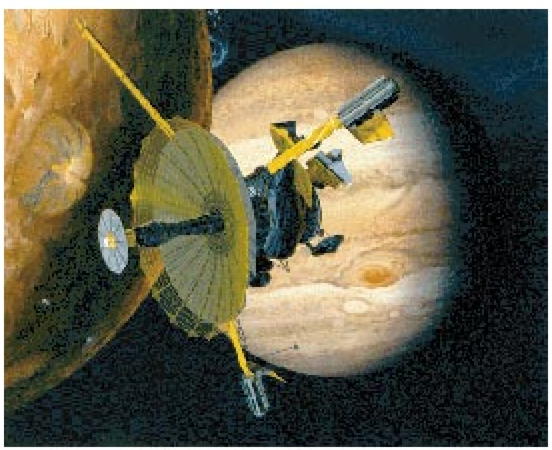
New Class for Spring Term 2000 !

CDS 270: Dynamical Systems and Space Mission Design

Instructors: **Marsden, Lo, Koon, Ross**

Mondays and Fridays, 1:30 to 3 pm, 125 Steele



Course Goals: Guided by a new understanding of the three-body problem, we apply basic tools of dynamical systems theory to new classes of space mission trajectories. Actual missions and new concepts will be covered.

Prerequisites: Some familiarity with mechanics, dynamical systems theory and numerical computation.

For more information, see http://www.cds.caltech.edu and follow course links to CDS 270-1

Dynamical Systems, and Space Mission Design

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Control and Dynamical Systems and JPL
California Institute of Technology

CDS 270-1, Lecture 1, March 27, 2000

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Structure of the Talk

- ***Part 1***: Homoclinic and heteroclinic structures related to the collinear libration points in the the three body problem and its relation to resonant transition of comets and the Genesis mission.
- ***Part 2***: Petit Grand Tour of the Moons of Jupiter (Ganymede and Europa) and Lunar Capture (Efficient missions to the Moon).
- ***Part 3***: Optimal control for halo orbit insertion. (Collaboration with ***Radu Serban, Linda Petzold and Roby Wilson***)

Acknowledgments

- *H. Poincaré, J. Moser*
- *C. Conley, R. McGehee*
- *R. Farquhar et al.*
- *C. Simó, J. Libre, R. Martinez*
- *G. Gómez, J. Masdemont*
- *E. Belbruno, B. Marsden, J. Miller*
- *K. Howell and the Purdue group*
- *P. Chodas, D. Yeomans, A. Chamberlin, R. Wilson*
- *Genesis mission design team*
- *M. Dellnitz & Paderborn dynamical systems group*

More Information and References

- Part 1:

  Koon, W.S., M.W. Lo, J.E. Marsden and S.D. Ross [1999] ***Heteroclinic connections between periodic orbits and resonance transitions in celestial mechanics***,

  available from
  http://www.cds.caltech.edu/~marsden/

- Parts 2 and 3 are covered in papers in preparation.

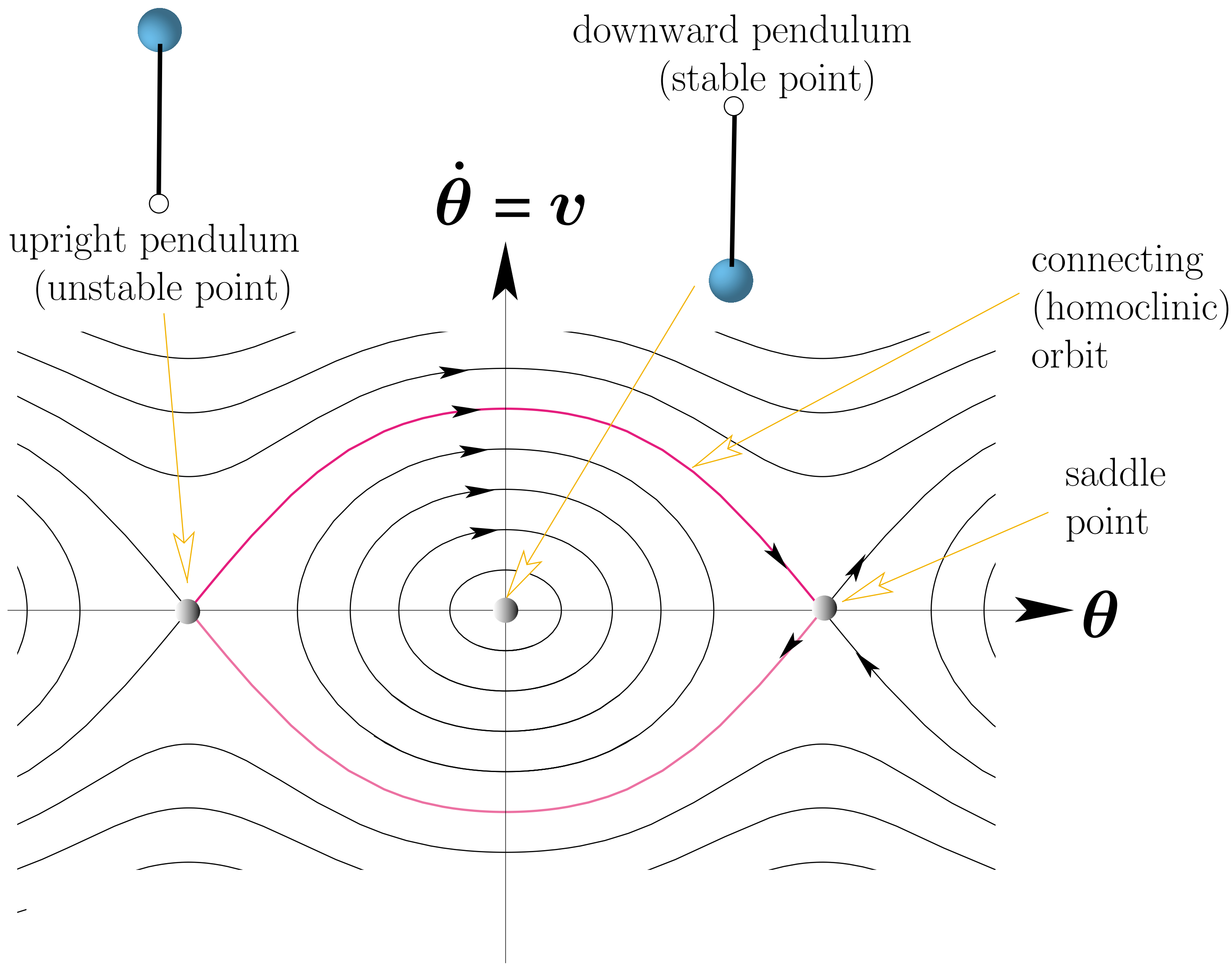
Connecting Orbits

Simple Pendulum

- Equations of a ***simple pendulum*** are $\ddot{\theta} + \sin\theta = 0$.
- Write as a ***system in the plane***;

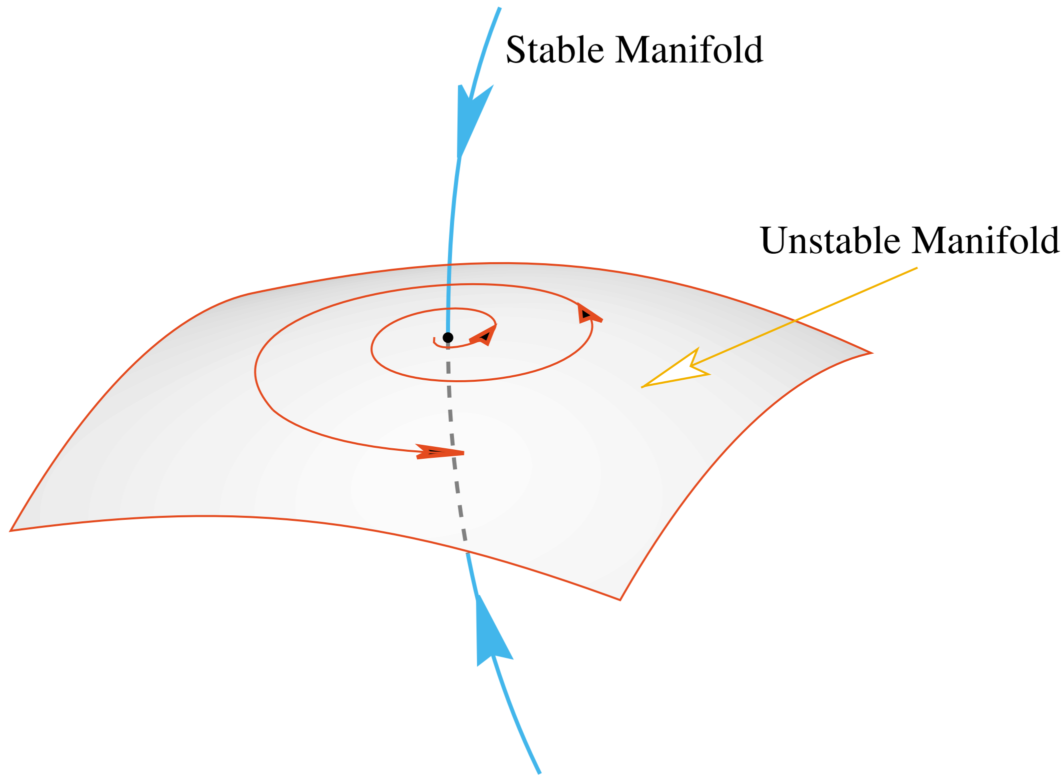
$$\frac{d\theta}{dt} = v$$
$$\frac{dv}{dt} = -\sin\theta$$

