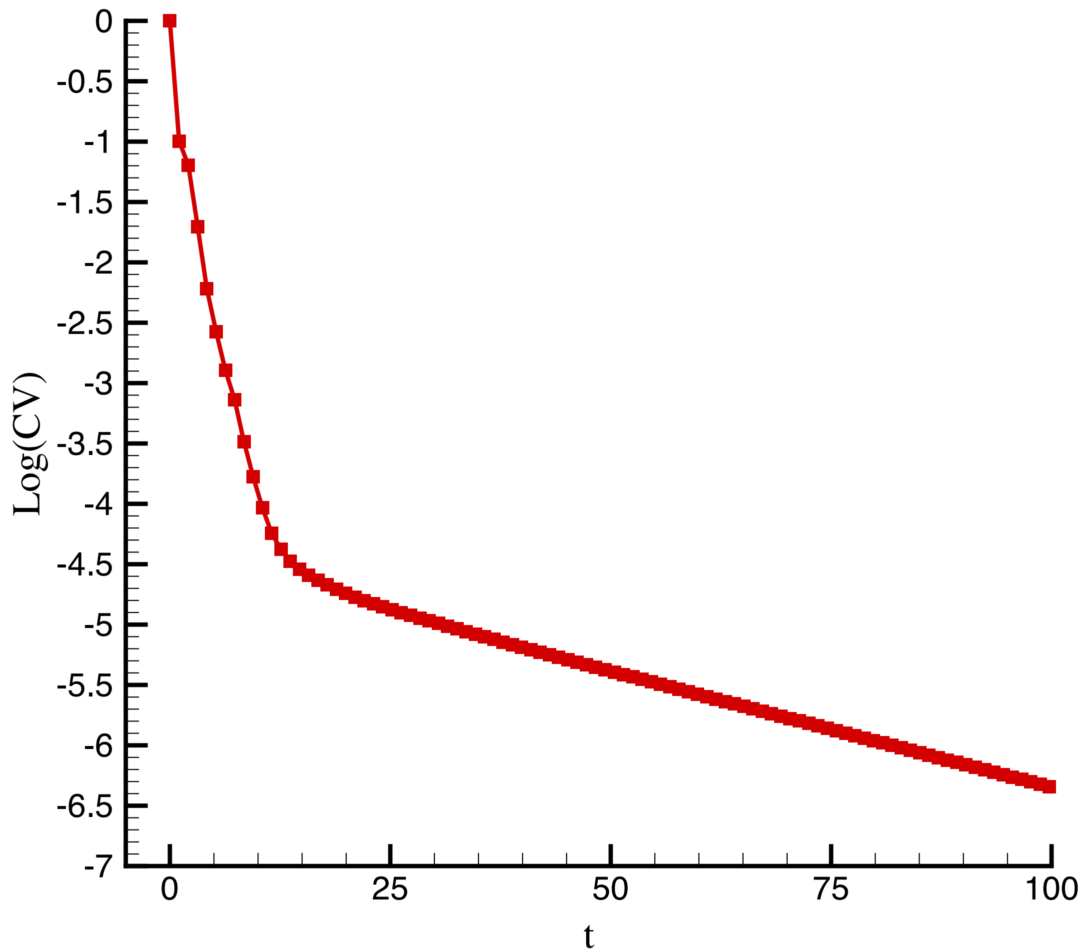


- Saddle manifolds and lobe dynamics provide template for motion

[2] Computations of Mohsen Gheisarieha and Mark Stremler (Virginia Tech)

Stable/unstable manifolds and lobes in fluids

Fluid example: time-periodic Stokes flow[2]



