

DESIGNING TRAJECTORIES IN A PLANET-MOON ENVIRONMENT USING THE CONTROLLED KEPLERIAN MAP

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Designing fuel efficient trajectories which visit different moons is best handled by breaking up the problem into multiple three-body problems. This approach, called the patched three-body approach has received considerable attention in recent years, and has proved to lead to substantial fuel savings compared to the traditional patched-conic approach. We consider the problem of designing fuel-efficient multi-moon orbiter trajectories in the Jupiter-Europa-Ganymede system with realistic transfer times. Previously, the authors devised a method to get fuel-optimal (i.e. near zero fuel) trajectories in the same system, but the trajectories so obtained are not feasible due to very long transfer times involved. The aim of the current paper is to describe a methodology which exploits the underlying structure of the dynamics of the two three-body problems, i.e. Jupiter-Europa-Spacecraft and Jupiter-Ganymede-Spacecraft, using the Hamiltonian structure preserving Keplerian map approximations derived earlier and using small control inputs in the form of instantaneous ΔV s to get substantially lower time-of-flight than obtained previously.

INTRODUCTION

Low energy spacecraft trajectories such as multi-moon orbiters for the Jupiter system can be obtained by harnessing multiple gravity assists by moons in conjunction with ballistic capture to drastically decrease fuel usage.^{1,2,3} These phenomena have been explained by applying techniques from dynamical systems theory to systems of n bodies considered three at a time.^{4,5,6,7,8} One can design trajectories with a predetermined future and past, in terms of transfer from one Hill's region to another. Using this approach, which has been dubbed the "Multi-Moon Orbiter" (MMO), a scientific spacecraft can orbit several moons for any desired duration, instead of flybys lasting only seconds. This approach should work well with existing techniques, enhancing interplanetary trajectory design capabilities for aggressive missions in planet-moon environments. The approach is quite flexible in the sense that the spacecraft can be made to respond to unforeseen events, and can be made to revisit any region.

Previously,⁹ the authors have been able to construct fuel optimal trajectories in the Jupiter-Europa-Ganymede system, but the trajectories so obtained are not feasible due to very long

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transfer times involved. The concern of this paper is to describe a methodology using the analytically derived Keplerian Map¹⁰ to obtain trajectories with a substantially lower time-of-flight, using small control inputs in the form of instantaneous ΔV s. The paper is arranged as follows. We first describe the Keplerian Map, which is the analytical tool used in the process of finding desired trajectories. We then review the framework of breaking down the multi-moon problem into 2 3-body problems and finding zero-fuel trajectories that go from one moon to the other. Using this framework, we then describe the methodology for generating low energy trajectories that can be completed in reasonable time using small control inputs. We also discuss the tradeoff between fuel consumption and time-of-flight for the family of trajectories obtained using the same method.

KEPLERIAN MAP FOR EVOLUTION UNDER NATURAL DYNAMICS OF PCR3BP

Each map, which we call the *periapsis Poincaré map* (or *Keplerian map*), is an update map for the angle of periapse ω in the rotating frame and Keplerian energy K , $(\omega_n, K_n) \mapsto (\omega_{n+1}, K_{n+1})$. The map has the form

$$\begin{pmatrix} \omega_{n+1} \\ K_{n+1} \end{pmatrix} = \begin{pmatrix} \omega_n - 2\pi(-2K_{n+1})^{-3/2} \\ K_n + \mu f(\omega_n; C_J, \bar{K}) \end{pmatrix} \quad (1)$$

i.e., a map of the cylinder $\mathcal{A} = S^1 \times R$ onto itself.

The map models a spacecraft on a near-Keplerian orbit about a central body of unit mass, where the spacecraft is perturbed by a smaller body of mass μ . The interaction of the spacecraft with the perturber is modeled as an impulsive kick at periapsis passage, encapsulated in the kick function f , see Figure 1(a), where (μ, C_J, \bar{K}) are considered bifurcation parameters.

The map captures well the dynamics of the full equations of motion; namely, the phase space, shown in Figure 1(b), is densely covered by chains of stable resonant islands, in between which is a connected chaotic zone. The more physically intuitive semimajor axis a is plotted for the vertical axis instead of Keplerian energy K , where $a = -1/(2K)$. The kick function obtained from the map shows that the biggest kicks are received for a very narrow range of periapse values. If the periapse occurs slightly ahead of the perturber, the particle is gets a negative a kick, and if the periapse is slightly behind the perturber, the kick is positive. Using similar methods as above, we can construct an apoapse kick map for the case when the spacecraft is in the interior realm, i.e. when its semi-major axis is less than that of the perturber. In case of an apoapse kick map, to receive the positive a kick, the apoapse needs to be slightly ahead of the perturber (or the periapse needs to be slightly less than π). Similarly, for a negative ‘a’ kick, the apoapse needs to be slightly behind the perturber (or the periapse needs to be slightly more than $-\pi$).