- Solutions are trajectories in the plane.
- The resulting ***phase portrait*** shows some important basic features:



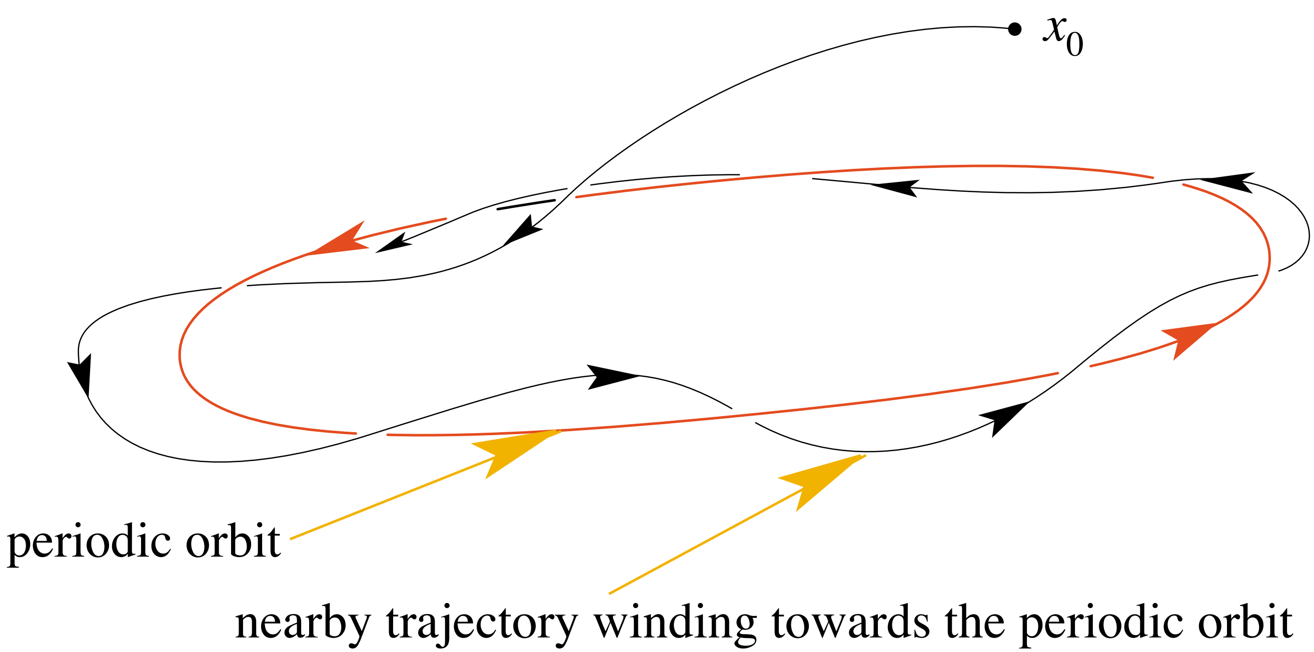
downward pendulum
(stable point)
$\dot{\theta} = v$
upright pendulum
(unstable point)
connecting
(homoclinic)
orbit
saddle
point
θ

Higher Dimensional Versions are Invariant Manifolds



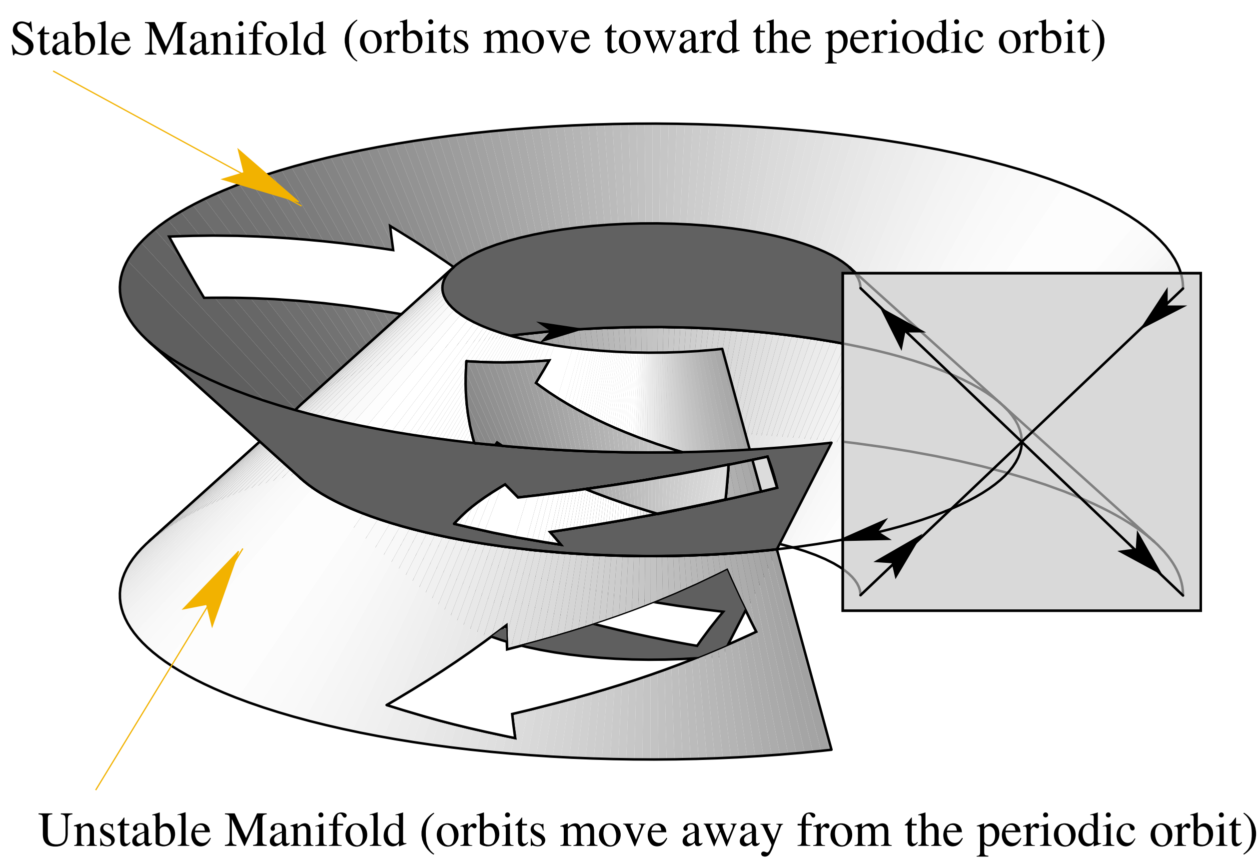
Periodic Orbits

- Can replace fixed points by ***periodic orbits*** and do similar things. For example, stability means nearby orbits stay nearby.



Invariant Manifolds for Periodic Orbits

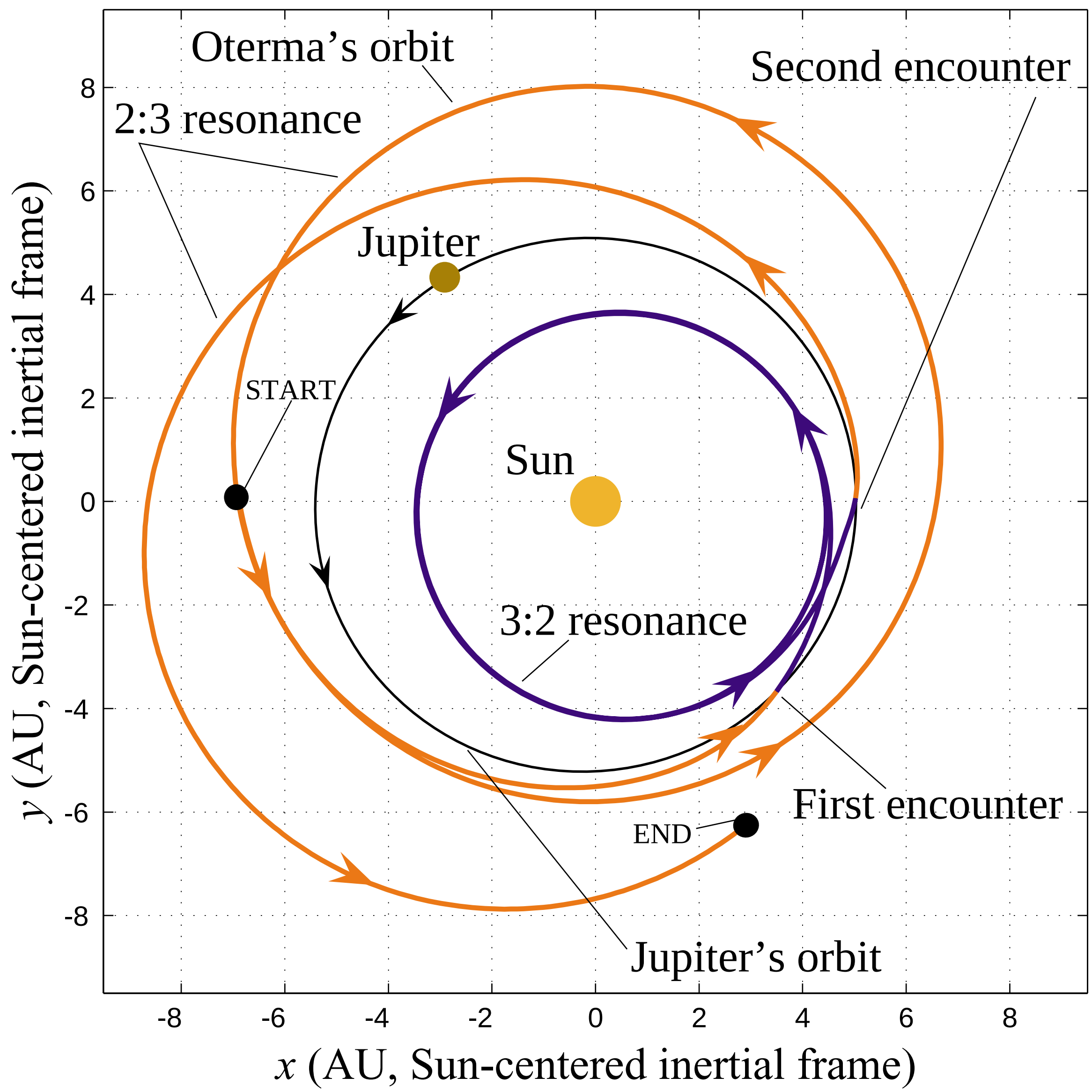
- Periodic orbits have ***stable and unstable manifolds***.



