FUEL-OPTIMAL TRAJECTORIES IN A PLANET-MOON ENVIRONMENT USING MULTIPLE GRAVITY ASSISTS

Shane D. Ross and Piyush Grover
Engineering Science and Mechanics
Virginia Polytechnic Institute and State University
Mail Code 0219, Blacksburg, VA, 24061, USA
E-mail: {`sdross,groverp`}`@vt.edu`

Abstract

For low energy spacecraft trajectories such as multi-moon orbiters for the Jupiter system, multiple gravity assists by moons could be used in conjunction with ballistic capture to drastically decrease fuel usage. In this paper, we outline a procedure to obtain a family of zero-fuel multi-moon orbiter trajectories, using a family of Keplerian maps derived by the first author previously. The maps capture well the dynamics of the full equations of motion; the phase space contains a connected chaotic zone where intersections between unstable resonant orbit manifolds provide the template for lanes of fast migration between orbits of different semimajor axes. Patched three body approach is used and the four body problem is broken down into two three-body problems, and the search space is considerably reduced by the use of properties of the Keplerian maps. We also introduce the notion of 'Switching Region' where the perturbations due to the two perturbing moons are of comparable strength, and which separates the domains of applicability of the corresponding two Keplerian maps.

1   Introduction

For low energy spacecraft trajectories such as multi-moon orbiters for the Jupiter system, multiple gravity assists by moons could be used in conjunction with ballistic capture to drastically decrease fuel usage.[1–3] These phenomena have been explained by applying techniques from dynamical systems theory to systems of n bodies considered three at a time.[4–8] One can design trajectories with a predetermined future and past, in terms of transfer from one Hill's region to another. One of the examples we have considered is an extension of the Europa Orbiter mission[9–11] to include an orbit around Ganymede.[12] More recently, we

have considered a mission in which a single spacecraft orbits *three* of Jupiter's planet-size moons—Callisto, Ganymede and Europa—one after the other, using very little fuel.[1] Using this approach, which has been dubbed the "Multi-Moon Orbiter" (MMO), a scientific spacecraft can orbit several moons for any desired duration, instead of flybys lasting only seconds. Our approach should work well with existing techniques, enhancing interplanetary trajectory design capabilities for aggressive missions in planet-moon environments.

The main concern of this study is not to construct flight-ready end-to-end trajectories, but rather to use simpler methods to determine families of multi-moon orbiter trajectories. Such information can supply, for example, the fuel consumption versus time of flight trade-off for a MMO mission. The fuel requirements in terms of the sum of all velocity changes (ΔV) are greatly reduced by including multiple gravity assist (GA) maneuvers with the jovian moons. For instance, by using multiple GAs, we have found tours with a deterministic ΔV as low as $\sim$20 m/s as compared to $\sim$1500 m/s using previous methods.[9,10] In fact, this extremely low ΔV is on the order of statistical navigation errors. By using small impulsive maneuvers totaling only 22 m/s, a spacecraft initially injected into a jovian orbit can be directed into an inclined, elliptical capture orbit around Europa. Enroute, the spacecraft orbits both Callisto and Ganymede for long duration using a ballistic capture and escape methodology developed previously.[12] This way of designing missions is called the *patched three-body approximation* (P3BA) and will be elaborated upon further in this paper.

We introduce the "switching region," the P3BA analogue to the "sphere of influence." Numerical results are given for the problem of finding a trajectory from an initial region of phase space (escape orbits from moon A) to a target region (orbits captured by moon B) using no control. This can be modified to accommodate small or large controls.

We use a family of symplectic twist maps to approximate a particle's motion in the planar circular restricted three-body problem, derived in recent work[13]. The maps capture well the dynamics of the full equations of motion; the phase space contains a connected chaotic zone where intersections between unstable resonant orbit manifolds provide the template for lanes of fast migration between orbits of different semimajor axes.

2 The Keplerian or periapsis Poincaré map

Each map, which we call the *periapsis Poincaré map* (or *Keplerian map*), is an update map for the angle of periapse ω in the rotating frame and Keplerian energy K, $(\omega_n, K_n) \mapsto (\omega_{n+1}, K_{n+1})$. The map has the form

$$\begin{pmatrix} \omega_{n+1} \\ K_{n+1} \end{pmatrix} = \begin{pmatrix} \omega_n - 2\pi(-2K_{n+1})^{-3/2} \\ K_n + \mu f(\omega_n; C_J, \bar{K}) \end{pmatrix} \tag{1}$$

i.e., a map of the cylinder $\mathcal{A} = S^1 \times \mathbb{R}$ onto itself.

The map models a spacecraft on a near-Keplerian orbit about a central body of unit mass, where the spacecraft is perturbed by a smaller body of mass μ. The interaction of the spacecraft with the perturber is modeled as an impulsive kick at periapsis passage, encap-

sulated in the kick function f, see Figure 1(a), where $(\mu, C_J, \bar{K})$ are considered bifurcation parameters.

