Optimal Control of Structural Dampers

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***Abstract*—Deformation due to wind is a significant obstacle in the design of tall buildings. Many large buildings include dampers in their upper floors to counteract oscillation due to wind, and recent designs permit controlling the damping properties of these weights with magnetorheological fluids. In this study, we consider the optimal control of these dampers. Our metric of interest is chosen to minimize tenant discomfort from lateral sway on the topmost floors of the building. A kinematic model for the building is proposed and stochastic descriptions of the wind are used to provide dynamic loading. We demonstrate that this formulation structures the optimization problem as a Markov Decision Process. A dynamic programming optimization scheme is developed to compute optimal control trajectories for the system, although computational complexity limits the method in practice. We present a summary of these techniques as well as preliminary results of the method.**

I. Introduction

High rise and flexible structures are now a part of every developing region for their efficient use of land area and energy. These come with a price of designing against excessive instability due to earthquake and wind excitation. Typically the goal is to dampen excessive vibrations using active or passive tuned mass dampers which are control systems designed for an assumed excitation. The design never accounts for the inhabitant discomfort resulting from dynamic loading due to wind excitations. While the idea of controlling buildings is relatively new, there is a very large body of research spanning more than a century concerning the response of tall buildings to wind.

The interest in understanding human discomfort was initiated in the transportation research community as in [1], [2]. In Smith et al. [1], one such study reports subjective rating of the passengers for different road conditions and automobiles and its correlation with root mean square acceleration. This study involved measuring vertical accelerations from the floorboard of a vehicle and lateral acceleration at the passenger/seat interface. The results also indicate that humans are more receptive towards lateral acceleration during discomfort from vibrations.

Fundamental initial work was laid down by Davenport, who studied basic statistical distribution for winds with the purpose of determining sufficient safety factors in building construction [3, 4]. These studies considered only static loads and winds sampled from distributions.

Within the various disciplines of civil engineering a wide variety of building models are used for different applications. These models are often too complicated for the sort of analysis we present because of the numerous degrees of freedom and the resulting complexity. Recent interest in wind turbines has resulted in considerable research in realistic statistical models for wind. Kaminsky et al. [5] summarizes several different approaches to model wind as a stochastic process, all of which attempt to accurately reproduce the autocorrelation of wind at varying time scales. However, even these methods are known to only partially reproduce the necessary properties of realistic wind as reported in [6].

Previous authors have considered the problem of the optimal control of damping systems [7, 8, 9, 10]. Their approaches provide considerable insight in to the system, but they exclusively consider static metrics for structural concerns, as opposed to dynamic metrics such as the one we consider here.

In Sect. II, we present the structural model with reduced no of degrees of freedom for proof of concepts of optimal control in minimizing the inhabitant discomfort. In Sect. III, we present the control strategies we envision for the problem and present the implementation details and conceptual challenges. Further in Sect. IV, we present dynamic response of the structural model for a test wind loading to indicate what metric should be used in optimizing the objective function and in Sect. V, we discuss future directions and current understanding of the model and control methods.

II. Modeling

Our approach aims at exploring the optimal control of a structure under wind excitation by using a reduced order model which still captures the dynamics of the structure relevant to the design of a damper. Hence we are faced with the following challenges:

- Previous research in structural health monitoring ([7, 8, 9, 10]) has focused on **ground excitation** (ie. earthquakes), rather than **wind excitation**. Ground excitation can be modeled as a family of periodic base perturbations, whereas wind excitation is inherently **stochastic**.
- Structural engineers have been analyzing wind gust response for a very long time([3, 4]), but their models are not directly applicable here.
  - They analyze the response to a **constant** gust, then use a statistical analysis to find the strongest gust that needs to be considered for a given factor of safety. We are concerned with a dynamic wind over time.
  - Their models tend to be excessively complex with many degrees of freedom.

Thus none of these methods are directly applicable to what were are trying to do here. We must adapt a combination of them for our optimal control study.