Resonant Transitions

■ Jupiter Comets–such as Oterma

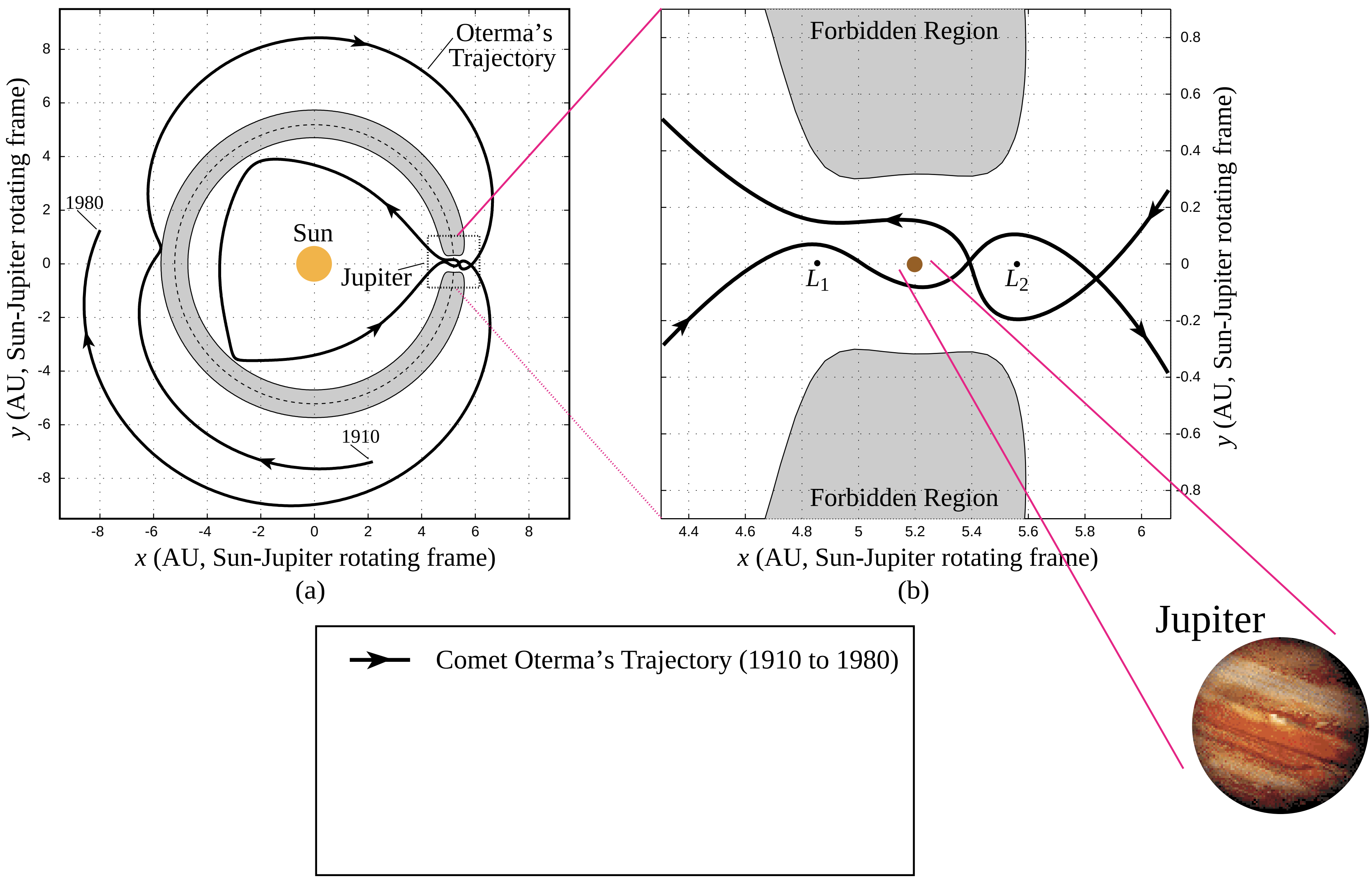
- Comets moving in the vicinity of Jupiter do so mainly under the influence of Jupiter and the Sun–*i.e.*, in a three body problem.
- These comets sometimes make a ***rapid transition*** from **outside** to **inside** Jupiter's orbit.
- ***Captured temporarily*** by Jupiter during transition.
- **Exterior** (2:3 resonance) $\longrightarrow$ **Interior** (3:2 resonance).
- The next figure shows the orbit of Oterma (AD 1915–1980) in an inertial frame



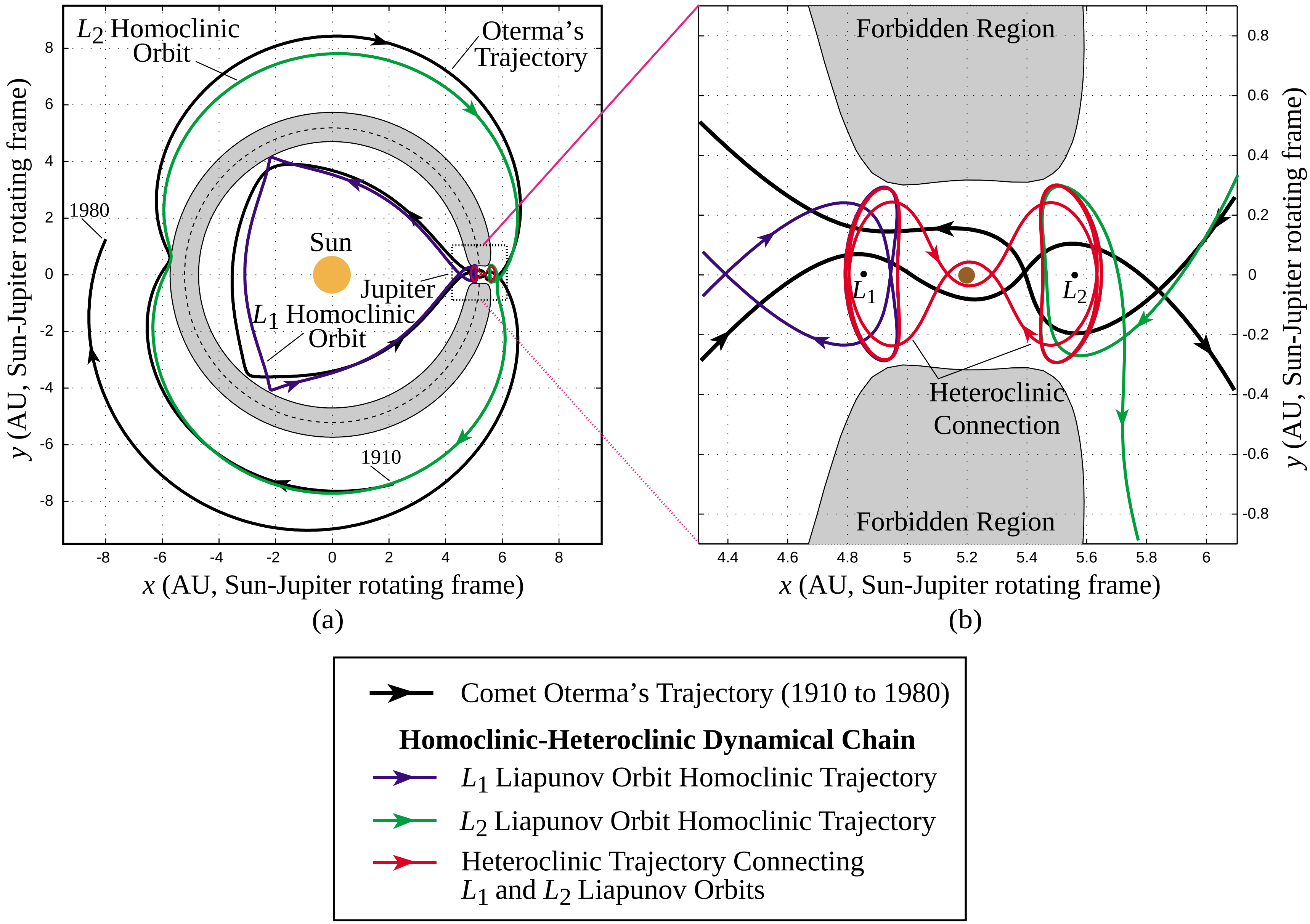
Oterma's orbit
Second encounter
2:3 resonance
Jupiter
START
Sun
3:2 resonance
First encounter
END
Jupiter's orbit
8
6
4
2
0
-2
-4
-6
-8
-8
-6
-4
-2
0
2
4
6
8
y (AU, Sun-centered inertial frame)
x (AU, Sun-centered inertial frame)