The map captures well the dynamics of the full equations of motion; namely, the phase space, shown in Figure 1(b), is densely covered by chains of stable resonant islands, in between which is a connected chaotic zone. The more physically intuitive semimajor axis a is plotted for the vertical axis instead of Keplerian energy K, where $a = -1/(2K)$. The kick function obtained from the map shows that the biggest kicks are recieved for a very narrow range of periapse values. If the periapse occurs slightly ahead of the perturber, the particle is gets a negative 'a' kick, and if the periapse is slightly behind the perturber, the kick is positive. Using similar methods as above, we can construct an apoapse kick map for the case when the spacecraft is in the interior realm, i.e. when its semi-major axis is less than that of the perturber. In case of an apoapse kick map, to recieve the positive 'a' kick, the apoapse needs to be slightly ahead of the perturber (or the periapse needs to be slightly less than π). Similarly, for a negative 'a' kick, the apoapse needs to be slightly behind the perturber (or the periapse needs to be slightly more than $-\pi$).
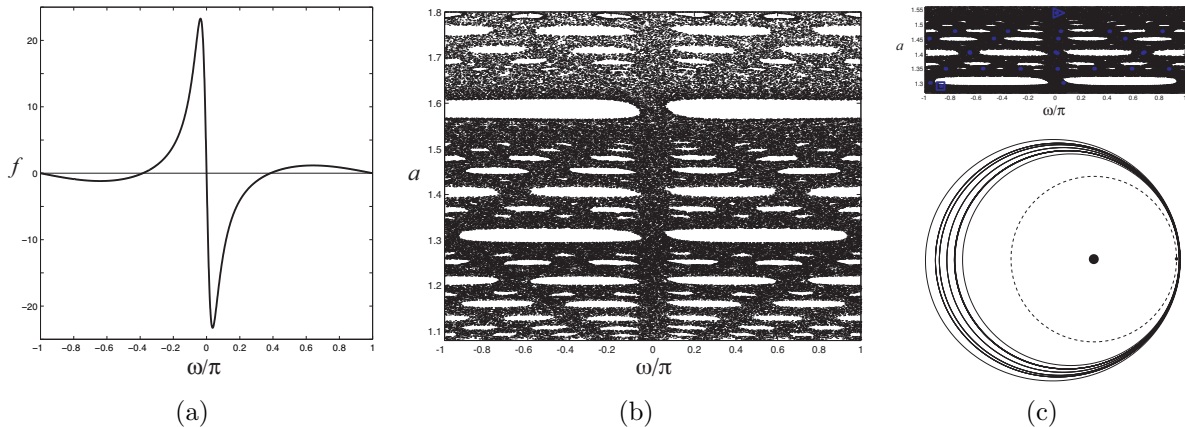

(a)   (b)   (c)

Figure 1: (a) The energy kick function f vs. ω for typical values of the parameters. (b) The connected chaotic sea in the phase space of the Keplerian map. The semimajor axis $a[= -1/(2K)]$ vs. the angle of periapsis ω is shown for parameters $\mu = 5.667 \times 10^{-5}$, $C_J = 2.995$, $\bar{a} = -1/(2\bar{K}) = 1.35$ appropriate for a spacecraft in the Jupiter-Callisto system. The initial conditions were taken initially in the chaotic sea and followed for 10^4 iterates, thus producing the 'swiss cheese' appearance where holes corresponding to stable resonant islands reside. (c) Upper panel: a phase space trajectory where the initial point is marked with a triangle and the final point with a square. Lower panel: the configuration space projections in an inertial frame for this trajectory. Jupiter and Callisto are shown at their initial positions, and Callisto's orbit is dashed. The uncontrolled spacecraft migration is from larger to smaller semimajor axes, keeping the periapsis direction roughly constant in inertial space. Both the spacecraft and Callisto orbit Jupiter in a counter-clockwise sense.

The engineering application envisioned for the map is to the design of low energy trajectories, specifically between moons in the Jupiter moon system. Multiple gravity assists

are a key physical mechanism which could be exploited in future scientific missions ([1]). For example, a trajectory sent from Earth to the Jovian system, just grazing the orbit of the outermost icy moon Callisto, can migrate using little or no fuel from orbits with large apoapses to smaller ones. This is shown in Figure 1(c) in both the phase space and the inertial configuration space. From orbits slightly larger than Callisto's, the spacecraft can be captured into an orbit around the moon. The set of all capture orbits is a solid cylindrical tube in the phase space, as shown in Figure 2(a) (for details of the tube computation, see, e.g.,[6]). Followed backward in time this solid tube intersects transversally our Keplerian map, interpreted as a Poincaré surface-of-section. The resulting elliptical region, Figure 2(b), is an *exit* from jovicentric orbits exterior to Callisto. It is the first backward Poincaré cut of the solid tube of capture orbits

The advantage of considering an analytical two-dimensional map as opposed to full numerical integration of the restricted three-body equations of motion is that we can apply all the theoretical and computational machinery applicable to phase space transport in symplectic twist maps[14]. For example, previous work on twist maps can be applied, revealing the existence of lanes of fast migration between orbits of different semimajor axes. These lanes can be used by a spacecraft sent from Earth to the Jovian system. A spacecraft whose trajectory just grazes the orbit of the outermost icy moon Callisto can migrate using little or no fuel from orbits with large apoapses to smaller ones.