The engineering application envisioned for the map is to the design of low energy trajectories, specifically between moons in the Jupiter moon system. Multiple gravity assists are a key physical mechanism which could be exploited in future scientific missions.¹ For

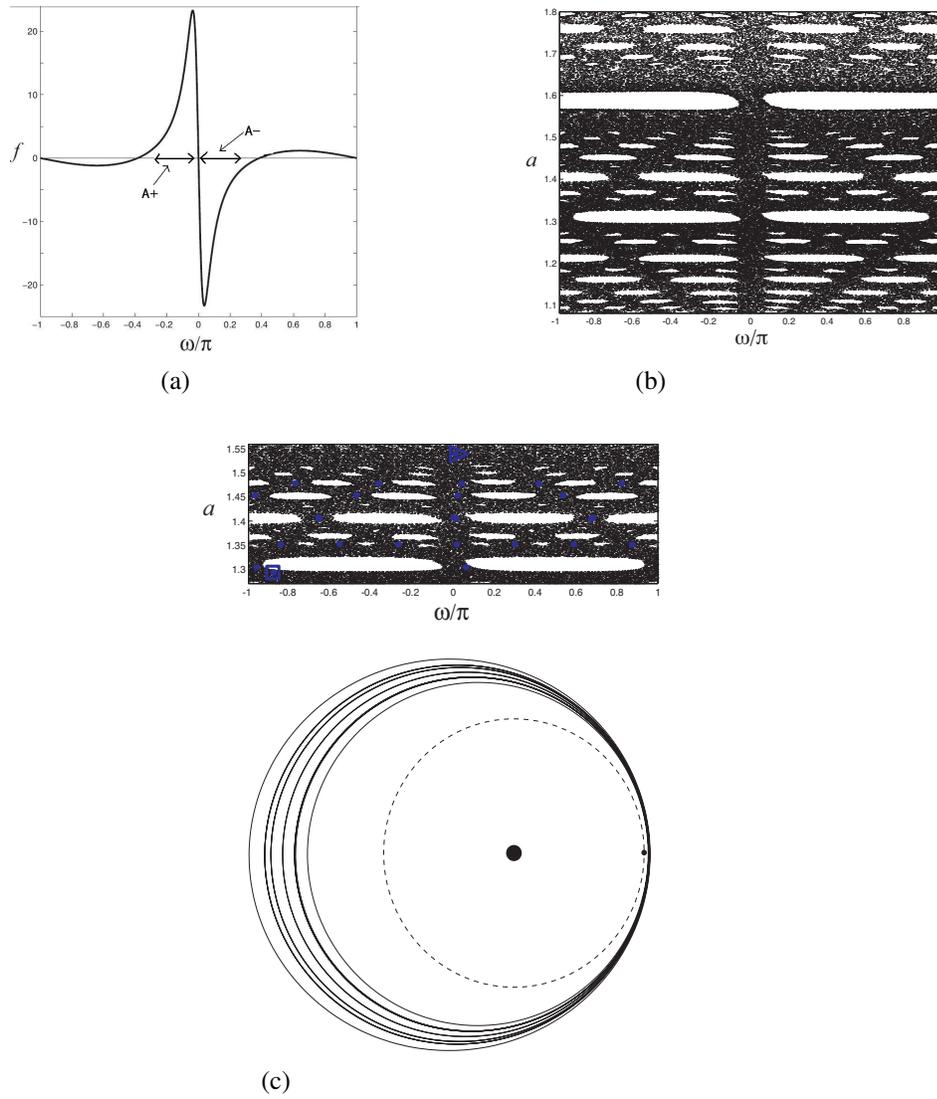


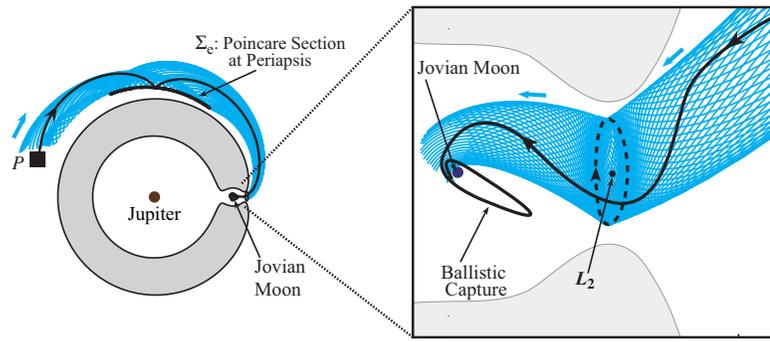
Figure 1 (a) The energy kick function f vs. ω for typical values of the parameters. (b) The connected chaotic sea in the phase space of the Keplerian map. The semimajor axis $a [= -1/(2K)]$ vs. the angle of periaapsis ω is shown for parameters $\mu = 5.667 \times 10^{-5}$, $C_J = 2.995$, $\bar{a} = -1/(2\bar{K}) = 1.35$ appropriate for a spacecraft in the Jupiter-Callisto system. The initial conditions were taken initially in the chaotic sea and followed for 10^4 iterates, thus producing the ‘swiss cheese’ appearance where holes corresponding to stable resonant islands reside. (c) Upper panel: a phase space trajectory where the initial point is marked with a triangle and the final point with a square. Lower panel: the configuration space projections in an inertial frame for this trajectory. Jupiter and Callisto are shown at their initial positions, and Callisto’s orbit is dashed. The uncontrolled spacecraft migration is from larger to smaller semi-major axes, keeping the periaapsis direction roughly constant in inertial space. Both the spacecraft and Callisto orbit Jupiter in a counter-clockwise sense.

