Proceedings of the ASME 2008 Summer Bioengineering Conference (SBC2008)
June 25-29, Marriott Resort, Marco Island, Florida, USA

SBC2008-193215

LOCATING SEPARATRICES AND BASINS OF STABILITY IN BIODYNAMIC SYSTEMS

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INTRODUCTION

In biomechanics, a separatrix or recovery envelope exists between standing and falling. Standing with postural sway is a distinctly different type of motion than falling. A comparable problem to standing postural sway is the challenge of maintaining torso stability. In this case, a separatrix exists delineating stable torso sway from unstable and potentially injurious motion. An approach is presented for identifying separatrices in state space generated from noisy time series data sets representative of those generated from experiments. We demonstrate how Lagrangian coherent structures (LCS), ridges in the state space distribution of finite-time Lyapunov exponents (FTLE), can be used to locate these separatrices. As opposed to previous approaches which required an entire vector field, this method can be performed using a single trajectory that evolves over time.

MATHEMATICAL MODEL

The wobble chair [1] has been used to isolate movement of the lumbar spine in order to gain a better understanding of the dynamics utilized to maintain torso stability and prevent injury (Figure 1). In order to begin to understand the behavior of the full dimensional system, a simplified model is developed for the wobble chair.

The simplified model is governed by the following differential equation,

$$\ddot{\theta} = \frac{mgh \sin\theta - kd^2 \sin\theta - C(\theta,\dot{\theta}) + N}{mh^2} \tag{1}$$

where, $\ddot{\theta}$, m, g, h, θ, $\dot{\theta}$, k, and d are the angular acceleration, mass, acceleration of gravity, height, rotation angle, angular velocity, spring stiffness, and the distance of the springs from the central ball joint, respectively.

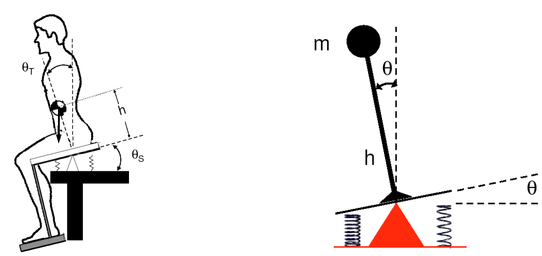

Figure 1: System schematic and simplified model

The equation for a limited gain proportional-derivative control, C, is given by,

$$C(\theta,\dot{\theta}) = G_d\dot{\theta} + \begin{cases} G_P\theta & if\ \theta < \theta_{cr} \\ G_{P\_max} & otherwise \end{cases} \tag{2}$$

where, G_d is the derivative gain constant, $\theta_{cr} = G_{p\ max}/G_p$ is the smallest angle at which the maximum gain is achieved, G_p is the proportional gain constant, and $G_{p\ max}$ is the maximum value of proportional gain. System noise, N, is introduced into the model as zero mean Gaussian random force perturbations.

NONLINEAR ANALYSIS

Lagrangian coherent structures are separatrices in state space delineating qualitatively different kinds of motion. The existing method to calculate LCS requires a vector field to be defined at each instant in time. Trajectories are generated from the vector field, and a FTLE field is determined by calculating the rate of local divergence. LCS are identified as the ridges in the FTLE field that separate two regions of flow.

LCS are found for the simplified model with and without system noise. First, the model is made deterministic and conservative by setting N and G_d equal to zero in the governing equation, Eq(1). This model is used to create a vector field. Next, the vector field is used to generate a FTLE field, and the LCS are found using established methods [2]. The location of the LCS can be seen as dark bands in Figure 2a. When system noise and damping are reintroduced, the results are similar (Figure 2b).

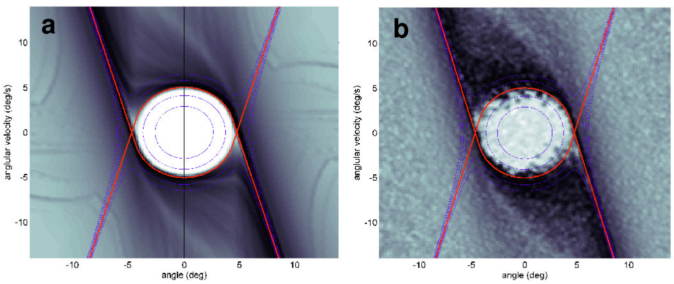

Figure 2: State space distribution of the FTLE field for the systems without (a) and with (b) noise

In both cases, the LCS are observed to align well with the iso-energy lines associated with the heteroclinic orbit. The heteroclinic orbit separates stable motion near the upright vertical position from unstable motion further from the origin in state space. In the case with system noise, the perturbations cause the LCS to be less distinct. However, the location of the LCS is unaltered. When the FTLE field is shown in three dimensions the LCS creates a "volcano" like structure forming a state space separatrix (Figure 3).