# 3 A method for determining inter-moon transfers

In order to make the search through the space of inter-moon transfers computationally tractable, one needs to use simplified models. The forward-backward method in the restricted three-body problem phase space is used.[12,15] The influence of only one moon at a time is considered. First, we review the P3BA and the dynamics in the circular restricted three-body problem.

**The Patched Three-Body Approximation (P3BA)**

The P3BA discussed by Ross et al.[1] considers the motion of a particle (or spacecraft, if controls are permitted) in the field of n bodies, considered two at a time, e.g., Jupiter and its ith moon, M_i. When the trajectory of a spacecraft comes close to the orbit of M_i, the perturbation of the spacecraft's motion away from purely Keplerian motion about Jupiter is dominated by M_i. In this situation, we say that the spacecraft's motion is well modeled by the Jupiter-M_i-spacecraft restricted three-body problem. Within the three-body problem, we can take advantage of phase space structures such as tubes of capture and escape, as well as lobes associated with movement between orbital resonances. Both tubes and lobes, and the dynamics associate with them, are important for the design of a MMO trajectory.

The design of a MMO of the jovian system is guided by three main ideas.[1,12]

1. The motion of the spacecraft in the gravitational field of the three bodies Jupiter, Ganymede, and Europa is approximated by two segments of purely three body motion
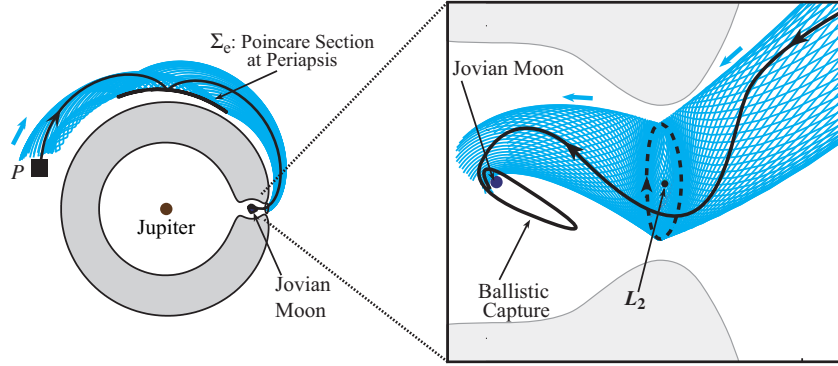
(a)
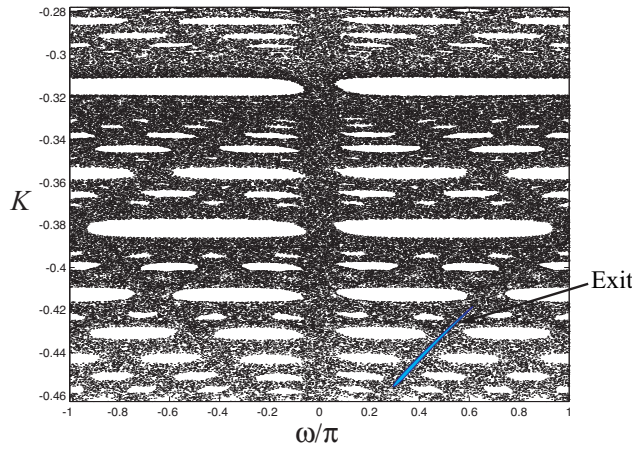
(b)

Figure 2: (a) A spacecraft P inside a tube of gravitational capture orbits will find itself going from an orbit about Jupiter to an orbit about a moon. The spacecraft is initially inside a tube whose boundary is the stable invariant manifold of a periodic orbit about L_2. The three-dimensional tube, made up of individual trajectories, is shown as projected onto configuration space. Also shown is the final intersection of the tube with Σ_e, a Poincaré map at periapsis in the exterior realm. (b) The numerically computed location of an exit on Σ_e, with the same map parameters as before. Spacecraft which reach the exit will subsequently enter the phase space realm around the perturbing moon. The vertical axis is the Keplerian energy K of the instantaneous conic orbit about Jupiter.

in the circular, restricted three-body model. The trajectory segment in the first three body system, Jupiter-Ganymede-spacecraft, is appropriately patched to the segment in the Jupiter-Europa-spacecraft three-body system.

2. For each segment of purely three body motion, the invariant manifolds tubes of L_1 and L_2 bound orbits (including periodic orbits) leading toward or away from temporary capture around a moon, are used to construct an orbit with the desired behaviors.

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Portions of these tubes are "carried" by the lobes mediating movement between orbital resonances. Directed movement between orbital resonances is what allows a spacecraft to achieve large changes in its orbit. When the spacecraft's motion, as modeled in one three-body system, reaches an orbit whereby it can switch to another three-body system, we switch or "patch" the three-body model to the new system.