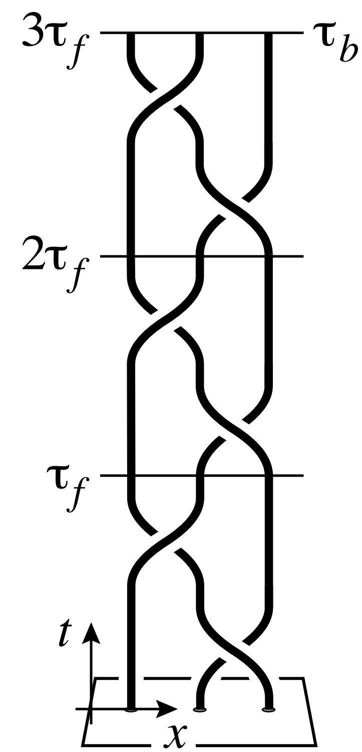
- Homogenization has two exponential rates

[2] Computations of Mohsen Gheisarieha and Mark Stremler (Virginia Tech)

Stable/unstable manifolds and lobes in fluids

Almost-cyclic sets stirring the surrounding fluid like ‘ghost rods’
— provides fast homogenization time scale
— **works even when periodic orbits are absent!**

Stable/unstable manifolds and lobes in fluids

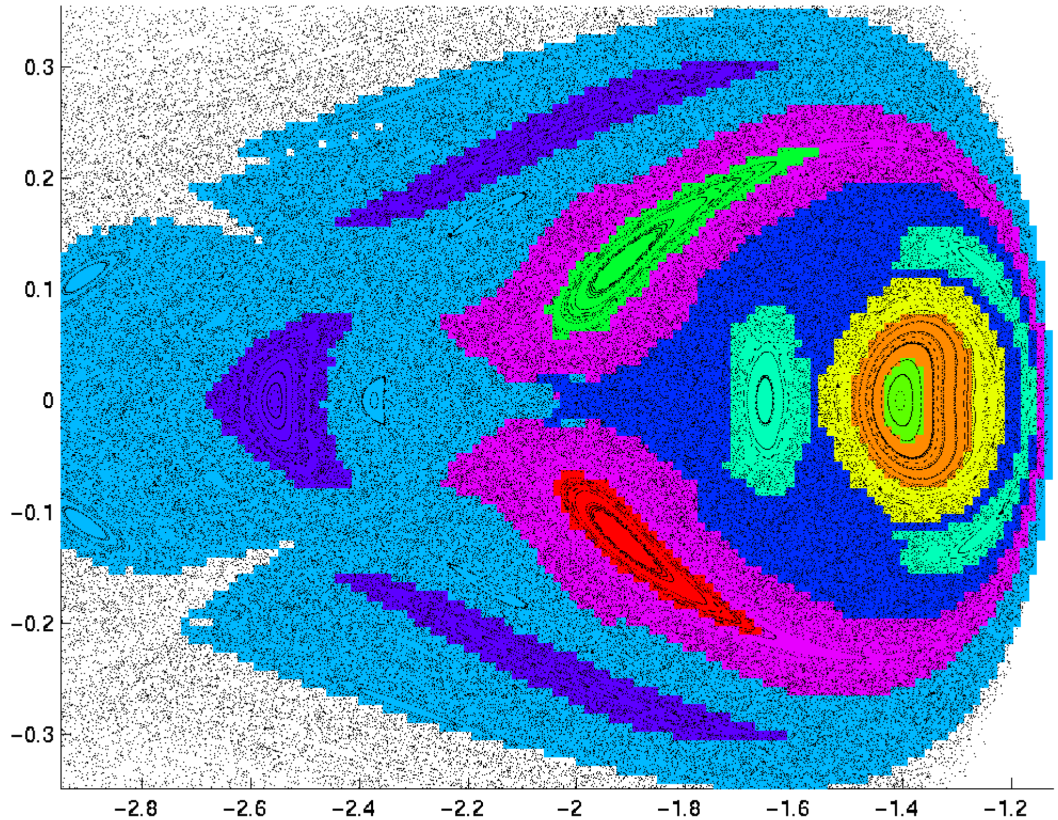


Can use with topological methods to estimate degree of mixing — braid word and Thurston-Nielsen classification theorem

- Stremler, Ross, Grover, Kumar [2011] Phys. Rev. Lett.