example, a trajectory sent from Earth to the Jovian system, just grazing the orbit of the outermost icy moon Callisto, can migrate using little or no fuel from orbits with large apoapses to smaller ones. This is shown in Figure 1(c) in both the phase space and the inertial configuration space. From orbits slightly larger than Callisto's, the spacecraft can be captured into an orbit around the moon. The set of all capture orbits is a solid cylindrical tube in the phase space, as shown in Figure 2(a) (for details of the tube computation, see, e.g., Ref⁶). Followed backward in time this solid tube intersects transversally our Keplerian map, interpreted as a Poincaré surface-of-section. The resulting elliptical region, Figure 2(b), is an *exit* from jovicentric orbits exterior to Callisto. It is the first backward Poincaré cut of the solid tube of capture orbits

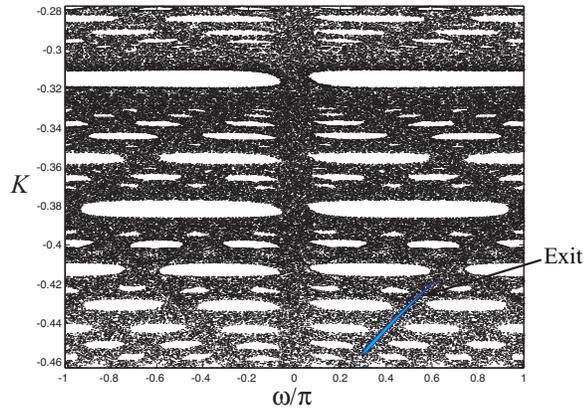
The advantage of considering an analytical two-dimensional map as opposed to full numerical integration of the restricted three-body equations of motion is that we can apply all the theoretical and computational machinery applicable to phase space transport in symplectic twist maps.¹¹ For example, previous work on twist maps can be applied, revealing the existence of lanes of fast migration between orbits of different semimajor axes. These lanes can be used by a spacecraft sent from Earth to the Jovian system. A spacecraft whose trajectory just grazes the orbit of the outermost icy moon Callisto can migrate using little or no fuel from orbits with large apoapses to smaller ones.

PATCHED THREE BODY APPROXIMATION

The P3BA discussed by Ross et al.¹ considers the motion of a spacecraft in the field of n bodies, considered two at a time, e.g., Jupiter and its i th moon, M_i . When the trajectory of a spacecraft comes close to the orbit of M_i , the perturbation of the spacecraft's motion away from purely Keplerian motion about Jupiter is dominated by M_i . In this situation, we say that the spacecraft's motion is well modeled by the Jupiter- M_i -spacecraft restricted three-body problem. Within the three-body problem, we can take advantage of phase space structures such as tubes of capture and escape, as well as lobes associated with movement between orbital resonances. Both tubes and lobes, and the dynamics associated with them, are important for the design of a MMO trajectory. The motion of the spacecraft in the gravitational field of the three bodies Jupiter, Ganymede, and Europa is approximated by two segments of purely three body motion in the circular, restricted three-body model. The trajectory segment in the first three body system, Jupiter-Ganymede-spacecraft, is appropriately patched to the segment in the Jupiter-Europa-spacecraft three-body system. For each segment of purely three body motion, the invariant manifolds tubes of L_1 and L_2 bound orbits (including periodic orbits) lead toward or away from temporary capture around a moon. Portions of these tubes are "carried" by the lobes mediating movement between orbital resonances. Directed movement between orbital resonances is what allows a spacecraft to achieve large changes in its orbit. When the spacecraft's motion, as modeled in one three-body system, reaches an orbit whereby it can switch to another three-body system, we switch or "patch" the three-body model to the new system. This initial guess solution is then refined to obtain a trajectory in a more accurate four-body model. Evidence suggests that these initial guesses are very good,¹ even in the full n -body model.



(a)



(b)

Figure 2 (a) A spacecraft P inside a tube of gravitational capture orbits will find itself going from an orbit about Jupiter to an orbit about a moon. The spacecraft is initially inside a tube whose boundary is the stable invariant manifold of a periodic orbit about L_2 . The three-dimensional tube, made up of individual trajectories, is shown as projected onto configuration space. Also shown is the final intersection of the tube with Σ_e , a Poincaré map at periapsis in the exterior realm. (b) The numerically computed location of an exit on Σ_e , with the same map parameters as before. Spacecraft which reach the exit will subsequently enter the phase space realm around the perturbing moon. The vertical axis is the Keplerian energy K of the instantaneous conic orbit about Jupiter.