3. This initial guess solution is then refined to obtain a trajectory in a more accurate four-body model. Evidence suggests that these initial guesses are very good,[1] even in the full n-body model and considering the orbital eccentricity of the moons.[16]

Tube Dynamics: Ballistic Capture and Escape

The tubes referred to above are cylindrical stable and unstable invariant manifolds associated to bounded orbits around L_1 and L_2. They are the phase space structures that mediate motion to and from the smaller primary body, e.g., mediating spacecraft motion to and from Europa in the Jupiter-Europa-spacecraft system. They also mediate motion between primary bodies for separate three-body systems, e.g., spacecraft motion between Europa and Ganymede in the Jupiter-Europa-spacecraft and the Jupiter-Ganymede-spacecraft systems. Details are discussed extensively in Koon et al.[6,7] and Gómez et al.[17]

Inter-Moon Transfer and Switching Region

During the inter-moon transfer—where one wants to leave a moon and transfer to another moon, closer in to Jupiter—we consider the transfer in two portions, shown schematically in Figure 3, with M_1 as the outer moon and M_2 as the inner moon. In the first portion, the transfer determination problem becomes one of finding an appropriate solution of the Jupiter-M_1-spacecraft problem which jumps between orbital resonances with M_1, i.e., performing resonant GA's to decrease the perijove.[1] M_1's perturbation is only significant over a small portion of the spacecraft trajectory near apojove (A in Figure 3(a)). The effect of M_1 is to impart an impulse to the spacecraft, equivalent to a ΔV in the absence of M_1.

The perijove is decreased until it has a value close to M_2's orbit, in fact, close to the orbit of M_2's L_2. We can then assume that a GA can be achieved with M_2 with an appropriate geometry such that M_2 becomes the dominant perturber and all subsequent GA's will be with M_2 only. When a particular resonance is reached, the spacecraft can then be ballistically captured by the inner moon.[6]We note that a similar phenomenon has been observed in previous studies of Earth to lunar transfer trajectories.[15,18]

The arc of the spacecraft's trajectory at which the spacecraft's perturbation switches from being dominated by moon M_1 to being dominated by M_2 is called the "switching orbit." A rocket burn maneuver need not be necessary to effect this switch. The set of possible switching orbits is the "switching region" of the P3BA. It is the analogue of the "sphere of influence" concept used in the patched-conic approximation, which guides a mission designer regarding when to switch the central body for the model of the spacecraft's Keplerian motion. The task of searching for trajectories that go from near-Ganymede to near-Europa Jovicentric
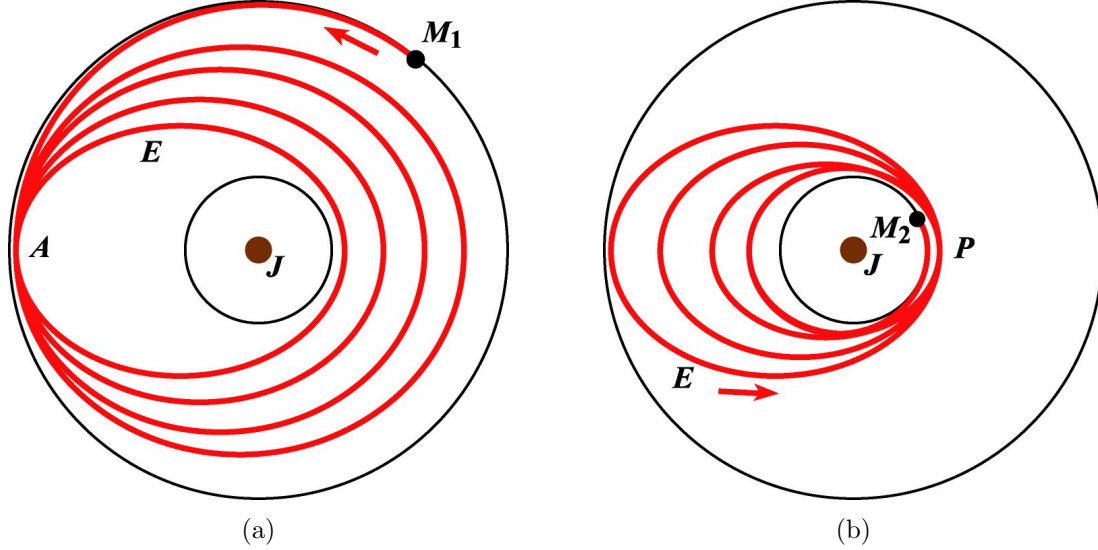
Figure 3: **Inter-moon transfer via resonant gravity assists.** (a) The orbits of two Jovian moons are shown as circles. Upon exiting the outer moon's (M_1's) sphere-of-influence, the spacecraft proceeds under third body effects onto an elliptical orbit about Jupiter. The spacecraft gets a gravity assist from the outer moon when it passes through apojove (denoted A). The several flybys exhibit roughly the same spacecraft/moon geometry because the spacecraft orbit is in near-resonance with the moon's orbital period and therefore must encounter the moon at about the same point in its orbit each time. Once the spacecraft orbit comes close to grazing the orbit of the inner moon, M_2 (in fact, grazing the orbit of M_2's L_2 point), the inner moon becomes the dominant perturber. The spacecraft orbit where this occurs is denoted E. (b) The spacecraft now receives gravity assists from M_2 at perijove (P), where the near-resonance condition also applies. The spacecraft is then ballistically captured by M_2.