Almost-invariant set (AIS) approach

- Partition phase space into **loosely coupled regions**
  "Leaky" regions with a long residence time[3]



The phase space is divided into several invariant and almost-invariant sets.

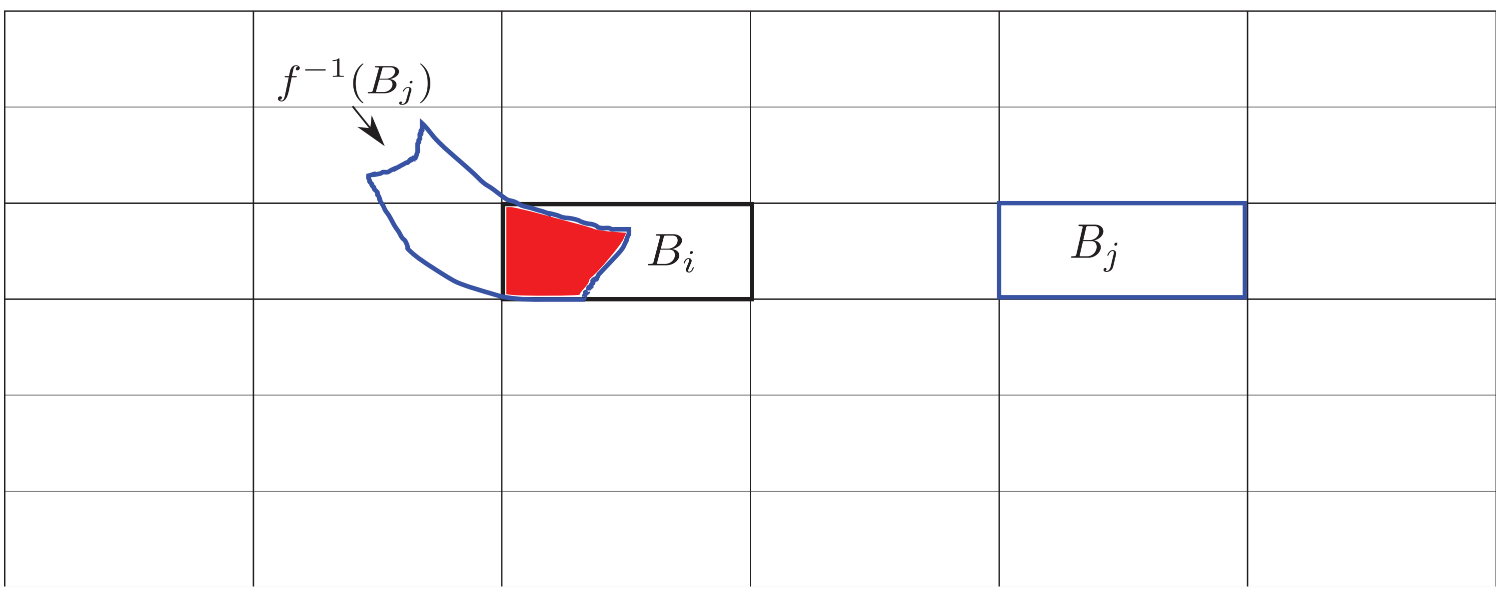
[3] work of Dellnitz, Junge, Froyland, Padberg, et al

Almost-invariant set (AIS) approach

- Create a fine box partition of the phase space $\mathcal{B} = \{B_1, \ldots B_q\}$, where q could be 10^7+
- Consider a (weighted) q-by-q **transition matrix**, P, for our dynamical system, where