Previously,⁹ the authors described a methodology to obtain fuel optimal trajectories for the MMO, with the help of the Keplerian Map. During the inter-moon transfer—where one wants to leave a moon and transfer to another moon, closer in to Jupiter—we consider the transfer in two portions, shown schematically in Figure 3, with M_2 as the inner moon. In the first portion, the transfer determination problem becomes one of finding an appropriate solution of the Jupiter- M_1 -spacecraft problem which jumps between orbital resonances with M_1 , i.e., performing resonant GA's to decrease the perijove.¹ M_1 's perturbation is only significant over a small portion of the spacecraft trajectory near apojove (A in Figure

3(a)). The effect of M_1 is to impart an impulse to the spacecraft, equivalent to a ΔV in the absence of M_1 .

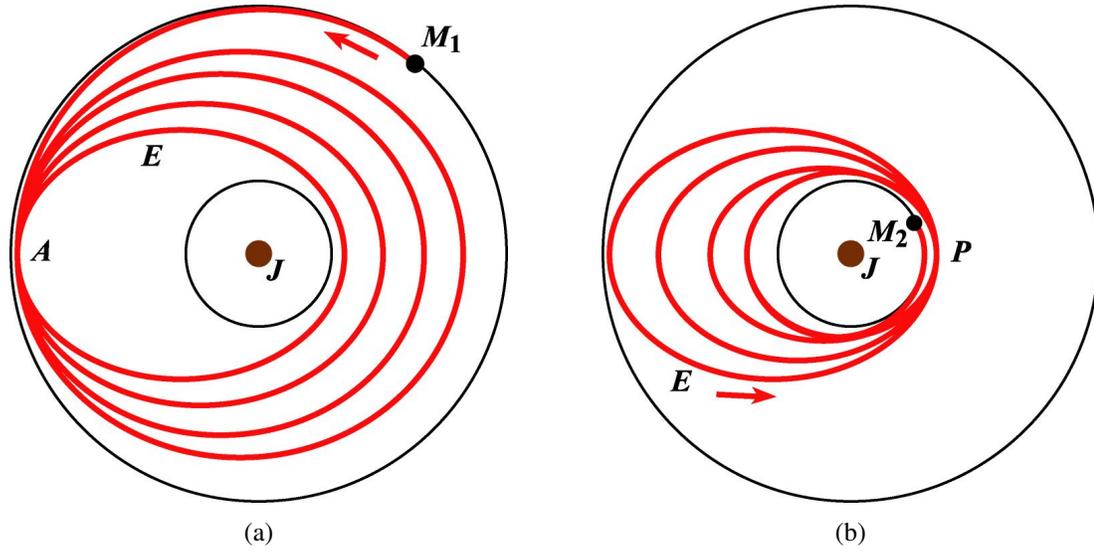


Figure 3 Inter-moon transfer via resonant gravity assists. (a) The orbits of two Jovian moons are shown as circles. Upon exiting the outer moon's (M_1 's) sphere-of-influence, the spacecraft proceeds under third body effects onto an elliptical orbit about Jupiter. The spacecraft gets a gravity assist from the outer moon when it passes through apojove (denoted A). The several flybys exhibit roughly the same spacecraft/moon geometry because the spacecraft orbit is in near-resonance with the moon's orbital period and therefore must encounter the moon at about the same point in its orbit each time. Once the spacecraft orbit comes close to grazing the orbit of the inner moon, M_2 (in fact, grazing the orbit of M_2 's L_2 point), the inner moon becomes the dominant perturber. The spacecraft orbit where this occurs is denoted E . (b) The spacecraft now receives gravity assists from M_2 at perijove (P), where the near-resonance condition also applies. The spacecraft is then ballistically captured by M_2 .

The perijove is decreased until it has a value close to M_2 's orbit, in fact, close to the orbit of M_2 's L_2 . We can then assume that a GA can be achieved with M_2 with an appropriate geometry such that M_2 becomes the dominant perturber and all subsequent GA's will be with M_2 only. When a particular resonance is reached, the spacecraft can then be ballistically captured by the inner moon.⁶ The arc of the spacecraft's trajectory at which the spacecraft's perturbation switches from being dominated by moon M_1 to being dominated by M_2 is called the "switching orbit." A rocket burn maneuver need not be necessary to effect this switch. The set of possible switching orbits is the "switching region" of the P3BA, see Figure 4. It is the analogue of the "sphere of influence" concept used in the patched-conic approximation, which guides a mission designer regarding when to switch the central body for the model of the spacecraft's Keplerian motion.