Structural Model

We model the system as a simple cantilever beam, with the damper mounted on the top floor. The displacements of the building and damper are measured at the top floor, from the centerline of equilibrium position of the building. Although the cantilever beam has a response along its entire length, we assume that the beam vibrates in its first few higher modes. Thus we can simplify the model using a lumped mass and assuming the wind forcing at the top of the building as shown in Fig. 1

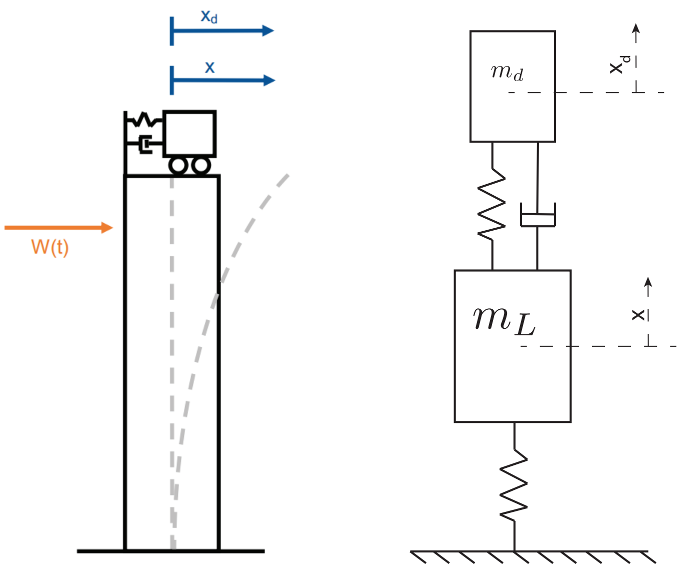


Figure 1. The building is transformed to a lumped mass cantilever beam, where the wind acts only on the top floor.

The Thompson lumped-mass equivalence says that if the distributed-mass beam has mass per unit length m and has length L, the dynamically equivalent lumped mass is given by

$$m_L = \frac{33}{140} mL \tag{1}$$

With these assumptions the model reduces to a coupled mass-spring system with no damping from the structure itself. The equations of motion has been derived in the appendix for interested readers and describes briefly subsequent reduction to first order form shown in Eqn. 9. Here we will briefly describe the approach for analyzing the linear time invariant system of Eqn. 9 under wind loading, $W(t)$ as input.

$$\begin{Bmatrix} \dot{x} \\ \dot{x}_d \\ \dot{y} \\ \dot{y}_d \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k+k_d}{m_L} & \frac{k_d}{m_L} & -\frac{c_d}{m_L} & \frac{c_d}{m_L} \\ \frac{k_d}{m_d} & -\frac{k_d}{m_d} & \frac{c_d}{m_d} & -\frac{c_d}{m_d} \end{bmatrix} \begin{Bmatrix} x \\ x_d \\ y \\ y_d \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_L} & 0 \\ 0 & \frac{1}{m_d} \end{bmatrix} W(t) \tag{2}$$

In this formulation, the state space consists of position and velocity of the structure as $(x, \dot{x}) = (x, y)$ and damper as $(x_d, \dot{x}_d = x_d, y_d)$ as shown in Eqn. 2. The co-efficient of damping denoted by c_d which is due to the damper alone has been assumed to have a form similar to Rayleigh damping i.e.,

$$c_d = \alpha m_d + \beta k_d$$

where α and β are real scalars with units 1/sec and sec respectively. In modal analysis method of vibration, this is not a conventional form which assumes a linear combination of mass and stiffness matrix. The major advantage from this assumption is to convert the damping matrix into an equivalent Rayleigh damping so that using orthogonal transformation a structure having N degrees of freedom can be reduced to N-number of uncoupled equations. However, for systems with large degrees of freedom, it is difficult to guess meaningful values of α and β at the start of the analysis. Hence, we have assumed a similar form to tackle this guessing problem using an optimal strategy. On the other hand, the simplified model of a high rise building is used to test optimal control of damper characteristics to minimize inhabitant discomfort and avoiding higher degree of freedom. This results in abstracting the problem for a proof of concept and so that design engineers can implement practical dampers with tunable parameters.