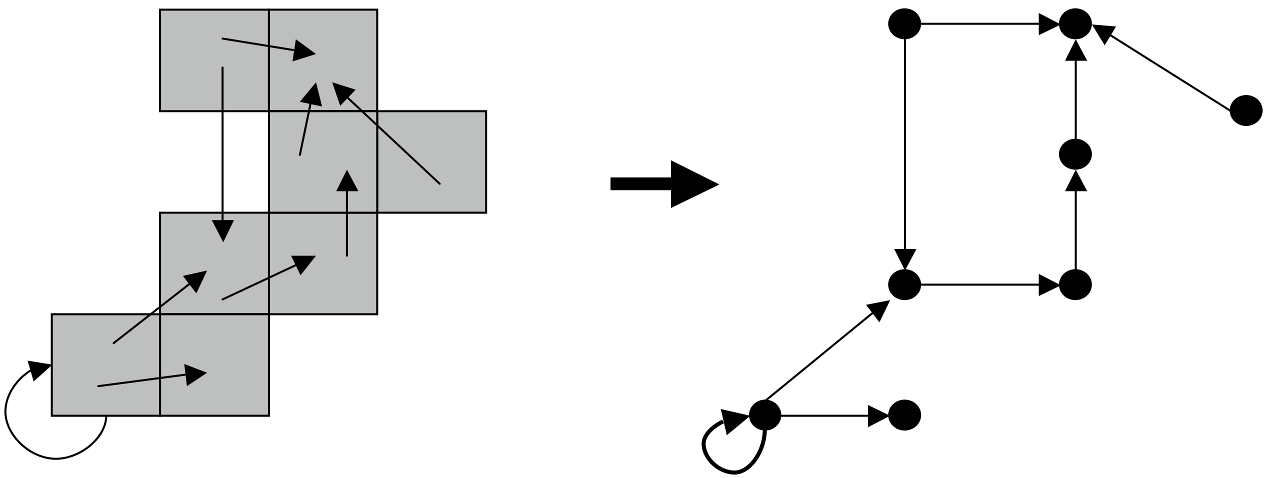
$$P_{ij} = \frac{\mu(B_i \cap f^{-1}(B_j))}{\mu(B_i)},$$

the *transition probability* from B_i to B_j using, e.g., $f = \phi_{t_0}^{t_0+T}$



- P approximates our dynamical system via a finite state Markov chain.

Almost-invariant set (AIS) approach



If $P_{ij} > 0$, then there is an edge between nodes i and j in the graph with weight P_{ij}.

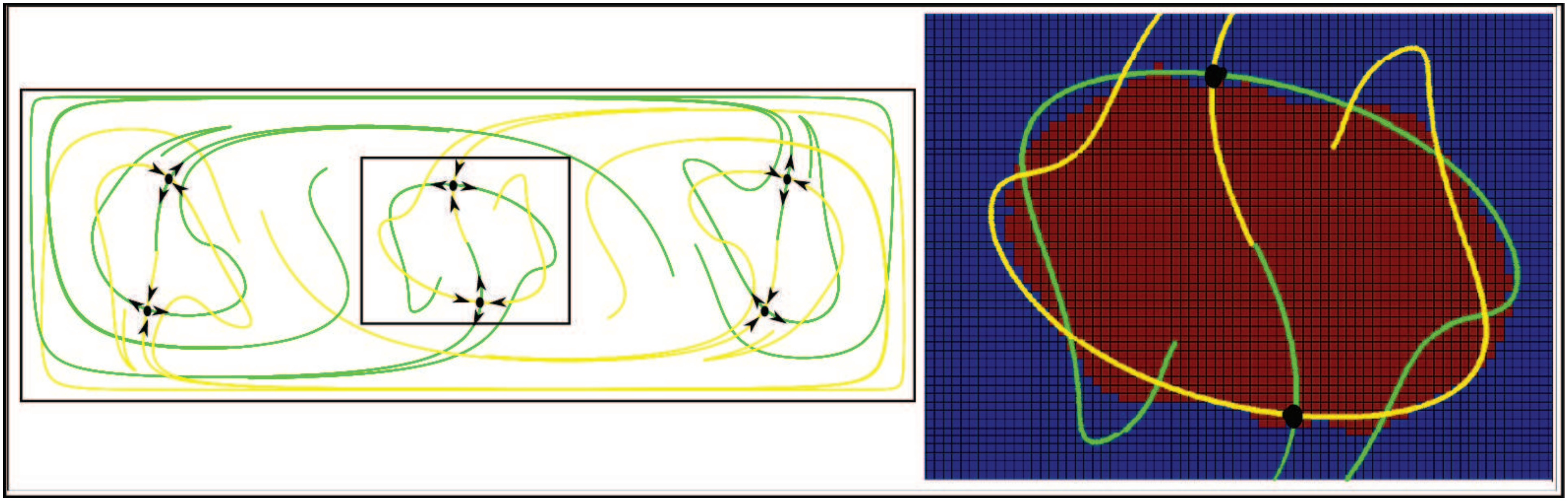
- A set B is called almost invariant over the interval $[t_0, t]$ if