The task of searching for trajectories that go from near-Ganymede to near-Europa Jovi-

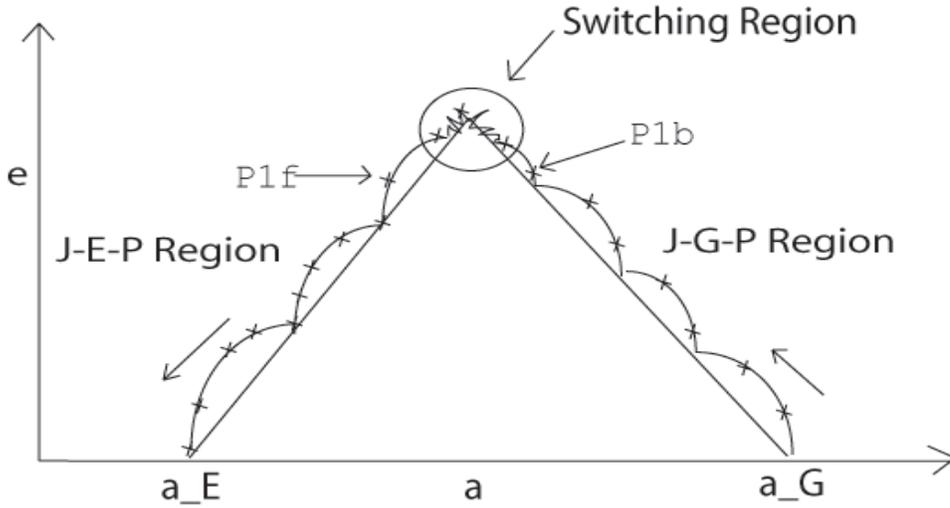


Figure 4 Schematic trajectory in a-e plane showing various regions. Various apoapses/periapses are marked 'x'

centric orbits was simplified using the Keplerian Maps for the two three body systems. The figures 5, reproduced from that paper, show the validity of the patched three body approximation. Figure 5a shows the semi-major axis time history, starting from exit from Ganymede to capture by Europa. Figure 5b shows the time history in an a-e plot, clearly showing that the particle follows constant energy contours in the two regimes. Figure 5c and Figure 5d show the three-body energy history of the particle for Jupiter-Ganymede-Particle and Jupiter-Europa-Particle systems. The time $t=0$ refers to point where we patch the two solutions. Clearly, the energy for former system is constant for $t \leq 0$ and energy for the latter system is almost constant for $t \geq 0$, further proof that the patched three body approach can be used to obtain such trajectories.

A METHOD FOR DESIGNING TRAJECTORIES USING CONTROLLED MAP

The controlled Keplerian map with a control u is an update map for the angle of periapse (or apoapse) ω in the rotating frame and Keplerian energy K , $F : \mathcal{A} \times U \rightarrow \mathcal{A}$

$$F \left(\left(\begin{array}{c} \omega_n \\ K_n \end{array} \right), u_n \right) = \left(\begin{array}{c} \omega_{n+1} \\ K_{n+1} \end{array} \right) = \left(\begin{array}{c} \omega_n - 2\pi(-2K_{n+1})^{-3/2} \\ K_n + \mu f(\omega_n) + \alpha u_n \end{array} \right) \quad (2)$$

where $u_n \in U = [-u_{\max}, u_{\max}]$, $u_{\max} \ll 1$. The term $\alpha = \alpha(C_J, \bar{K})$ is approximated as constant. The control strategy employed to get desired trajectories is two-fold: it involves a coarse control part where the aim is to get rapid decrease in the semi-major axis value of the spacecraft, and a fine control part, where we target specific regions of interest in the phase space. The reason for this two-pronged strategy is that the traditional forward-backward approach¹² works best only if there are no big resonances in between the starting point and the target region. As was mentioned earlier, and is evident from Figure 1b, the phase