- Next figure shows Oterma's orbit in a ***rotating frame*** (so Jupiter looks like it is standing still) and with some invariant manifolds in the ***three body problem*** superimposed.

Orbit of Oterma in a Rotating Frame



Orbit of Oterma in a Rotating Frame with the Homoclinic-Heteroclinic Chain



- Now lets look at two ***movies of the trajectory of comet Oterma***, first in an ***inertial frame*** and then in a frame ***rotating with the sun and Jupiter***.

$$\boxed{\texttt{Movie: Oterma in inertial frame}}$$

$$\boxed{\texttt{Movie: Oterma in a rotating frame}}$$

The Planar Restricted Three Body Problem–PCR3BP

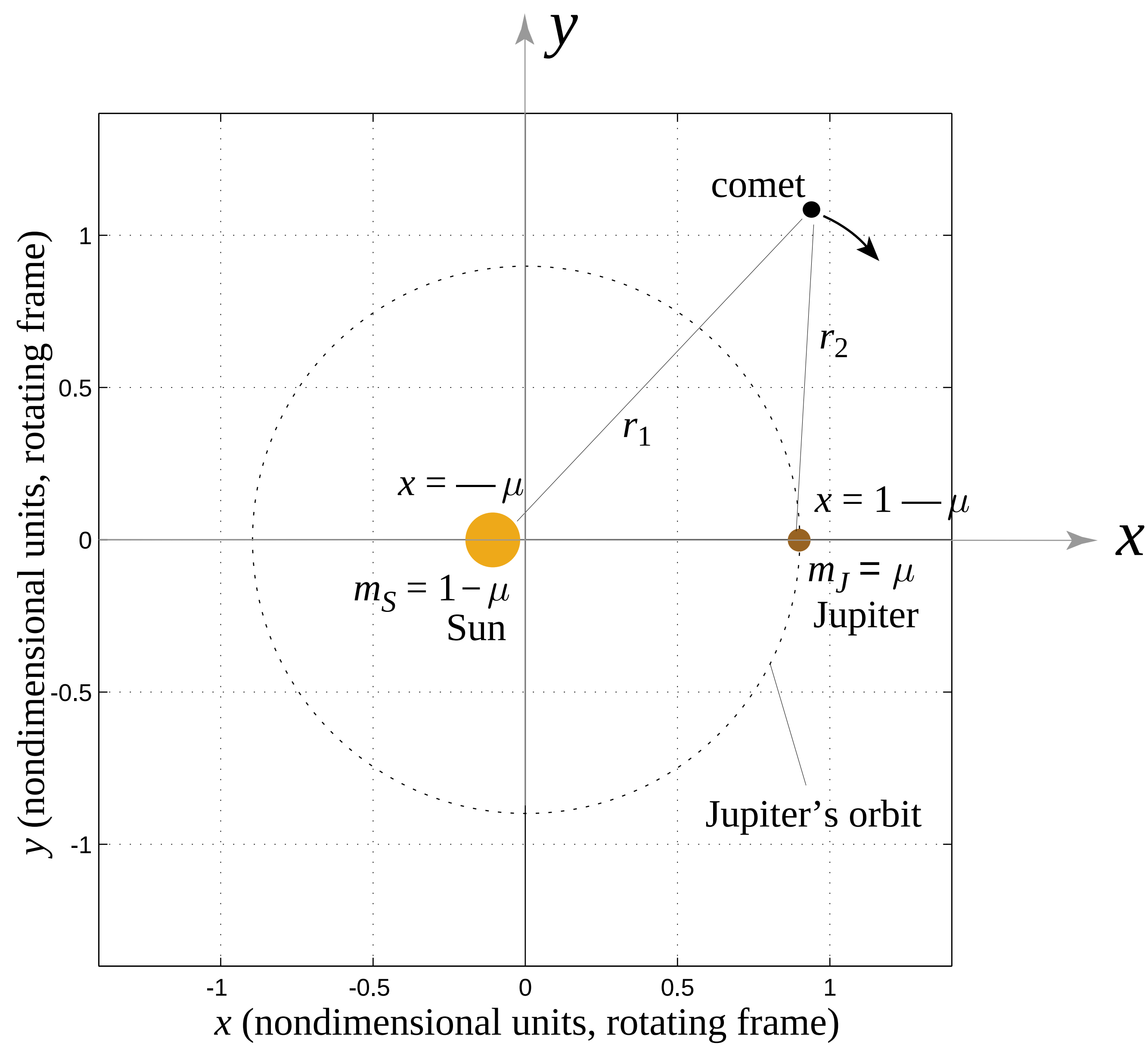
General Comments

- The two main bodies could be the ***Sun and Jupiter***, or the ***Sun and Earth***, etc. The total mass is normalized to 1; they are denoted $m_S = 1 - \mu$ and $m_J = \mu$, so $0 < \mu < 1$.
  - The two main bodies rotate in the plane in circles counterclockwise about their common center of mass and with angular velocity ω (also normalized to one).
  - The third body, the ***comet or the spacecraft***, has mass zero and is free to move in the plane.
- The ***planar*** restricted three body problem is used for simplicity. Generalization to the ***three dimensional problem*** is of course important, but many of the effects can be described well with the planar model.

Equations of Motion

- ***Notation:*** Choose a ***rotating coordinate system*** so that
  - the origin is at the center of mass
  - the Sun and Jupiter are on the x-axis at the points $(-\mu, 0)$ and $(1-\mu, 0)$ respectively–i.e., the distance from the sun to Jupiter is normalized to be unity.
  - Let (x, y) be the position of the comet in the plane relative to the positions of the Sun and Jupiter.
  - distances to the Sun and Jupiter:

$$r_1 = \sqrt{(x+\mu)^2 + y^2} \quad \text{and} \quad r_2 = \sqrt{(x-1+\mu)^2 + y^2}.$$



y
x
comet
r_1
r_2
$x = -\mu$
$x = 1 - \mu$
$m_S = 1 - \mu$
Sun
$m_J = \mu$
Jupiter
Jupiter's orbit
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1
x (nondimensional units, rotating frame)
y (nondimensional units, rotating frame)

- ***Kinetic energy*** (wrt inertial frame) in rotating coordinates:

$$K(x, y, \dot{x}, \dot{y}) = \frac{1}{2}\left[(\dot{x} - \omega y)^2 + (\dot{y} + \omega x)^2\right]$$

- The ***Lagrangian*** is K.E. $-$ P.E., given by

$$L(x, y, \dot{x}, \dot{y}) = K(x, y, \dot{x}, \dot{y}) - V(x, y),$$

where

$$V(x, y) = -\frac{1-\mu}{r_1} - \frac{\mu}{r_2}.$$

- ***Euler-Lagrange equations***:

$$\boxed{\ddot{x} - 2\omega\dot{y} = -\frac{\partial V_\omega}{\partial x}, \qquad \ddot{y} + 2\omega\dot{x} = -\frac{\partial V_\omega}{\partial y}}$$

where the ***augmented potential*** is

$$V_\omega = V - \frac{\omega^2(x^2 + y^2)}{2}.$$

- ***Legendre transform*** to get ***Hamiltonian form***.
- The ***Hamiltonian*** ($\neq K.E. + P.E.$) is

$$H = \frac{(p_x + \omega y)^2 + (p_y - \omega x)^2}{2} + V_\omega(x, y),$$

- Relationship between ***momenta*** and ***velocities***:

$$\dot{x} = \frac{\partial H}{\partial p_x} = p_x + \omega y; \qquad \dot{y} = \frac{\partial H}{\partial p_y} = p_y - \omega x.$$