$$\rho_\mu(B) = \frac{\mu(B \cap \phi^{-1}(B))}{\mu(B)} \approx 1. \qquad (1)$$

  Can maximize the value of ρ_μ over all possible combinations of sets $B \in \mathcal{B}$.

- In practice, AIS identified via **eigenvectors** of P or graph-partitioning
- Appropriate for non-autonomous, aperiodic, finite-time settings

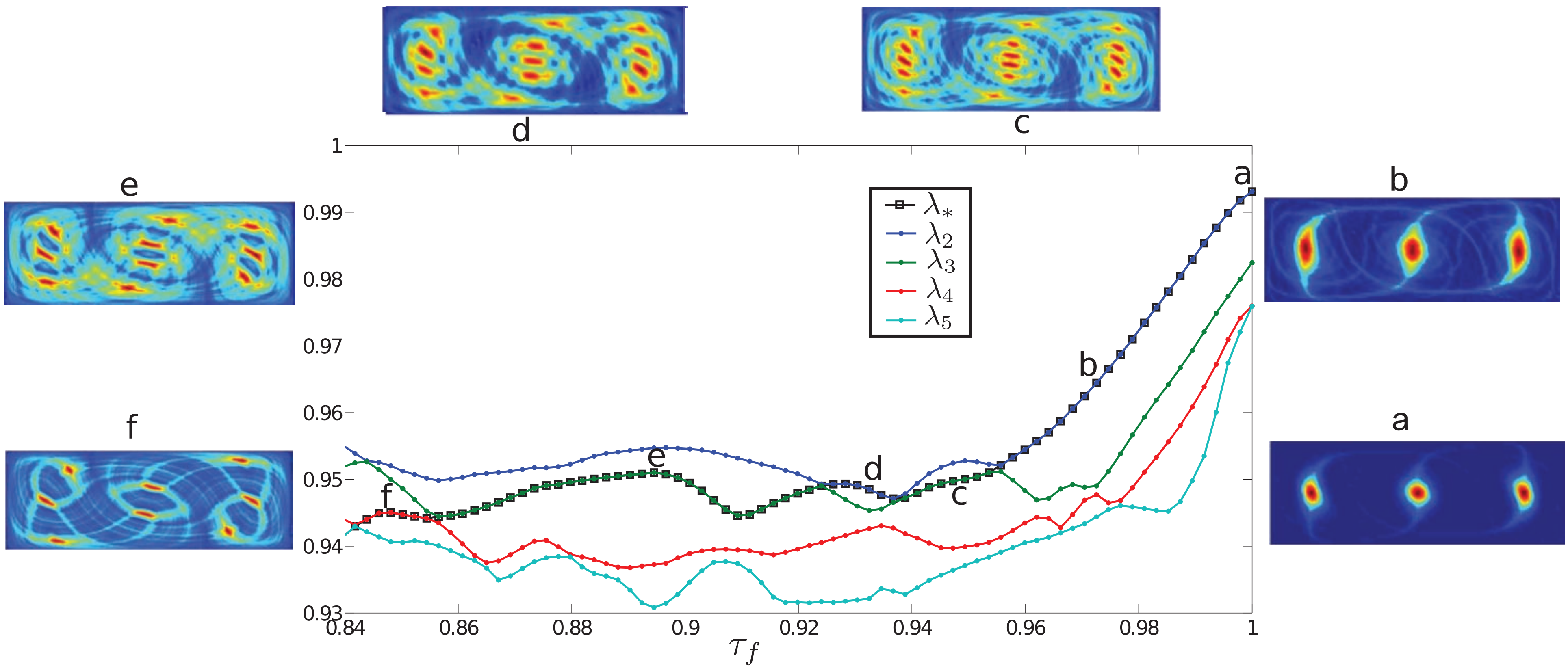
Almost-invariant set (AIS) approach



- Link between AIS boundaries and invariant manifolds of fixed points, and more generally, of normally hyperbolic invariant manifolds (NHIMs)[4]