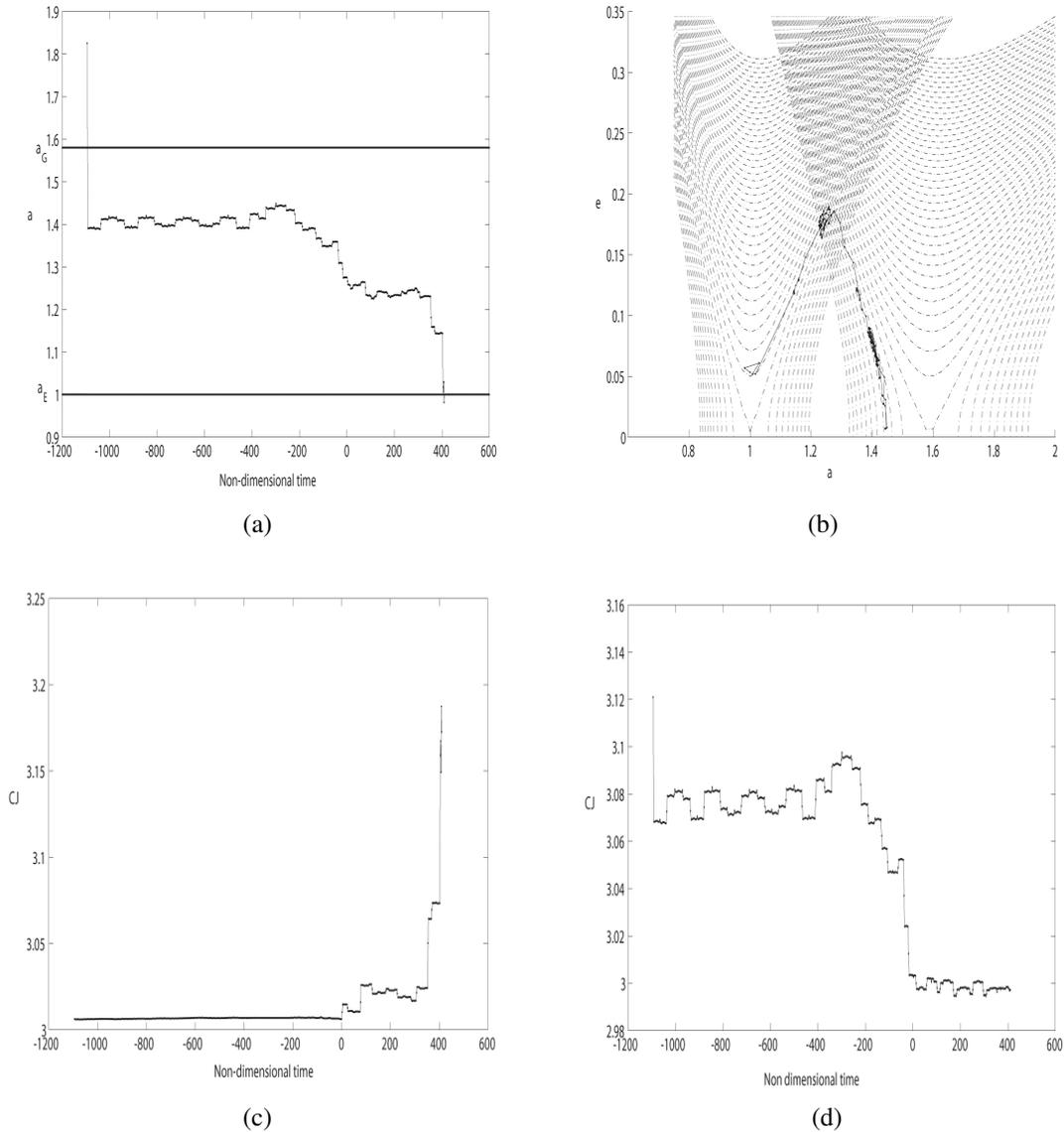


Figure 5 Trajectory found using the Patched Three Body Approximation. a). Semi-major axis time history b). Trajectory in 'a-e' plane. c) Jacobi Constant for J-G-P system d). Jacobi Constant for J-E-P system

space for our problem is populated with big resonances resulting in a mixed phase space, and hence, the forward-backward approach alone would not be sufficient for our purposes. The coarse control algorithm is based on the fact that large changes in semi-major axis(i.e. action) occurs for a very small range of values of the periaapse angle(i.e. angle), see Figure 1a. We adopt the policy of 'going with the flow', until we approach a region where there is going to be large change in the semi-major axis. The outline of the algorithm for a single three body system is as follows:

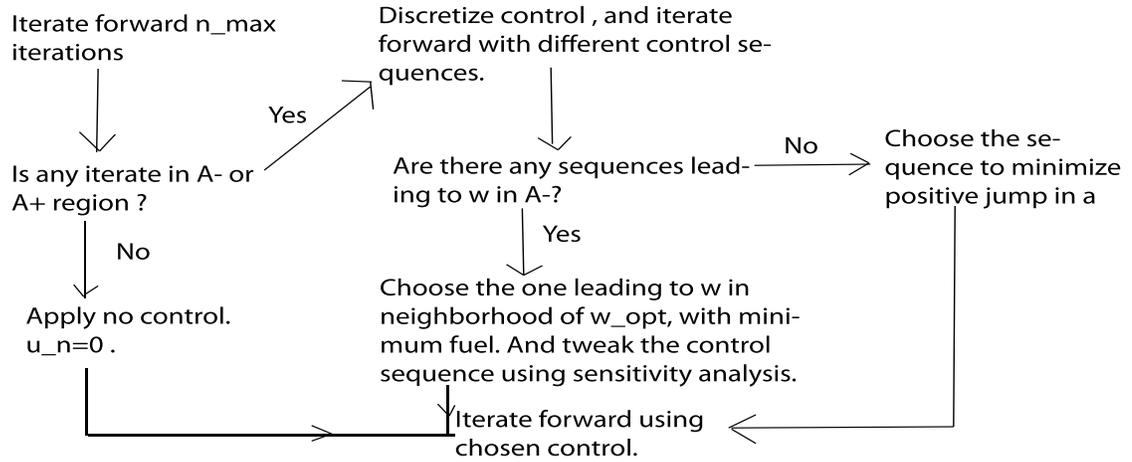


Figure 6 The coarse control algorithm

1). At (ω_n, a_n) , we iterate forward n_{max} steps with $u = 0$, where n_{max} holds an inverse relationship with u_{max} . If any of the calculated iterates lies with the region of high increase, labelled A_+ or the region of high decrease, labelled A_- , we calculate the control (2a and 2b, respectively). Else, we do not employ any control at the current iteration step (i.e, $u_n = 0$). The size of both these regions (i.e A_- and A_+) is directly proportional to the parameter $\delta\omega$.