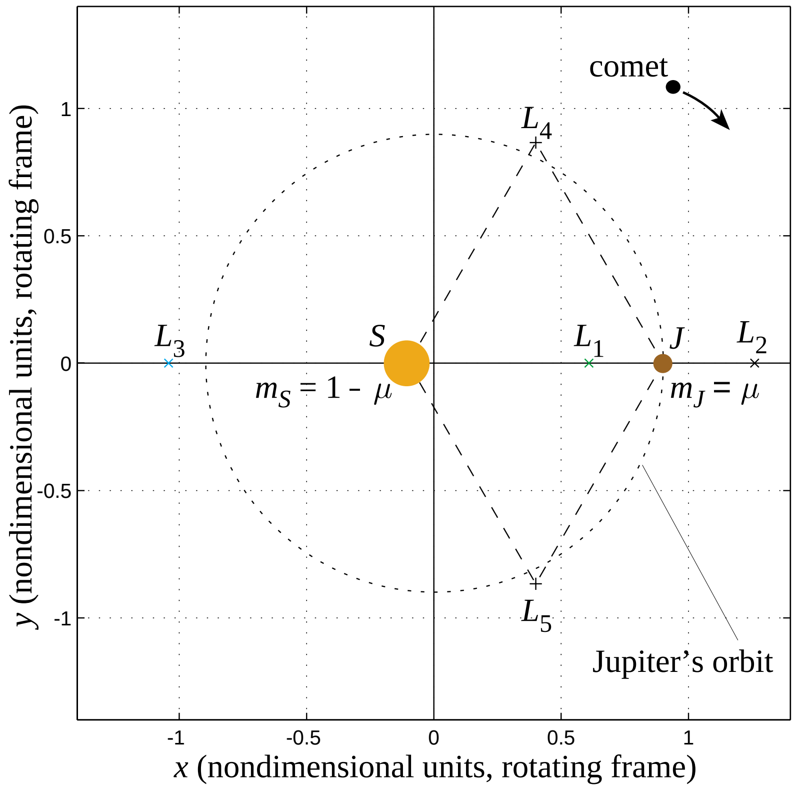
- ***Remaining dynamical equations***:

$$\begin{aligned} \dot{p}_x &= -\frac{\partial H}{\partial x} = p_y - \omega x - \frac{\partial V_\omega}{\partial x} \\ \dot{p}_y &= -\frac{\partial H}{\partial y} = -p_x - \omega y - \frac{\partial V_\omega}{\partial y}. \end{aligned}$$

- ***Jacobi constant*** is often written $C = -2H$.

Five Equilibrium Points

- Three ***collinear*** (Euler, 1750) on the x-axis— L_1, L_2, L_3
- Two ***equilateral points*** (Lagrange, 1760)— L_4, L_5.



Stability

- Eigenvalues of the linearized equations at L_1, L_2, L_3 have one real and one imaginary pair. The ***stable and unstable manifolds*** of these equilibria play an important role.
- Associated periodic orbits are called the ***Liapunov orbits***–Genesis targets a 3D counterpart, the ***halo orbits***. Their stable and unstable manifolds are also important.
- For space mission design, the most interesting equilibria are the ***unstable*** ones, not the stable ones!

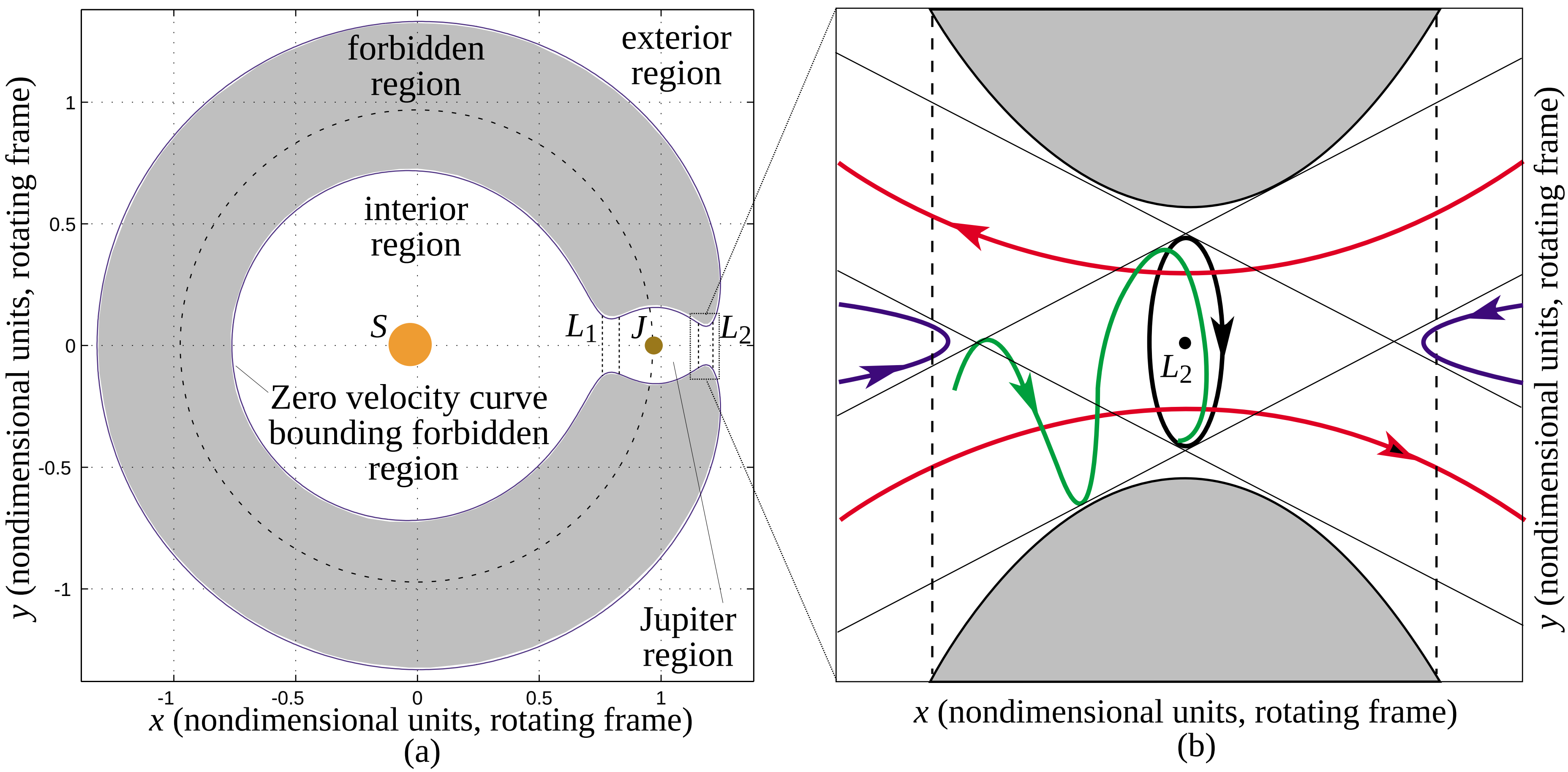
Consider the dynamics plus the control!

Control often makes unstable objects, not attractors, of interest![1] Under proper ***control management*** they are incredibly ***energy efficient***.

[1]This is related to ***control of chaos***; see Bloch, A.M. and J.E. Marsden [1989] Controlling homoclinic orbits, *Theor. & Comp. Fluid Mech.* **1**, 179–190.

Hill's Regions

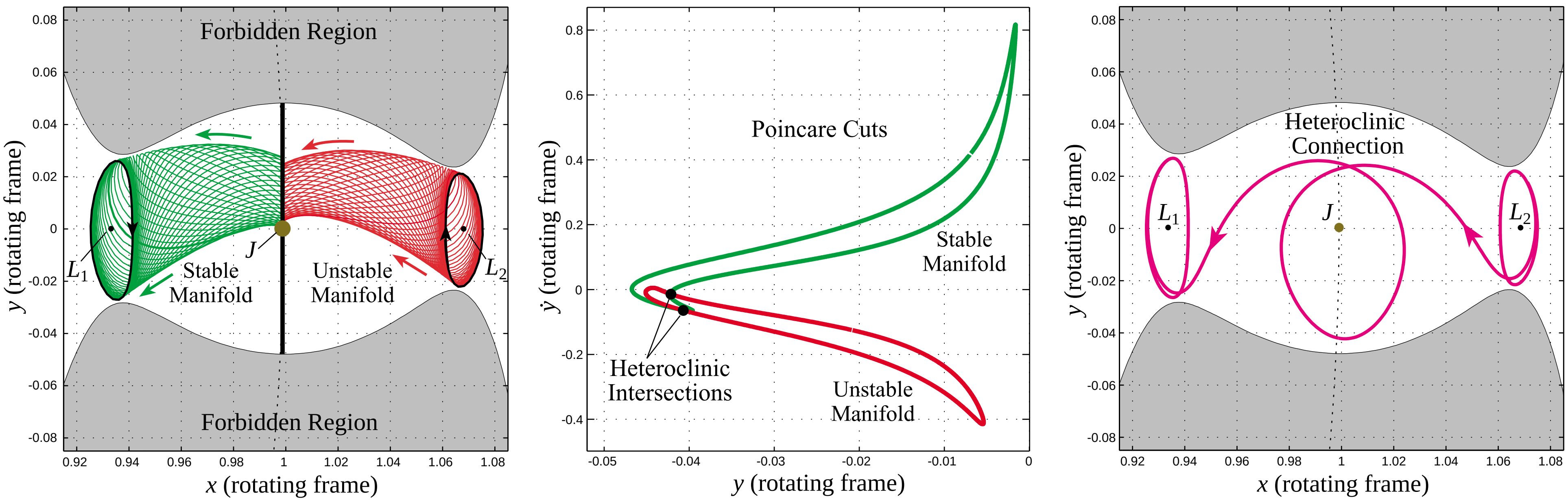
- Our main concern is the behavior of orbits whose energy is ***just above*** that of L_2.
- The ***Hill's region*** is the projection of this energy region onto position space.
- For this case, the Hill's region contains a "neck" about L_1 and L_2. This neck region and its relation to the global orbit structure is critical: it was studied in detail by ***Conley, McGehee and the Barcelona group***.
- Orbits with energy just above that of L_2 can be **transit orbits**, passing through the neck (Jupiter) region between the ***interior region*** (inside Jupiter's orbit) and the ***exterior region*** (outside Jupiter's orbit). They can also be **nontransit orbits** or **asymptotic orbits**