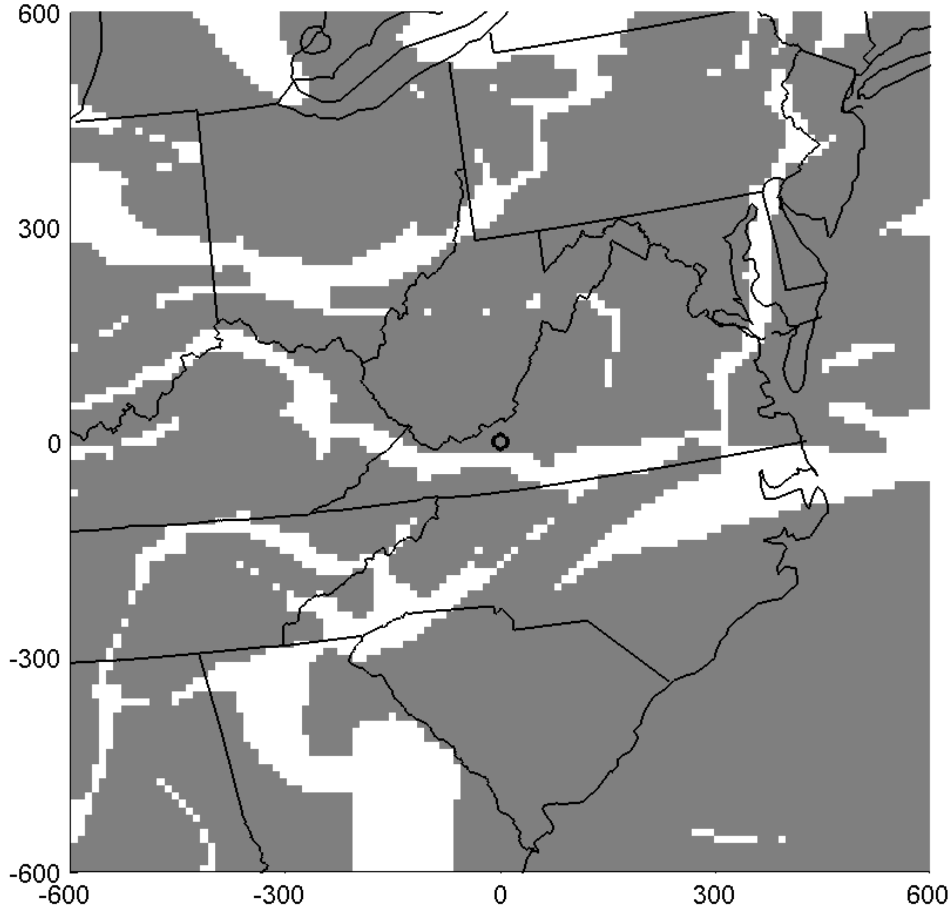
[4]Similar to link between AIS and invariant manifolds: Dellnitz, Junge, Lo, Marsden, Padberg, Preis, Ross, Thiere [2005] Phys. Rev. Lett.; Dellnitz, Junge, Koon, Lekien, Lo, Marsden, Padberg, Preis, Ross, Thiere [2005] Int. J. Bif. Chaos