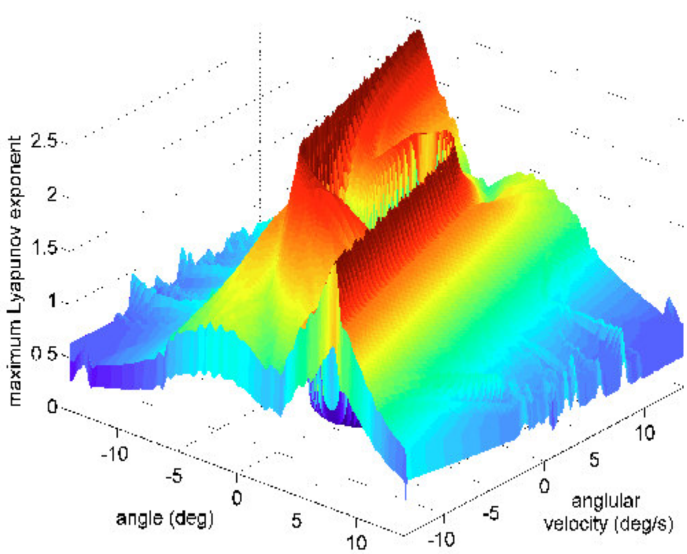

Figure 3: The LCS is revealed as a ridge in the FTLE field

LCS have previously been used to analyze dynamical systems defined by fluid flow fields from data, analytical biochemical models, and low degree of freedom mechanical systems. To our knowledge, the work presented herein is the first application of LCS to time series data with the absence of a vector field. Data in this form is commonly generated from biomechanical experiments.

A model of human postural control is used to generate simulated experimental data in the form of a single trajectory. The locations of these boundaries can be found by extracting additional information contained within the time series data. Rather than averaging the Lyapunov exponents over state space to obtain a single scalar value as traditionally done in biomechanics, one can generate a FTLE field and find LCS (Figure 4). Unlike previous methods, the approach described herein constructs a FTLE field using only trajectory data. This eliminates the need to generate a vector field which may not be accessible in an experimental setting.

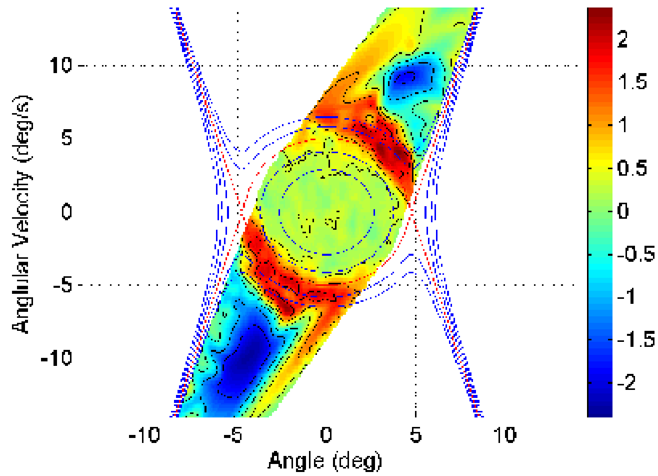

Figure 4: LCS found using only trajectory data

CONCLUSIONS

We have shown how boundaries that separate qualitatively different kinds of motion can be found using the method of Lagrangian coherent structures applied to time series data. We also demonstrated that the location of the LCS aligned well with the heteroclinic orbit even in the presence of system noise. In this biological example, the LCS forms the boundary of a basin of stability. Defining the basin of stability in state space provides a much richer understanding of the system dynamics over previous methods that calculate a single scalar value. The boundary, or recovery envelope, could be used in conjunction with sway data to define new measures of individual fall risk, e.g., the average distance of an individual's state from the boundary. In general, we believe the method demonstrated in this study provides a fruitful approach for extracting additional information from noisy experimental data, namely boundaries between qualitatively different kinds of motion.

ACKNOWLEDGMENTS

The authors would like to express their gratitude to Dr. Kevin P. Granata for his contributions and guidance during the early portions of this research. Tragically, Dr. Granata was killed in the April 16th shootings at Virginia Tech. Although he left this world far too early, his visionary spirit has guided the authors throughout this research.

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