Synthetic wind speed and loading

As discussed above, we model the wind loading as a point load and assume it is a one-step Markov process i.e., it only remembers the current wind state[1] for the transition to the next state. In terms of probability of transition, it can be stated as:

$$P(X(t_n) = x_n | X(t_1) = x_1, ..., X(t_{n-1}) = x_{n-1})) = P(X(t_n) = x_n | X(t_{n-1} = x_{n-1}))$$

We then use the following steps to synthesize wind speed for the model:

1) Constructing state transition matrix(or transition probability matrix,$\mathbf{P}$) for all the available wind states. For our purpose, we have used the empirical matrix given in [5]. Obtaining this from a time series of wind speed is dealt with more rigor in the community of wind energy harvesting and structural health monitoring. In those applications, the characteristics of the locally prevalent wind is required for design of wind turbines and structures which is dependent on topography, local seasonal and annual weather.
2) Compute the cumulative probability matrix($\mathbf{C}$) for the state transition matrix as:

$$C_{ij} = \sum_{k=1}^{j} P_{ik}$$

3) Initialize the state(s) with a random integer in $(1, N_s)$, where N_s is the number of wind states.
4) The current state s corresponds to a row in $\mathbf{C}$ and to obtain the next wind interval, we generate a random number R from the uniform random distribution over $(0, 1)$ and the next state will be K where

$$C_{ik-1} \leq R \leq C_{ik}$$

Thus, iterating the final step we can synthesize a time series of wind speed as shown in Fig 2. For the analysis

[1]For our purpose, wind state is defined as a closed interval of wind speed after partitioning the range of wind prevalent at a given site in equal diameter.

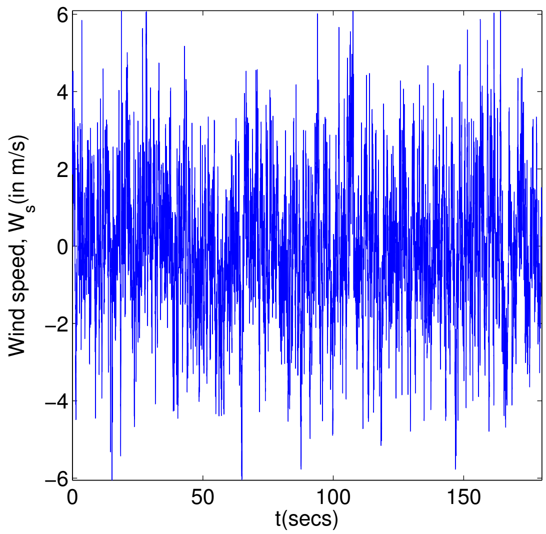


Figure 2. A synthetic wind loading generated using the state transition matrix given in [5] for Windsor, MA with a sampling frequency of 50 Hz.

shown, we are using the empirical state transition matrix given in Kaminsky et al. [5]. For a more practical approach, time series data for a particular location should be used for computing the state transition matrix for the synthetic wind speed.

In structural design, wind is classically modeled as a static wind pressure varying along the vertical profile of the building and the time variation is introduced by a gust factor. This converts the wind excitation to a static equivalent load and as such becomes obsolete for dynamic response analysis. Since we are interested in the discomfort of inhabitants due to excessive sway resulting from vibrations, a static model of wind is useless for this analysis. Although the spatial distribution of wind load makes this model complete by introducing other floors into the objective function of human discomfort but we will rest this extension until further studies.

III. Optimal Control

The problem then turns to computing optimal control (or trajectories) in this system. We have selected the metric *minimize the maximum velocity on the top floor*. Velocity and acceleration both affect the induced nausea in a complex and interconnected manner [11]. We choose to minimize the velocity because it has a direct representation in this model, and in a continually forced oscillating system such as this low velocities generally correspond to low accelerations. A more advanced metric could consider a function of both acceleration and velocity, giving their combined effect on nausea.

However, this deterministic statement of the metric is not actually well-defined, because the system is stochastic. Instead, we seek to *minimize the expected value of the maximum speed on the top floor over the next T time*.