Almost-invariant set (AIS) approach



- See Piyush Grover's poster for more

Almost-invariant set (AIS) approach



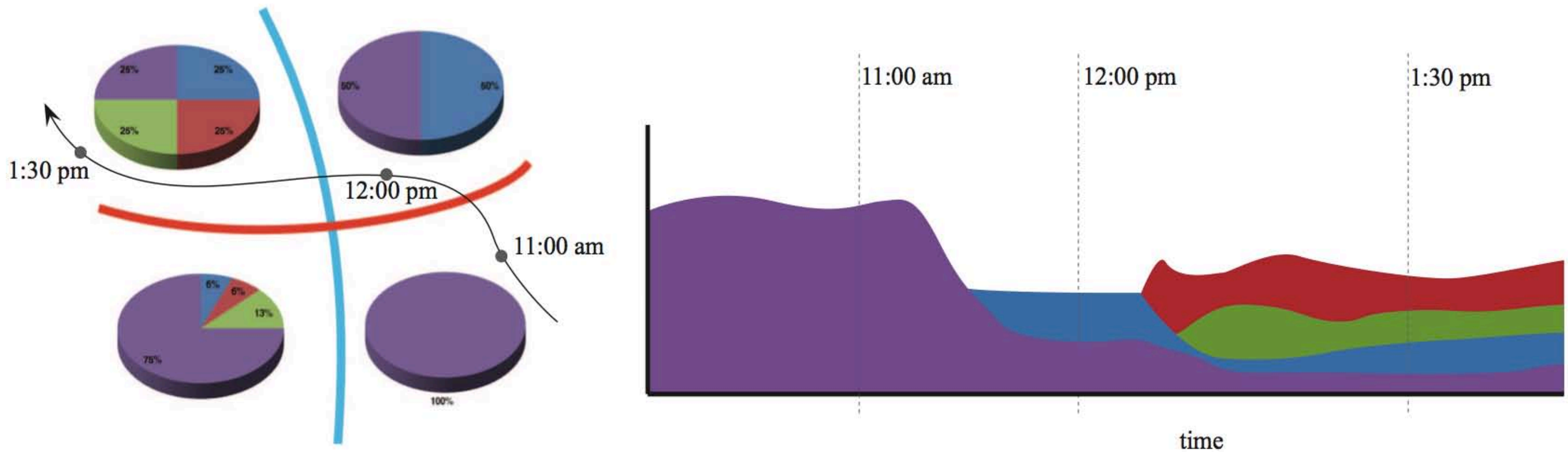
- Atmosphere: maximally coherent sets over 24 hours — boundaries seem to coincide with LCSs

Final words on chaotic transport

- What are the robust descriptions of transport which work in aperiodic, finite-time settings?
- Lobe dynamics, finite-time symbolic dynamics may work.
- Is there a generalization of Melnikov's method which would work for LCSs to establish homoclinic and heteroclinic-like tangles?
- Links between LCS, AIS/coherent sets, and topological methods?

Final words on the biological side

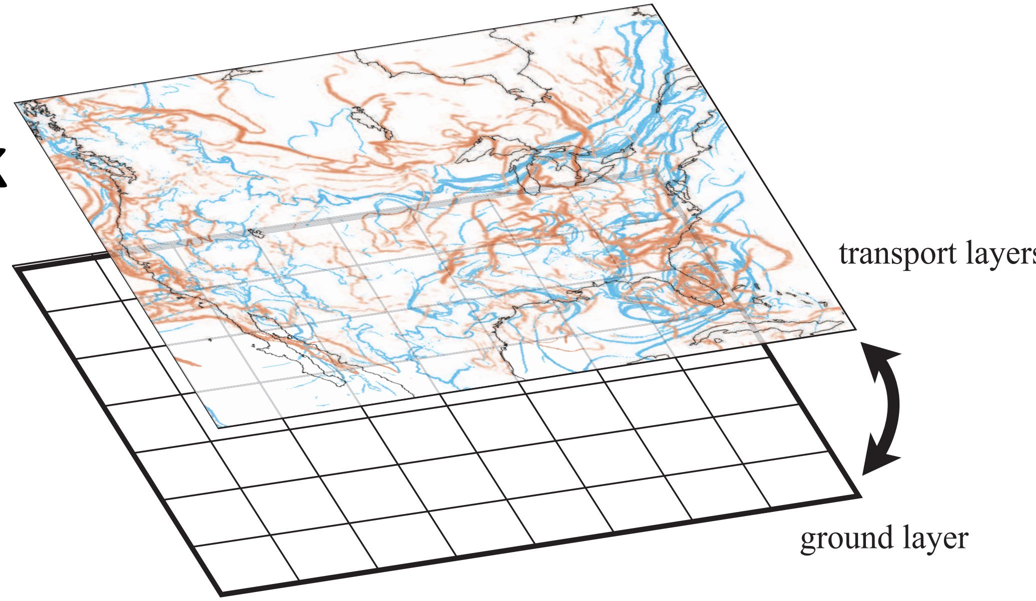
- **Airborne collections may encode recent atmospheric mixing events**