orbits can be simplified using the Keplerian Maps for the two three body systems. Given the size of the periodic orbit around L1 of Jupiter-Ganymede-Particle(J-G-P) system, we can find the three-body energy of the same. Similarly, given a target periodic orbit around L2 of the Jupiter-Europa-Particle(J-E-P) system, we can find its three-body energy. The small neighbourhood around the point where J-G-P and J-E-P contour lines interect for a given set of energies, represents the switching region. Figure 4 shows the various regions. The search for probable trajectories is done as follows:

1). We choose a point in the switching region, call it P(a_0,e_0) and time t=0. Without loss of generality, we assume the particle is currently at apojove. To uniquely define a trajectory in the four body system, we need to specify the periapse angle w.r.t the Jupiter-Ganymede line,w_g and w.r.t the Jupiter-Europa line,w_e, at time t=0.

2). The chosen point will be the result of significant kick from Ganymede towards Europa. Recall from the previous section that this implies that the previous apoapse should have occurred with the periapse slightly more than $-\pi$ w.r.t J-G line. Also, we want to be able to escape the switching region and go deep into the J-E-P region within the next few

iterations. Again, this implies that the next periapse should occur with periapse sligthly greater than zero w.r.t J-E line, which will lead to a negative 'a' kick. These two conditions are used to narrow down the values of the two unknowns, w_g and w_e. Then, using the full four-body equations we can find a set of values of these 2 unknowns that lead to entry into switching region from J-G-P region and exit from the switching region to J-E-P region, in a few iterations. This set is labelled S_w. Recall that the path of the particle in this region is called the "switching orbit". The first forward iterate at periapse into the J-E-P region is labelled as $P1_f$ and the first back iterate at apoapse into the J-G-P region is labelled as $P1_b$.

3). Now we need to search for the conditions from the set S_w that will lead to a successive decrease in the semi-major axis when iterated forward, and will lead to increase in the semi-major axis value when iterated back. Now this task of iterating in the J-E-P and J-G-P regions, can be efficiently handled by the Keplerian maps. For each 2-tuple (w_g,w_e) and a point P(a_0,e_0) in the switching region, we iterate forward the corresponding point $P1_f$ using the Periapse map which is valid only in the J-E-P region, and iterate backward the corresponding point $P1_b$ using the Apoapse map, which is valid only in the J-G-P region.

4). Once we have found out which values among the set S_w will result in an Jovicentric orbit from near-Ganymede to near-Europa, we use the full four-body equations to refine and tweak the intial guess. Also, we can cycle through various nearby (a,e) values in the switching region to get appropriate trajectories.
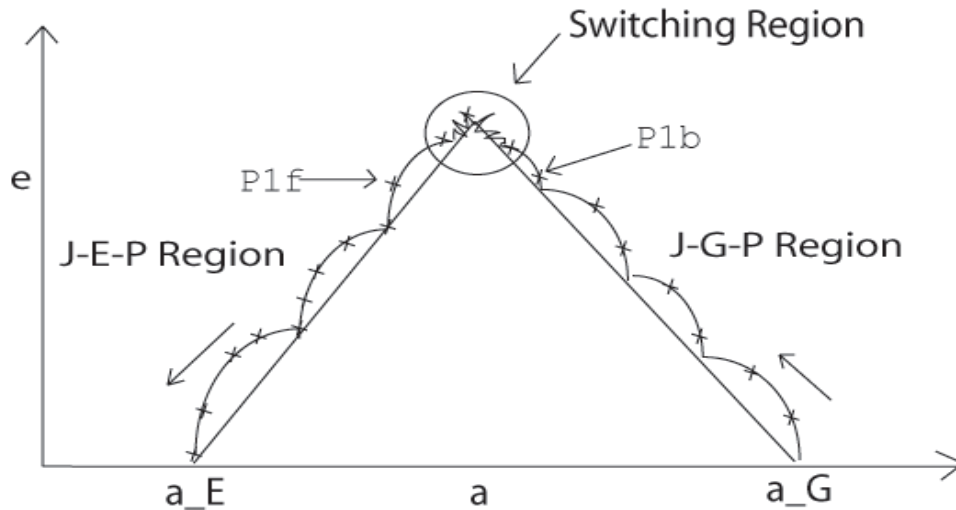

Figure 4: Schematic trajectory in a-e plane showing various regions. Various apoapses/periapses are marked 'x'

Numerical results

Figure 5(a) shows the time history of the semi-major axis for a trajectory obtained by the method described above. Figure 5(b) shows the same trajectory plotted in the 'a-e' plane.

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Note that for most part, it lies on the three-body-energy contours of one of the two three-body systems. Figures 5(c) and (d) show the 'Jacobi Constant' time history for the J-G-P and J-E-P system, respectively.

Its important to note here that the trajectory doesn't seem to be time optimal, since it spends a long time in certain resonances before moving on to other resonances. Lobe dynamics is the underlying mechanism by which transport across different resonances takes place, and we can use it to devise appropriate control strategies which can reduce the time of flight significantly.