forbidden region
exterior region
interior region
S
J
L_1
L_2
Zero velocity curve bounding forbidden region
Jupiter region
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1
y (nondimensional units, rotating frame)
x (nondimensional units, rotating frame)
(a)
L_2
x (nondimensional units, rotating frame)
y (nondimensional units, rotating frame)
(b)

Heteroclinic Connection

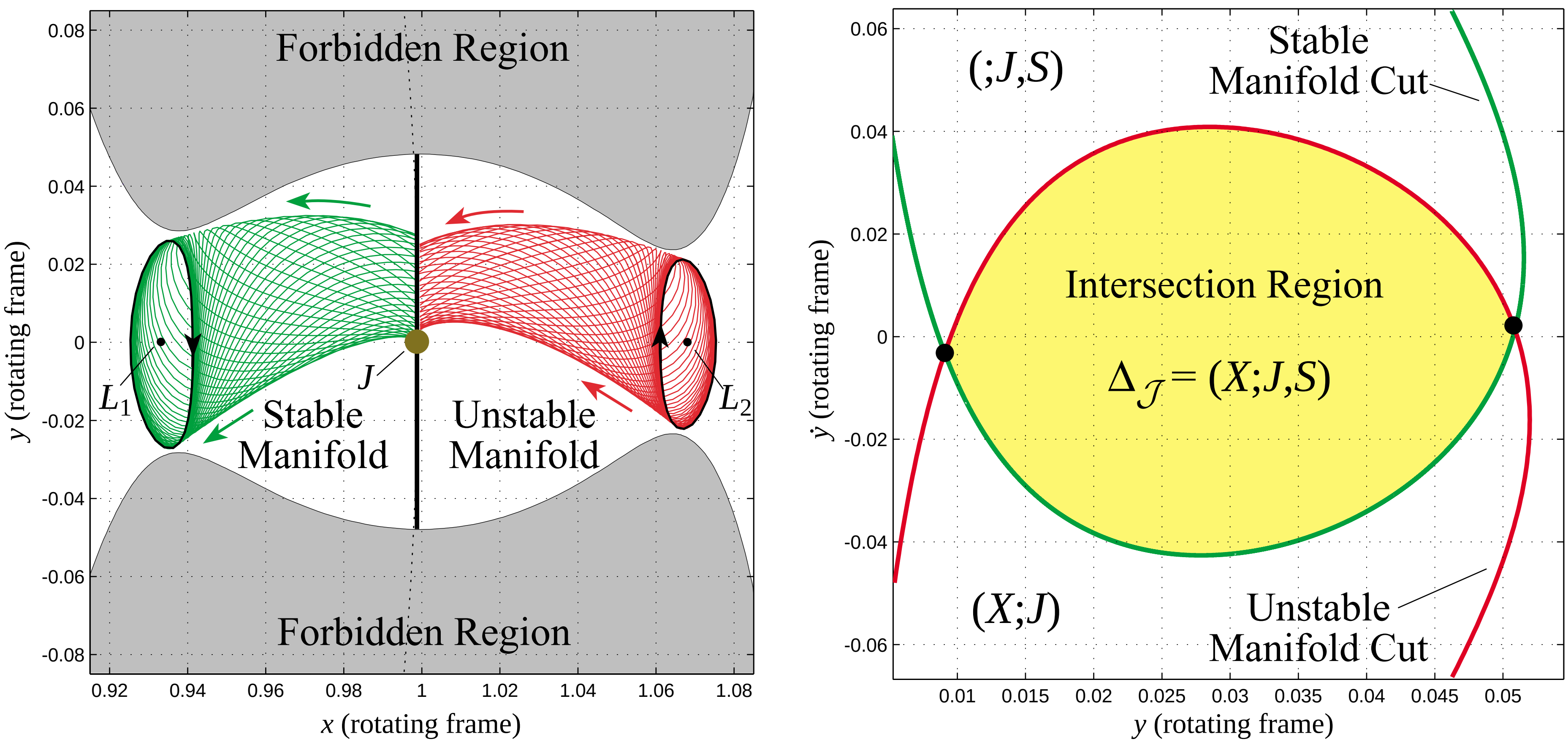
- These are located ***numerically*** by finding an intersection of the ***stable*** and ***unstable*** manifolds using a Poincaré cut.
- The stable and unstable manifolds are those of Liapunov orbits with ***the same Jacobi constant***.



Forbidden Region
Forbidden Region
L_1
L_2
J
Stable Manifold
Unstable Manifold
x (rotating frame)
y (rotating frame)
Poincare Cuts
Heteroclinic Intersections
Stable Manifold
Unstable Manifold
y (rotating frame)
$\dot{y}$ (rotating frame)
Heteroclinic Connection
L_1
J
L_2
x (rotating frame)
y (rotating frame)

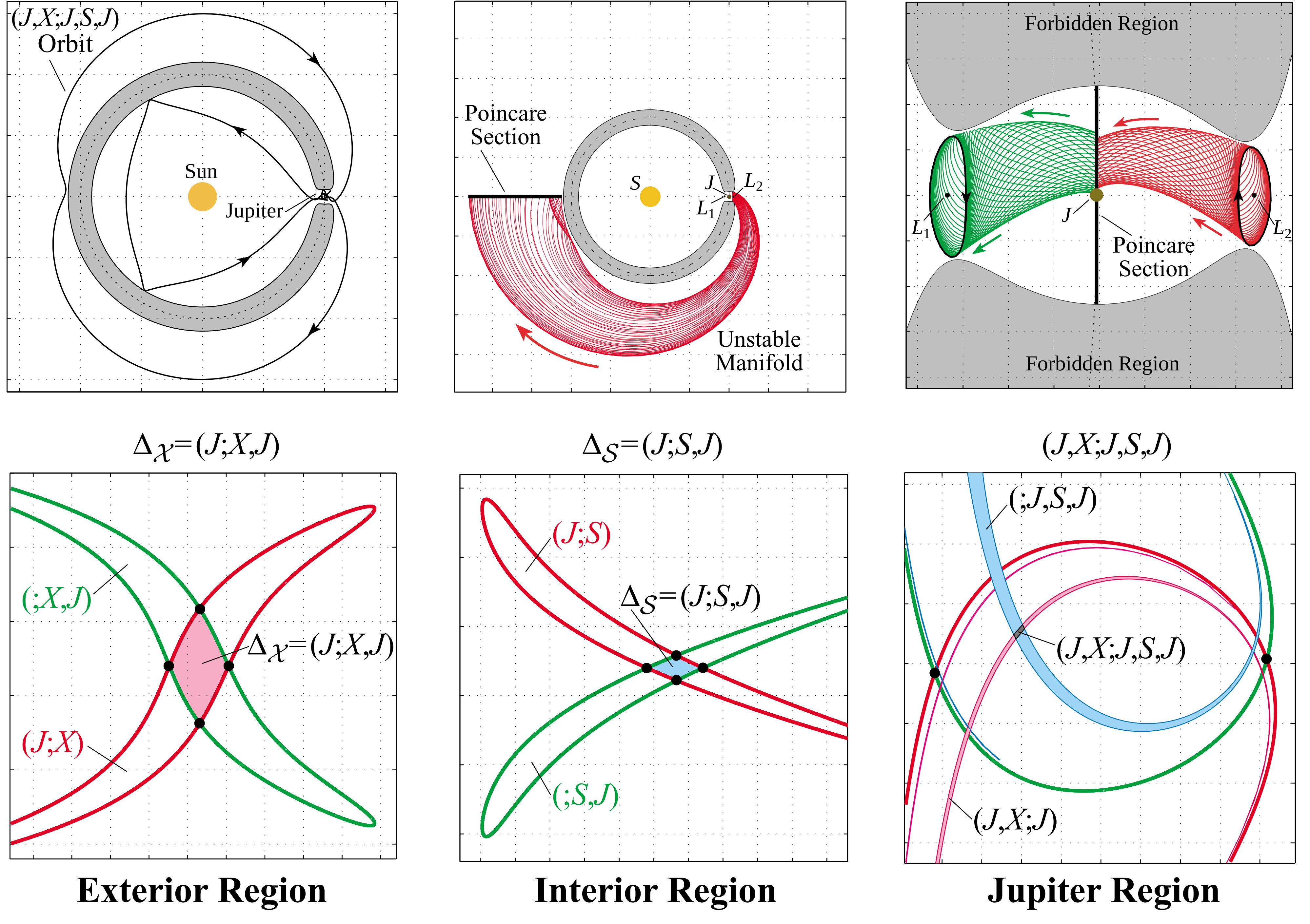
Example: Construction of $(\mathrm{J}, \mathrm{X}; \mathrm{J}, \mathrm{S}, \mathrm{J})$ Orbits

- Illustrate the construction of orbits with ***given itineraries***
- Closely related to constructions in ***symbolic dynamics***
- Invariant manifold ***tubes*** separate ***transit*** from ***nontransit*** orbits—these tubes and their properties are important!
- ***Green curve*** = Poincaré cut of L_1 ***stable manifold***.
- ***Red curve*** = cut of L_2 unstable manifold.
- Any point inside the intersection region Δ_J gives rise to an $(\mathbf{X}; \mathbf{J}, \mathbf{S})$ orbit.