Before we state an optimal control scheme for the system under this metric, we will note several properties of this system that must be considered to find optimal trajectories:

- **State includes wind** The idea of a *state* for the building must include the wind it is currently experiencing. This is because the distribution of future loads the wind could give depends on the current wind, and thus our control strategy must also depend on the current wind. Therefore we must include the wind in our concept of state. A state is then given by $s = \{x, y, x_d, y_d, W\}$.

- **Stochastic wind is not a distribution** The Markov chain formulation of wind describes transitions between a finite set of wind states. This can be considered as an approximation of a continuous processes, but even then it is state-dependent and not described by any distribution. This excludes the use of techniques in traditional stochastic optimal control, such as Linear-Quadratic-Gaussian control, which assume the random input is described by some distribution, usually Gaussian

- **The optimal trajectory is only followed for one timestep** Even though we phrase our optimal control in terms of trajectories, and the optimal solution will be a trajectory, we are still effectively developing a single step control rule. This is because as soon as we observe the wind at time $t + \Delta t$, we will update the control policy accordingly. It would be unreasonable to attempt to follow an entire future trajectory when new wind values are constantly being observed.

The stochastic process nature of the wind input excludes the use of differential techniques like Pontryagin's Minimum Principle. Instead, we will make use of the discrete formulation of the Hamilton-Jacobi-Bellman equation, referred to as simply the Bellman Equation, which is stated generically as:

$$V(x_t) = \max_{a_t}\{F(x_t, a_t) + \beta V(x_{t+1})\} \tag{3}$$

When the state is suitably discretized and the random wind input is included, this transforms the problem to a *Markov Decision Process*. These problems are studied in finance and operations research, although there is no straightforward general process for their direct solution.

For the discrete solution to be applicable, all components of the state vector for the system are discretized to create a finite set of states S. Each continuous component is divided in to N equally size state buckets within some bounded range. Together, the four discretizations result in N^4 discrete states for the entire system. This is a significant shortcoming which is expanded on in the results and conclusions.

The problem is framed by considering the set of all possible states at all future times within the horizon distance T. Due to the discretizations of the state and time, this set is finite. Each of these states has a probability p_s associated with it, which gives the likelihood that the system will be in that state at that time. At the current time the system is in some state s_0 with probability $P = 1$. We wish to find the control for the current state s_0 which minimizes the expected value of the cost for the current state. However, that cost depends on the cost of all of the subsequent states, and so forth to time T. This recursive formulation admits a discrete dynamic programming algorithm.

Dynamic Programming Algorithm

We follow the dominant approach in the analysis of Markov Decision Processes and design a dynamic programming algorithm to solve the problem. Dynamic programming algorithms are characterized by utilizing the overlapping substructure of the problem, in this case the recursive cost formulation, to build up a solution piece by piece.

As a general outline, the algorithm begins by considering all of the possible states at time T, for each of these states, the cost is exactly known. The algorithm then considers each of the states at time $T - \Delta t$. The cost at each of these states is minimized by choosing the control that minimizes the expected cost over the next timestep, where the costs have already been computed, in addition to the cost from the current step. This process continues iteratively backwards in time until the cost and optimal control of the initial state s_0 are computed.

`Optimal_Parameter_Dynamic_Programming`
Computes the optimal damping control at time t_0 by minimizing the expected trajectory cost over a horizon with length T.

- Compute an initial cost array at the horizon time $t_0 + T$ (5 dimensional with an entry for every discretized state). Each state has a cost given by its y value.
- Iterate backwards from time $t_0 + T$ to t_0:
  - For each state s at time t_i
    - Consider each possible damping value, initialize a cost to 0 for that value.
    - For every next wind, which occurs with probability p, find the next state s_{next} that would be transitioned to.
    - Look up the cost of s_{next}, c_{next} in the array for t_{i+1}, and add $p * c_{next}$ to the cost for this damping value. If the current state's y value is greater than c_{next}, add $p * y$ instead.
    - Select the damping value with minimal cost as optimal and store the cost in the array for this timestep.
- When the outer loop runs for the last timestep (the current time), the actual optimal control is the control selected for the actual state s_0.