Resonant Structure of Phase Space, Lobe Dynamics and Use of Control

Solutions to the four-body problem which lead to the behavior shown schematically in Figure 3 have been found numerically and the phenomena partially explained in terms of the P3BA.[1] The switching region between neighboring pairs of moons can only be accessed by traversing several subregions of the three-body problem phase space, known as "resonance regions," where the resonance is between the spacecraft orbital period and the dominant moon's orbit period around Jupiter, respectively.Early investigation into the phase space of the restricted three-body problem using Poincaré sections has revealed a phase space consisting overlapping resonance regions.[15,19] This means that movement amongst resonances is possible.

Lobe dynamics provides a general theoretical framework, based on invariant manifold ideas from dynamical systems theory, for discovering, describing and quantifying the transport "alleyways" connecting resonances.[20] A resonance region and the lobes of phase space associated with movement around it are shown in Figure 6 on a Poincaré section in quasi-action-angle coordinates. The lobes are defined using the stable and unstable manifolds associated to unstable resonant orbits. Starting in one of the lobes above the resonance, an initial condition can get transported to below the resonance, and vice versa. This corresponds decrease or increase in the spacecraft's semimajor axis for zero fuel cost.

Lobe dynamics tells us the most important spacecraft trajectories, i.e., the uncontrolled trajectories which traverse the resonance regions in the shortest time and serves as the underlying mechanism for uncontrolled spacecraft trajectories, such as the one shown in Figure 7(a). An initial condition in the upper right hand side of Figure 7(a) moves through the phase space as shown, jumping between resonance regions under the natural dynamics of the three-body problem, i.e., at zero fuel cost. Figure 7(b) shows a schematic of the corresponding trajectory in inertial space.

Essentially, the lobes act as templates, guiding pieces of the tube across resonance regions. We can numerically determine the fastest trajectory from an initial region of phase space (e.g., orbits which have just escaped from moon M_2) to a target region (e.g., orbits which will soon be captured by a neighboring moon M_2). Using small control inputs at appropriate points (the search being guided by Lobe dynamics computations), we can reduce the time taken for the inter-moon transfers drastically. Quite possibly, this task would be made easier by the use of the maps mentioned in this paper and will be taken up in the future. The
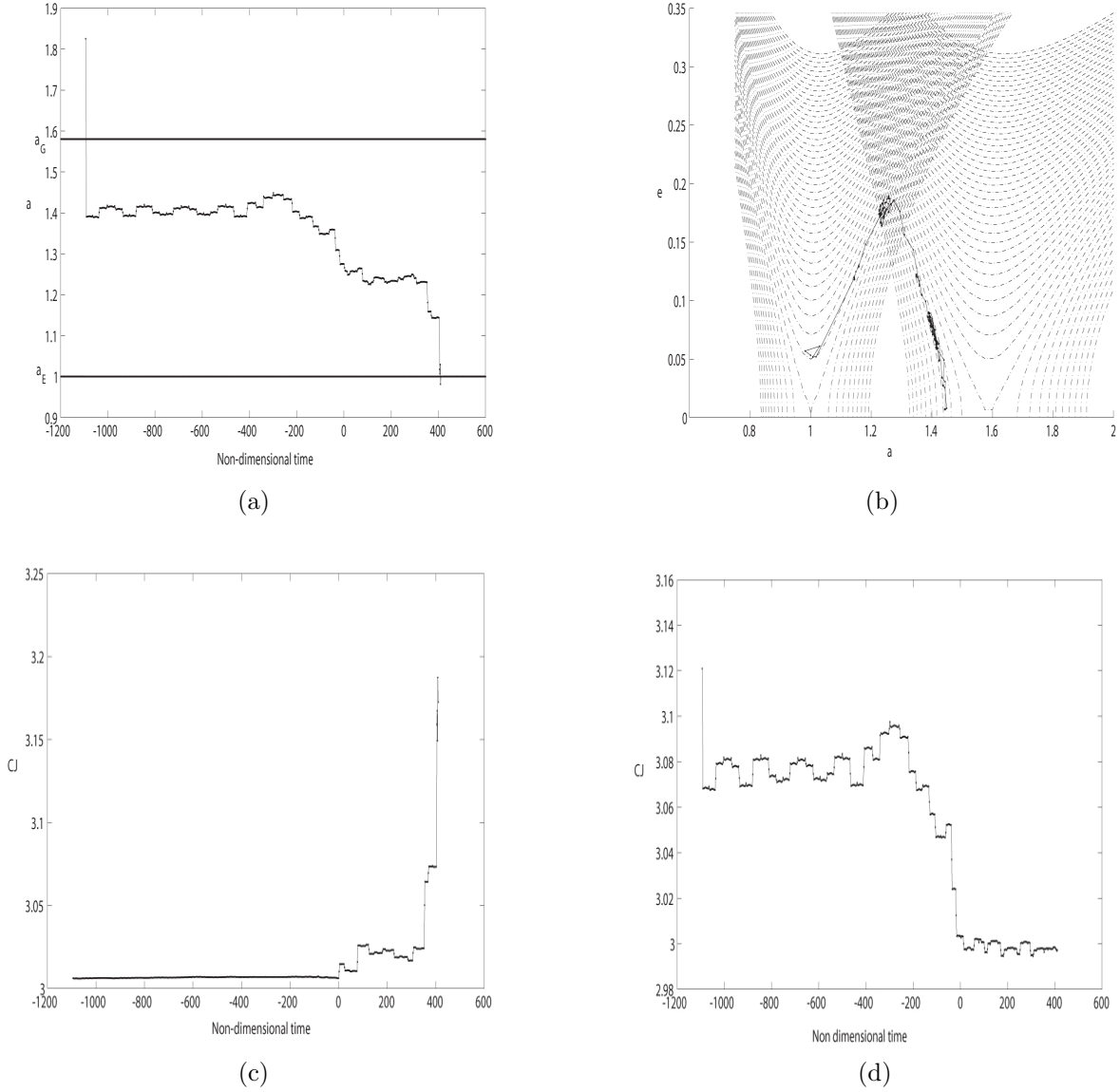
(a)