Forbidden Region
Stable
Manifold
Unstable
Manifold
L_1
L_2
J
Forbidden Region
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
0.92
0.94
0.96
0.98
1
1.02
1.04
1.06
1.08
x (rotating frame)
y (rotating frame)
$(;J,S)$
Stable
Manifold Cut
Intersection Region
$\Delta_{\mathcal{J}} = (X;J,S)$
$(X;J)$
Unstable
Manifold Cut
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05
y (rotating frame)
$\dot{y}$ (rotating frame)

- Continue with further Poincaré sections on the inside or outside as appropriate to locate points that satisfy the conditions for the remaining itinerary.


(J,X;J,S,J)
Orbit
Sun
Jupiter
Poincare
Section
S
J
L2
L1
Unstable
Manifold
Forbidden Region
L1
J
Poincare
Section
L2
Forbidden Region
Δ𝒳=(J;X,J)
Δ𝒮=(J;S,J)
(J,X;J,S,J)
(;X,J)
Δ𝒳=(J;X,J)
(J;X)
(J;S)
Δ𝒮=(J;S,J)
(;S,J)
(;J,S,J)
(J,X;J,S,J)
(J,X;J)
Exterior Region
Interior Region
Jupiter Region

- One can ***numerically implement*** this procedure.
- Conley [1968] and McGehee [1969] proved the ***existence of homoclinic orbits*** for both the ***interior and exterior region***.
- Llibre, Martinez and Simó [1985] showed ***analytically*** the existence of ***transversal symmetric (1,1)-homoclinic orbits in the interior region*** under appropriate conditions.
- For our problem, we need to find transversal homoclinic orbits in both interior and exterior regions as well as transversal heteroclinic cycles for the L_1 and L_2 Liapunov orbits.
- Numerical techniques of Gómez, Jorba, Masdemont and Simó [1993] play an important role.

- Comets like Oterma are
  - mostly heliocentric, with the key perturbations are dominated by Jupiter's gravitation.
  - Motion is very nearly in Jupiter's orbital plane
  - Jupiter's small eccentricity (.0483) plays little role during the fast resonance transition (less than or equal to one Jupiter period in duration).
  - The PCR3BP is an adequate starting model for illuminating the essence of the resonance transition process.
- We were motivated by the paper of Belbruno and (Brian) Marsden [1995]; related reference is that of Liao, Saari and Xia [1996], etc. (In our explanations, we don't invoke anything about Arnold diffusion—resonant transition is much simpler than this).

The Genesis Discovery Mission

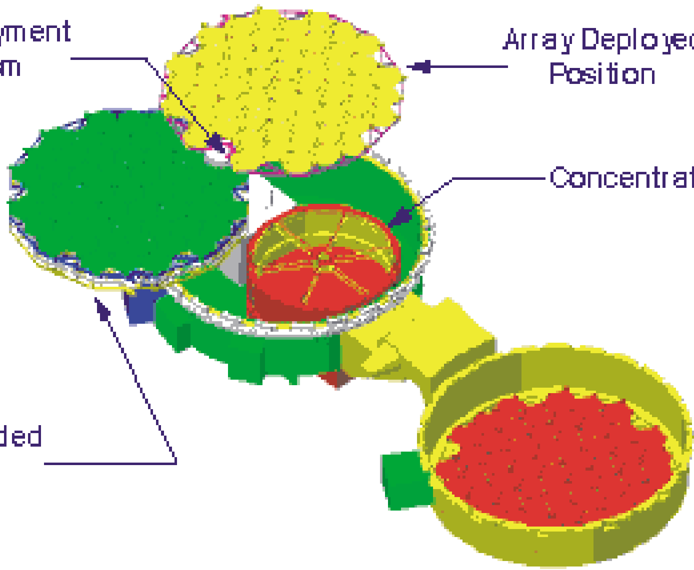
Some General Comments

- ***Mission Purpose.*** To gather solar wind samples and to return them to Earth for analysis.
- ***Mission Constraint.*** Must return in Utah during the daytime
- Will descend with a parachute for a ***helicopter snatch***
- Must have a ***lunar swingby contingency*** in case of bad weather
- ***Highly energy efficient*** (very small Δv required).
- One wants to make use of the dynamical sensitivity to design low cost trajectories—the Genesis trajectory is one example.

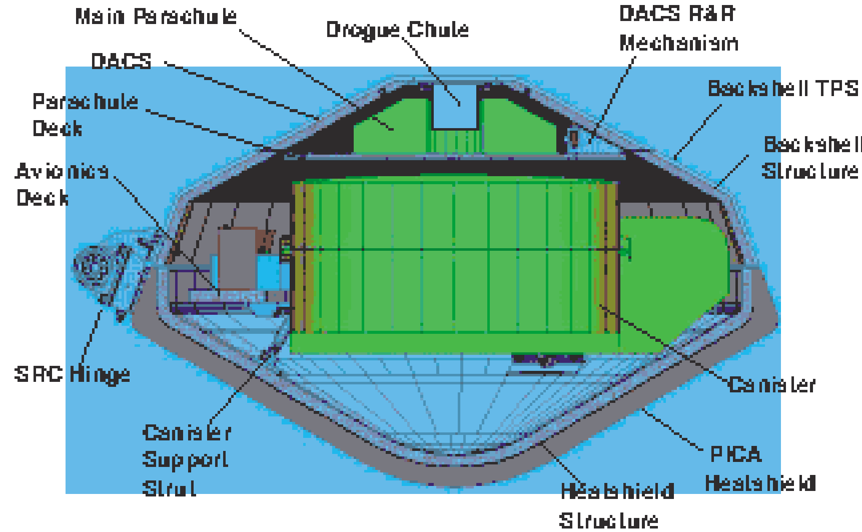
Mission Trajectory

- Final trajectory closely resembles the following four part mission design:
  - **1.** insertion onto an L_1 halo orbit ***stable manifold***.
  - **2.** using saddle point controllers, remain on the halo orbit for about 2 years (4 revolutions)
  - **3.** return to a near halo orbit around L_2 via a near ***hetero-clinic connection***
  - **4.** return to Earth on a near impact orbit (unstable manifold of a halo orbit around L_2.
- Final trajectory uses three dimensional problem and takes into account all the major bodies in the solar system.