2a). Ideally, if one of the future iterates calculated above is in A_+ , we want to apply control so as to move the iterate away from it to the neighboring A_- region. If the i th iterate (where $i < n_{max}$) is in A_+ , we calculate a control sequence over the next i iterates. The control domain (i.e. $[-u, u] \times [-u, u] \times \dots i$ times) is coarsely discretized and we obtain the iterates using each control sequence resulting from the discretization. If there are some sequences that result in the final iterate being in A_- , we choose the sequence which results in the final iterate being in a small neighborhood of ω_{opt} . Here ω_{opt} is the value of ω which leads to the maximum decrease in semi-major axis over one iteration. Then, using sensitivity analysis w.r.t the control at the last iterate in the sequence, we can tweak its value so that resultant $\omega \approx \omega_{opt}$.

On the other hand, if there is no such sequence that results in the final iterate being in A_- , we need to move the final periapse angle in the other direction so as to minimize the increase in 'a'. This local optimization can be handled by a similar discretization procedure as above.

2b). If one of the future iterates is in A_- , we again use the discretization of the control domain, and choose the control sequence in the same way as before. Sensitivity analysis is used to get final $\omega \approx \omega_{opt}$.

3). Once a threshold value of 'a' is reached, we switch to fine-control. Fine control can be handled by forward-backward method, where the target is the interior of the first intersection of the stable invariant manifold of a periodic orbit around L2 with the Poincare section at periapse.

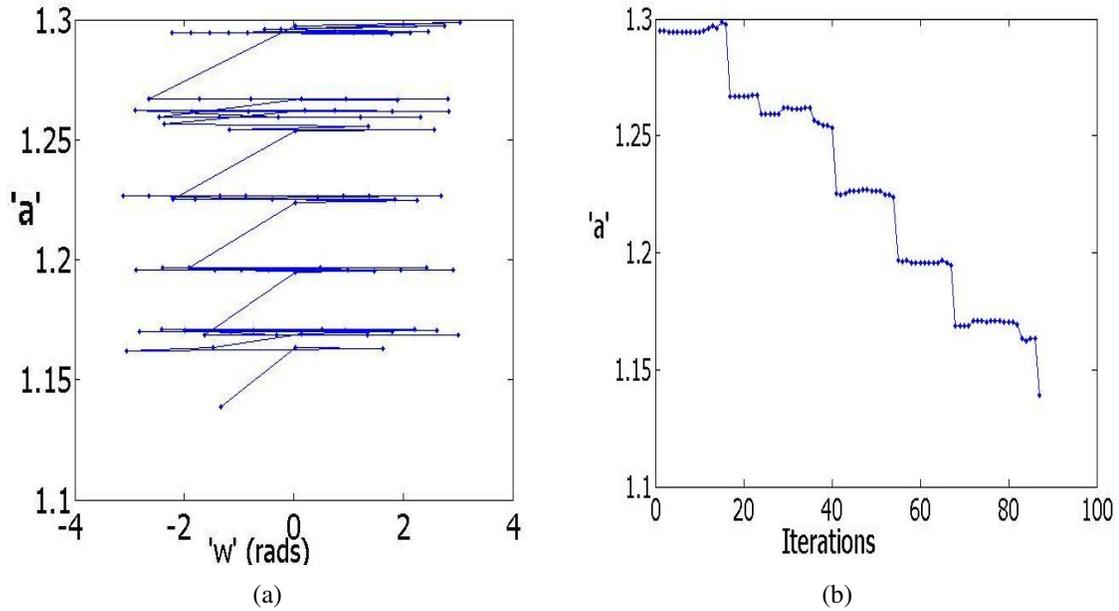


Figure 7 Sample trajectory designed using the algorithm. (a) Plot of Semi-major axis Vs Periapse angle. (b) Time history of semi-major axis. The spacecraft repeatedly visits the region of large decrease in semi-major axis.

In Figure 7 we show a sample trajectory obtained by the above mentioned algorithm, for the Jupiter-Europa-Spacecraft system. Note that as a result of appropriately timed control inputs, the spacecraft visits the region of high decrease of semi-major axis.

We can now use this algorithm within the framework of patched three body approximation. We use the controlled apoapse map to get the trajectory from near Ganymede to the switching region, and then patch it with another trajectory obtained by using controlled periapse map which leads to capture around Europa. A sample trajectory for such a case is shown in Figure 8. The spacecraft completes this trajectory using 160 m/s of fuel in 1.7 years, which included 116 revolutions around Jupiter (periapse/apoapse passages). The time taken for this mission is less than 10% of that taken for the previously shown optimal (zero) fuel trajectory for the same four body system.