Figure 3. The dynamic programming algorithm developed for optimal trajectories.

A. Algorithm Runtime Analysis

The main pitfall of this is the computational cost. If we discretized each of the state dimensions in to n_{disc} segments, the model has n_{disc}^4 total states. Performing each update step requires considering the transition from each state under each possible control value, for a total cost of n_{disc}^5. Repeating this process over n_{time} time steps gives an algorithmic cost of $\mathcal{O}(n_{time}\dot{n}_{disc}^5)$.

This runtime renders the algorithm completely infeasible for all but the smallest discretizations. Even if heuristic improvements were made, the requirement to consider every state at each timestep places algorithm limits on the performance. Clearly, the value of this formulation is more theoretical than practical in its current form.

IV. Results

For preliminary numerics, we make use of combining the state space model with a measurement equation which is a standard technique ([see 12]) for obtaining response from a linear time invariant(LTI) system. This can be stated in a matrix form as:

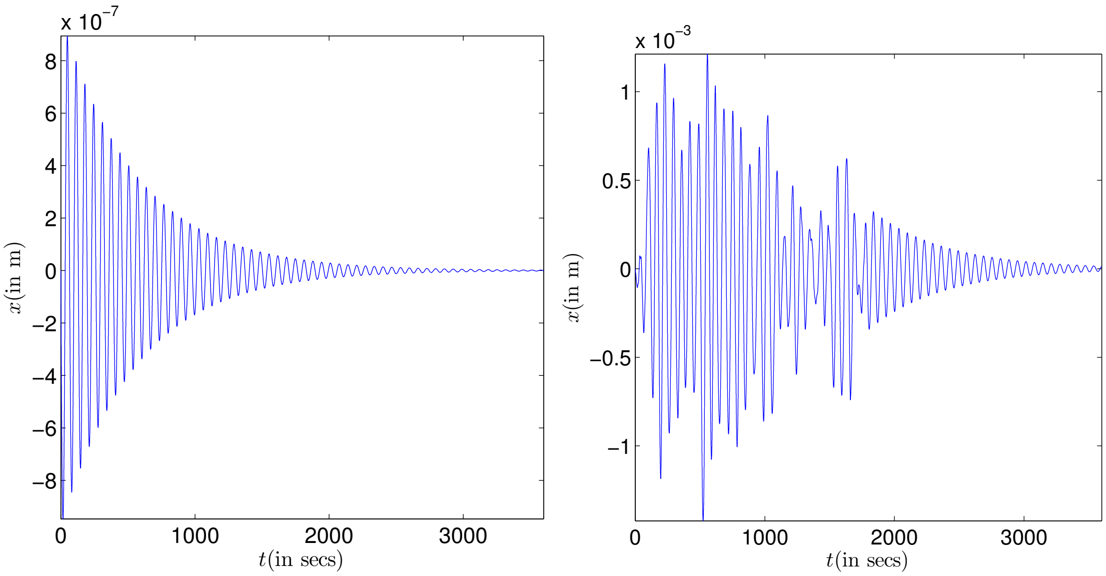
$$\dot{s} = \mathbf{A}s + \mathbf{B}w \tag{4}$$

$$z = \mathbf{P}s + \mathbf{Q}w \tag{5}$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{I}_{N\times N} & \mathbf{0}_{N\times N} \\ \mathbf{0}_{N\times N} & \mathbf{I}_{N\times N} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \ \mathbf{Q} = \begin{bmatrix} \mathbf{0}_{N\times N} \\ \mathbf{0}_{N\times N} \\ -\mathbf{M}^{-1} \end{bmatrix}$$

where, N is the number of degrees of freedom. In this formulation, we are measuring all the three motion variables i.e., position, velocity and acceleration for a given degree of freedom. Here, Eqn. 4 can be identified from Eqn. 2 with states, $s = \{x, x_d, \dot{x}, \dot{x}_d\}$. For the structural model used in this study, $\mathbf{A}$ and $\mathbf{B}$ is given by Eqn. 10 as shown in the Appendix. A. Eqn. 5 represents the measurement equation with $z = \{x, x_d, \dot{x}, \dot{x}_d, \ddot{x}, \ddot{x}_d\}$ and matrices $\mathbf{P}_{6\times 4}$ and $\mathbf{Q}_{6\times 2}$. This form is well-established in structural health monitoring and filter design where the input vector w represents control applied to the state space model and the measurement equation. The measurement equations record evolution of the variables in numerical computations and represent variables which can be measured using sensors during experiments.