(b)

(c)

(d)

Figure 5: Trajectory found using the Patched Three Body Approximation. a). Semi-major axis time history b). Trajectory in 'a-e' plane. c)Jacobi Constant for J-G-P system d). Jacobi Constant for J-E-P system

problem could be defined by a family of controlled Keplerian maps $F : \mathcal{A} \times U \rightarrow U$

$$F\left(\left(\begin{array}{c} \omega_n \\ K_n \end{array}\right), u_n\right) = \left(\begin{array}{c} \omega_{n+1} \\ K_{n+1} \end{array}\right) = \left(\begin{array}{c} \omega_n - 2\pi(-2K_{n+1})^{-3/2} \\ K_n + \mu f(\omega_n) + \alpha u_n \end{array}\right) \quad (2)$$

where $u_n \in U = [-u_{\max}, u_{\max}]$, $u_{\max} \ll 1$, and the parametric dependence of f is understood. The term $\alpha = \alpha(C_J, \bar{K})$ is approximated as constant. Physically, the control could be modeled as a small impulsive thrust maneuver performed at periapsis n changing the speed
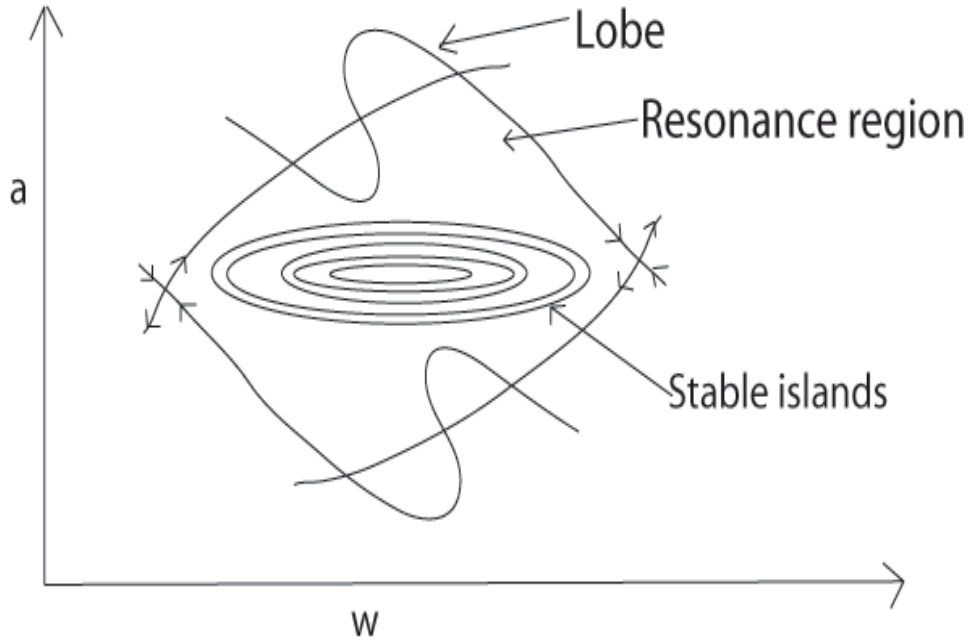
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Figure 6: Schematic of a resonance region and the lobes responsible for transport near that region. Also shown are the stable elliptical islands that are disconnected from the chaotic sea.

by u_n. This changes K_n by an energy αu_n in addition to the natural dynamics term $\mu f(\omega_n)$. This yields the ΔV vs. TOF trade-off for the inter-moon transfer between Ganymede and Europa. The important fact that Keplerian map captures the topology of the trajectories will allow more freedom to optimization schemes. Optimization at the level of discrete map has the capability of finding topologically new trajectories. The study of control in this framework may also shed light on the mechanism by which some minor bodies get handed off between planets[21–23].