Tradeoff Between Fuel and Time-of-Flight

We now discuss the tradeoff issues between time-of-flight and fuel(control). The amount of fuel used for providing ΔV s is expected to be proportional to the parameter δw upto a saturation point, since this parameter decides the size of the region in phase space where the control is actively applied, as discussed previously. More proactive control is also expected to decrease the time-of-flight upto a certain limit. In Figure 9, we show plots illustrating the time-of-flight Vs fuel tradeoff for a single three body system (Jupiter-Europa-Spacecraft). Several random initial conditions were taken near $a = 1.5$, and we show the plots for two of those conditions, that take the most and the least amount of iterations to reach the exit

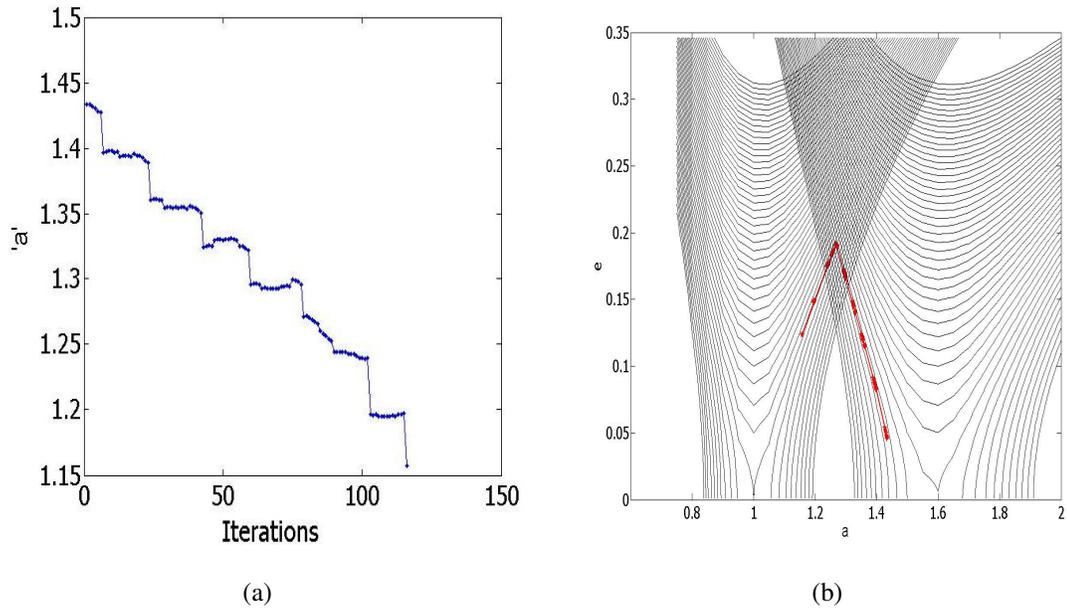


Figure 8 Trajectory for the Jupiter-Europa-Ganymede system using patched three body approach. (a) Time history of semi-major axis. b). Semi-major axis Vs eccentricity plot with three-body energy contours lines in the background.

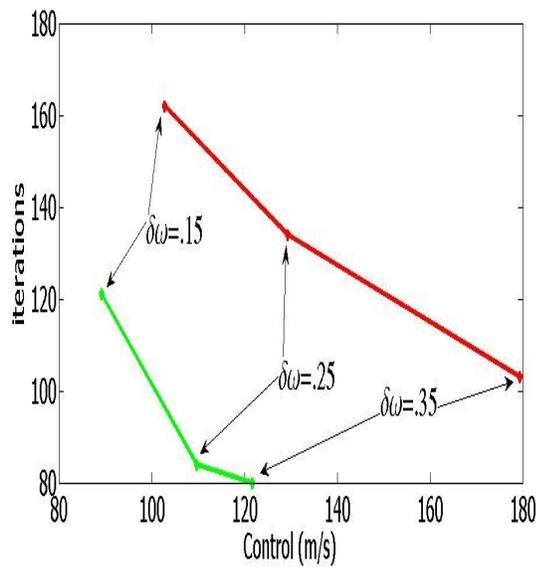


Figure 9 Plot showing the tradeoff between control used and number of iterations required for two different initial conditions leading to capture around Europa for the Jupiter-Europa-Spacecraft PCR3BP

region, for three different values of $\delta\omega$. This also gives us an estimate of fuel required for the each of the three missions. The basic characteristics of this system are expected to be similar for patched three body systems.

SUMMARY AND CONCLUSION

We have presented a methodology to obtain families of low fuel trajectories with realistic time-of-flights in the four body problem using the analytical Keplerian map. The controlled analytical map, used within the framework of patched three body approach, helps in reducing the search space considerably, as compared to using full equations of motion, and can be used to obtain appropriate low thrust control inputs that lead to rapid changes in the semi-major axis. The approach helps us in getting good initial approximations quickly, which can serve as inputs to the more sophisticated end-to-end trajectory design methods. We can also get lower order estimates on the control input required to complete the missions in a given time frame. A possible extension to this work would be to consider continuous low thrust input model instead of the discrete input model discussed in this paper.

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