Being a LTI system, the numerical errors scale linearly with time steps and possess numerical stability but it is the stochastic wind load w which renders the complexity and interesting dynamics underlying this system. First, we test our model with an impulse forcing (using impulse function in MATLAB) which physically represents the unit delta force at the initial time and for high enough damping should result in a decay of vibrations. This is shown in Fig. 4(a) and shows a typical decay response expected from a damped vibration system. While, the Fig. 4(b) shows the response due to wind excitation (using lsim function in MATLAB) which is acting on the structure for 30 minutes and then the oscillations die out. The parameters chosen for this simulation are such that the structure is only 10 times heavier than the damper and hence the damping is due to, for the most part, the a heavy mass attached to the structure. This is an obvious finding which is used for absorbing vibration energy in tall structures all over the world and we are able to capture using the simplified model and very crude assumptions.



(a) (b)

Figure 4. Fig. 4(a) shows displacement response for an impulse loading and Fig. 4(b) shows the same for the synthetic wind load shown in Fig. 5(a) sampled at 30 Hz for 30 minutes Parameters used for this response: $\alpha = 0.2$ and $\beta = 10^{-3}$, $m_L = 10^7$,$m_d = 10^6$, $k = 10^5$, $k_d = 1$ for the mass and stiffness of the structure and damper respectively in SI units. Using these parameters, the damping ratio corresponding to the first two mode is 1%

Further, we show some response from the model with parameters that represent a structure with very light damper in Fig. 5(b), 5(c), 5(d). As the numerics indicate this system doesn't have enough mass to absorb energy and the oscillations are not damped. The other observation is that the rate of change in the response of displacement, velocity and acceleration increase in the same order. The acceleration has the most correlation with human discomfort when traveling or in a seated posture as was indicated in the Sect. I. This has implications towards selecting a norm based on acceleration and its rate of change to construct the objective function for our study.

The dynamic programming method suffers significantly from excessive computational cost. Runtimes for the method are infeasible unless the values are discretized very coarsely ($n < 5$). With this level of discretization, the entire system breaks down and the proper dynamics are not recovered. When the algorithm is applied and the response is computed, the motion of a structure appears to be a straight line. This is an artifact caused by the discretization. When the state "buckets" are so large, the system can never make it out of its initial bucket in a single timestep. Unfortunately, the dynamic programming algorithm appears completely inapplicable to realistic problems due to its computational complexity.

V. Discussions

In this work, we provide a thorough and rigorous analysis of a swaying building subject to realistic stochastic wind loads.

A reduced order model of a damped building is developed. This model has few degrees of freedom, which permits effective optimal control, but still resolves the dynamics of the more complex structural models.

The relationship between the stochastic wind input and the resulting control is crucial in the development of control technique. Although Gaussian models are easier to analyze, they do not reproduce important physical characteristics of wind. We demonstrate the use of Markov models for more realistic wind modeling and analyze the implications for optimal control. We conclude that differential approaches such as Pontryagin's principles are not applicable, and instead turn to dynamic programming. While the dynamic programming approach permits the optimization of an otherwise intractable system, its computational cost impedes its use on realistic problems. So we present a brief overview of the approach. Future work will seek to modify the method algorithmically to allow it to be used with discretizations sufficiently fine to reproduce the desired dynamics. The authors are planning to follow up the work by Jerg et al. [13] which is an extension to stochastic input for two-degree of freedom system and uses feedback controllers for stabilization.