4 Discussion and conclusions

We have outlined a method to determine fuel optimal trajectories using multiple-gravity assists. Our main aim has been to describe a method that can be used to efficiently search the phase space for such trajectories. Using Picard's method of successive approximations, we derive a family of two-dimensional symplectic twist maps to approximate a particle's motion in the planar circular restricted three-body problem with Jacobi constant near 3.The maps model a particle on a near-Keplerian orbit about a central body of unit mass, where the spacecraft is perturbed by a smaller body of mass μ. The interaction of the particle with the perturber is modeled as an impulsive kick at periapsis(or apoapse) passage, encapsulated in a kick function f. The maps are identified as an approximation of a Poincaré return map
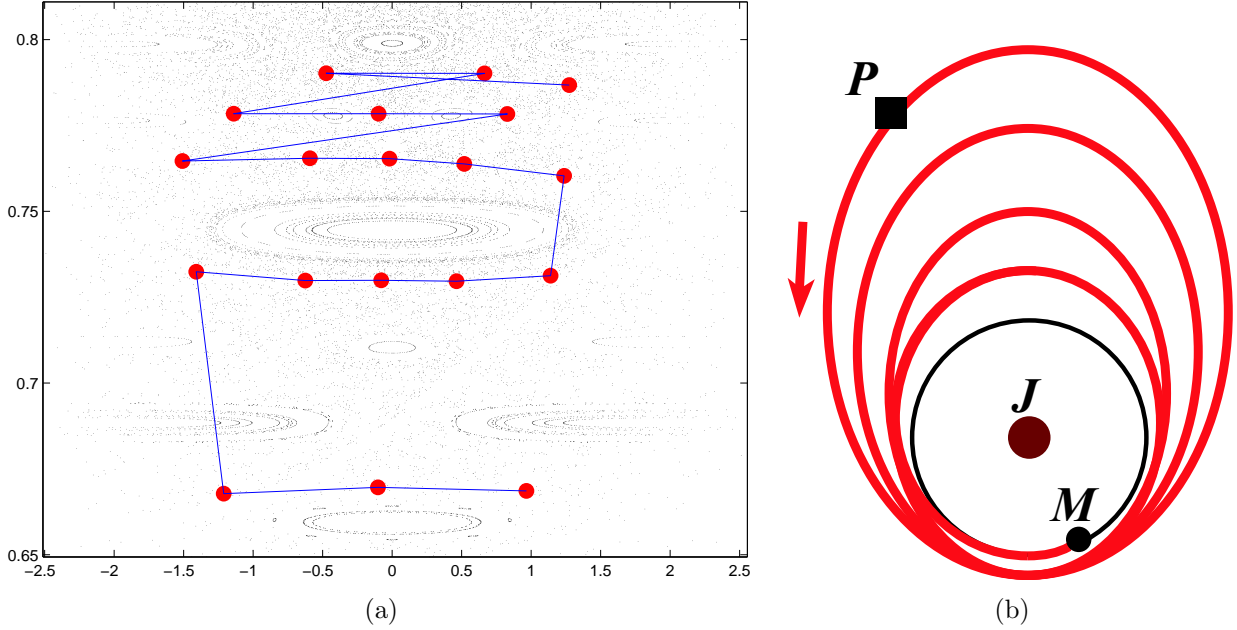
Figure 7: **Jumping between resonance regions leads to large orbit changes at zero cost.** An initial condition in the upper right hand side of (a) moves through the phase space as shown, jumping between resonance regions. In (b), a schematic of the corresponding trajectory of a spacecraft P in inertial space is shown. Jupiter (J) and one of its moons (M) are also shown schematically.

of the full equations of motion where the surface of section is taken at periapsis(apoapsis), mapping each periapsis(apoapsis) passage to the next in terms of the azimuthal separation of the particle and perturber ω and the Keplerian orbital energy of the particle about the central body. The map captures well the dynamics of the full equations of motion; namely, the phase space is densely covered by chains of stable resonant islands, in between which is a connected chaotic zone. The chaotic zone, far from being structureless, contains lanes of fast migration between orbits of different semimajor axes. The advantage of having an analytical two-dimensional map over full numerical integration is that we can apply all the machinery of the theory of transport in symplectic twist maps. We apply our Keplerian map to the identification of transfer trajectories applicable to spacecraft transfers in a planet-moon system. The use of subtle gravitational effects described by the map may be feasible for future missions to explore the outer planet moon systems where the timescale of orbits is measured in days instead of years and low-energy trajectories may be considered for inter-moon transfers. Physically, particles in the regime we study undergo multiple gravity assists of a different kind than the hyperbolic flybys of, say, the Voyager mission. The gravity assists we study are for particles on orbits with semimajor axes greater than the perturber's and whose periapsis passages occur close to but beyond the sphere of influence of the perturbing body. The effect of gravity assists is largest for particles whose passages occur slightly behind (resp. in front of) the perturbing body, resulting in a larger (resp. smaller) semimajor axis.

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This makes the apoapsis distance grow (resp. shrink) while keeping the periapsis distance relatively unchanged.

Low-thrust trajectories will be increasingly used in future space missions, e.g., outer planet moon tours. The trajectories required for these missions are complex and challenging to design. Strong multi-body effects combined with low-thrust control of capture and escape orbits around moons make trajectory generation difficult, to say nothing of optimization. Individual complete trajectories require a great deal of time to construct. To speed up the early design phases, we intend to combine the tools mentioned here for low-order estimates of low-thrust trajectories in order to obtain approximate propellant and time of flight requirements.

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