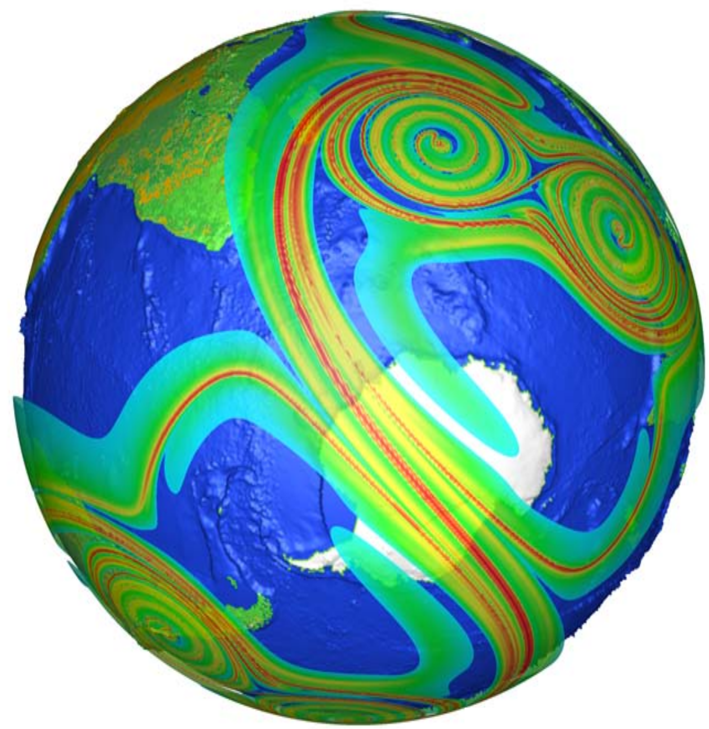


- **LCSs may reveal important transport / invasion events which are *unrelated* to obvious weather phenomena like storms or fronts**
- **What are the effects of climate change?**

Final words on the biological side

- **Incorporate transport network into framework for modeling transport of biota**
- **With management, this becomes a *complex coupled natural and human system***
- **Have organisms evolved to take advantage of the *global* transport network?**





The End

For papers, movies, etc., visit:
www.shaneross.com

Main Papers:

- Schmale, Ross, Fetters, Tallapragada, Wood-Jones, Dingus [2011] Isolates of *Fusarium graminearum* collected 40-320 meters above ground level cause Fusarium head blight in wheat and produce trichothecene mycotoxins. *Aerobiologia*, published online.
- Stremler, Ross, Grover, Kumar [2011] Topological chaos and periodic braiding of almost-cyclic sets. *Physical Review Letters* 106, 114101.
- Senatore & Ross [2011] Detection and characterization of transport barriers in complex flows via ridge extraction of the finite time Lyapunov exponent field, *International Journal for Numerical Methods in Engineering* 86, 1163.
- Lekien & Ross [2010] The computation of finite-time Lyapunov exponents on unstructured meshes and for non-Euclidean manifolds. *Chaos* 20, 017505.
- Tallapragada & Ross [2008] Particle segregation by Stokes number for small neutrally buoyant spheres in a fluid, *Physical Review E* 78, 036308.