Appendix

Equations of motion

$$\mathcal{L} = T - V = \left\{\frac{1}{2}m_d\dot{x}_d^2 + \frac{1}{2}m_L\dot{x}^2\right\} - \left\{\frac{1}{2}kx^2 + \frac{1}{2}k_d(x - x_d)^2\right\} \quad (6)$$

For the structure with lumped mass m_L, the Lagrange's equation is given by:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = Q_{nc_1} + W(t)$$

$$(m_L\ddot{x}) - \{-(kx + k_d(x - x_d))\} = -c_d(\dot{x} - \dot{x}_d) + W(t)$$

$$m_L\ddot{x} + c_d(\dot{x} - \dot{x}_d) + kx + k_d(x - x_d) = W(t) \quad (7)$$

For the damper with mass m_d, the Lagrange's equation is

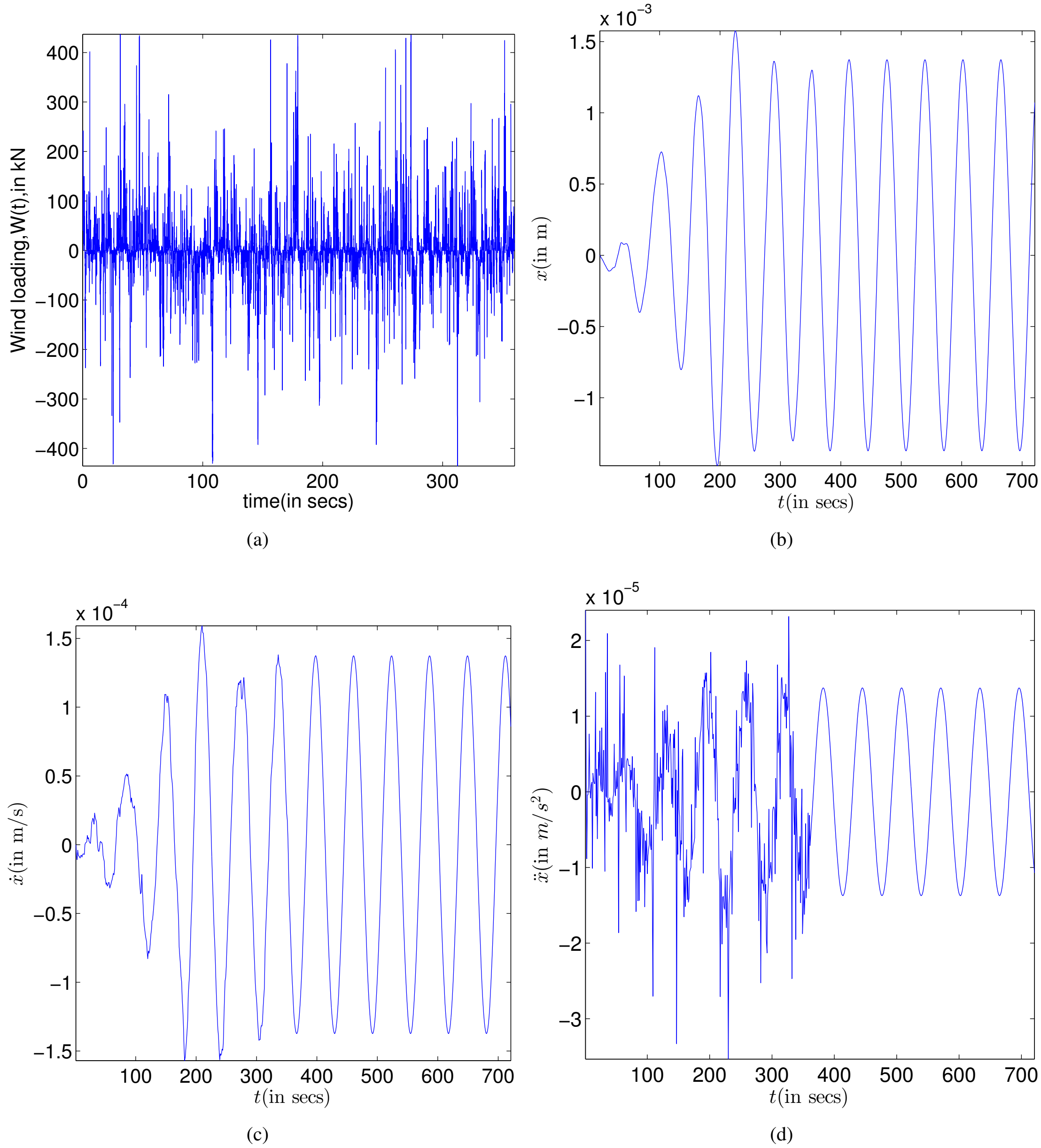


Figure 5. Fig. 5(a) shows the time series of wind loading used in the simulation, sampled at 30 Hz for 6 minutes. This corresponds to 6 minutes of wind loading after which the wind ceases. Fig. 5(b), Fig. 5(c) and Fig. 5(d) shows the displacement, velocity and acceleration response with $\alpha = 10^{-3}$ and $\beta = 10^{-3}$ for Rayleigh damping. $m_L = 10^7$,$m_d = 10^4$, $k = 10^5$, $k_d = 1$ for the mass and stiffness of the structure and damper respectively in SI units. Using these parameters, the damping ratio corresponding to the first two mode is $5.1 \times 10^{-4}\%$

given by:

$$\begin{aligned}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_d}\right) - \frac{\partial L}{\partial x_d} &= Q_{nc_2} \\ (m_d \ddot{x}_d) - \{-(k_d(x_d - x))\} &= -c_d(\dot{x}_d - \dot{x}) \\ m_d \ddot{x}_d + c_d(\dot{x}_d - \dot{x}) + k_d(x_d - x) &= 0 \end{aligned} \tag{8}$$

Generally in dynamical systems, a set of $M-$equations in the form of second order ODEs is first non-dimensionalized to isolate the parameters of the system and then converted to $2M-$first order ODEs. Here we will convert these equations to first order form but avoid non-dimensionalizing it, as we will identify the parameters from classical vibration problems.

Writing the equations in terms of position and velocity results in:

$$\begin{aligned}\ddot{x} &= -\frac{c_d}{m_L}(\dot{x} - \dot{x}_d) - \frac{k}{m_L}x - \frac{k_d}{m_L}(x - x_d) + \frac{W(t)}{m_L} \\ \ddot{x}_d &= -\frac{c_d}{m_d}(\dot{x}_d - \dot{x}) - \frac{k_d}{m_d}(x_d - x)\end{aligned}$$

Let us assume a form, similar to Rayleigh damping, for the damper to be

$$c_d = \alpha m_d + \beta k_d$$

where α and β are real scalars with units 1/sec and sec respectively. Now, we introduce the following variables to transform it into first order forms:

$$\dot{x} = y,\ \dot{x}_d = y_d,\ \ddot{x} = \dot{y}, \ddot{x}_d = \dot{y}_d$$

Thus, after a bit of rearranging and linear algebra, we can express this as Eqn. 9 with $\mathbf{A}$ and $\mathbf{B}$ defined in Eqn. 10 and N denotes the degrees of freedom in the system.

$$\dot{s} = \mathbf{A}s + \mathbf{B}w \tag{9}$$

with,

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_{N\times N} & \mathbf{I}_{N\times N} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix},\ \mathbf{B} = \begin{bmatrix} \mathbf{0}_{N\times N} \\ -\mathbf{M}^{-1} \end{bmatrix} \tag{10}$$

where,

$$\mathbf{M} = \begin{bmatrix} m_L & 0 \\ 0 & m_d \end{bmatrix} \text{is the mass matrix} \tag{11}$$

$$\mathbf{C} = \begin{bmatrix} c_d & -c_d \\ -c_d & c_d \end{bmatrix} \text{is the damping matrix} \tag{12}$$

$$\mathbf{K} = \begin{bmatrix} k + k_d & -k_d \\ -k_d & k_d \end{bmatrix} \text{is the stiffness matrix} \tag{13}$$

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