Sample Recovery Capsule



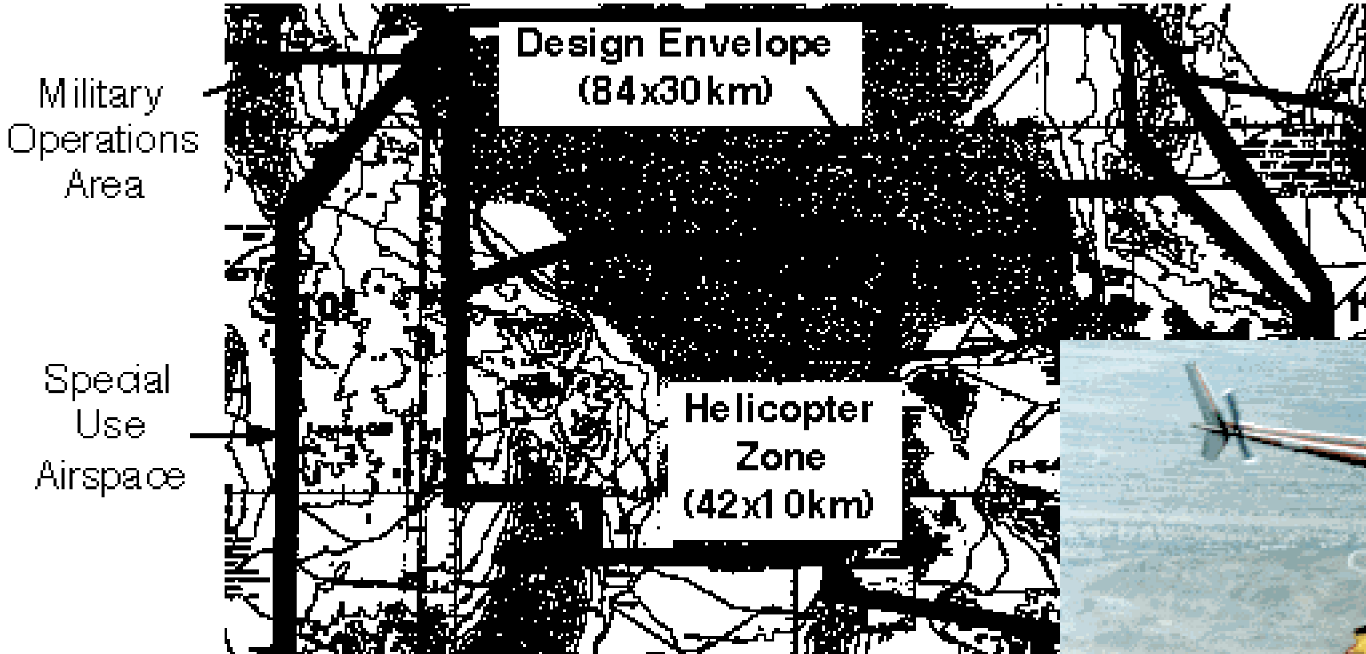
Collector Array Deployed



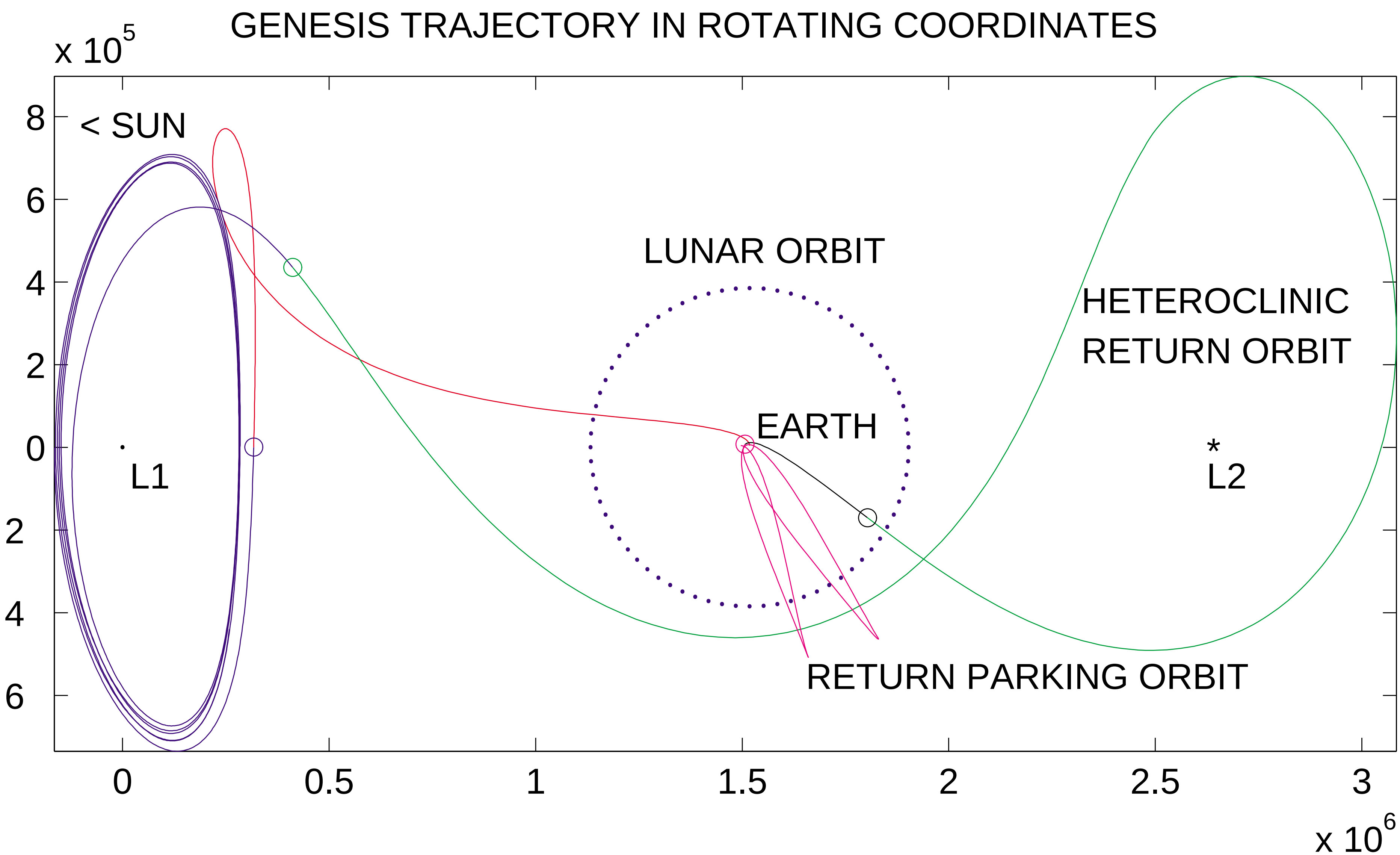
Sample Return Capsule

- Collectors Mostly Ultra-pure Silicon Wafers .6 mm Thick
- Some Metallic Foils, Diamond & Germanium
- Automated Solar Wind Collection In 3 Different Regimes

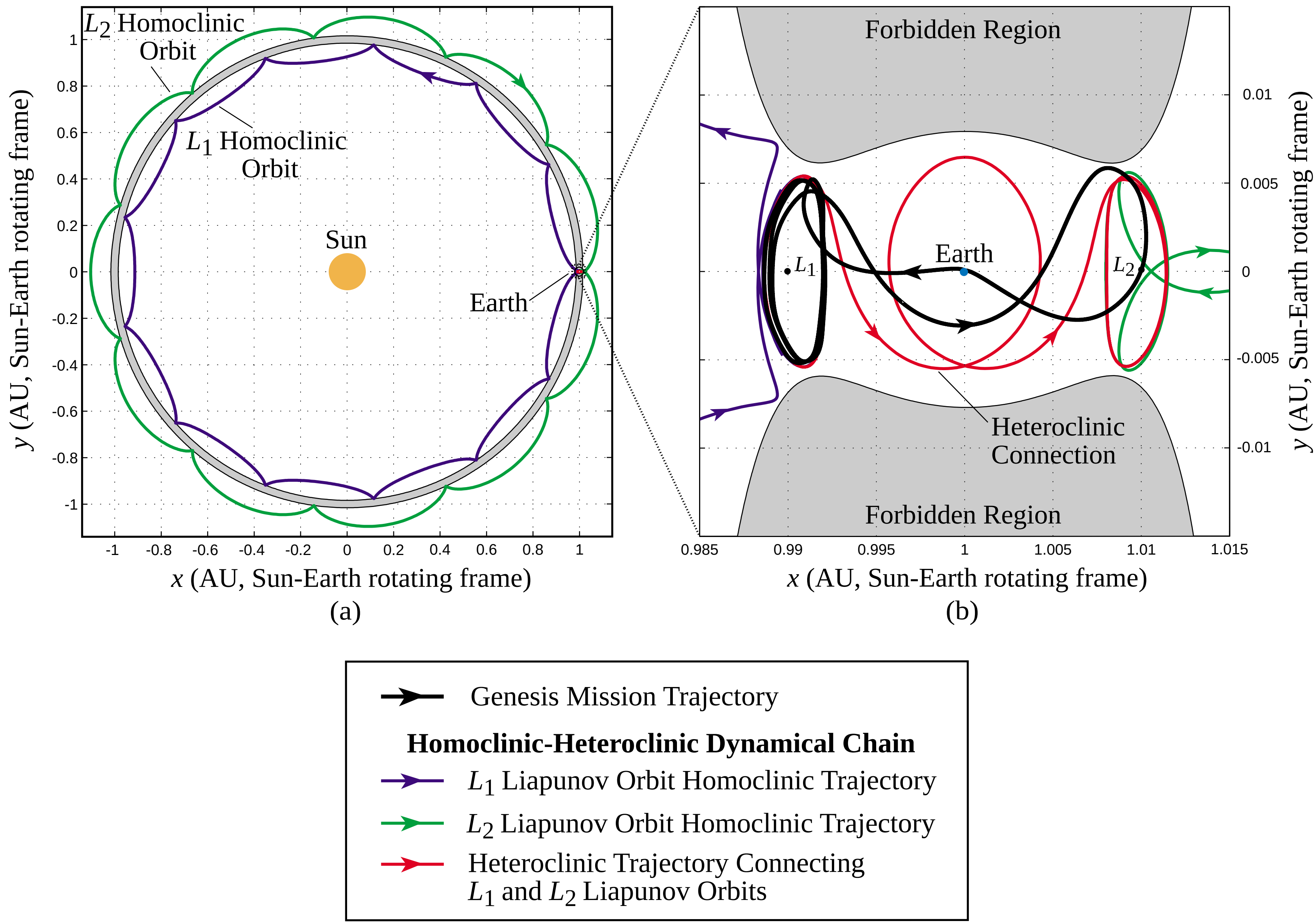
UTTR Mid-Air Recovery

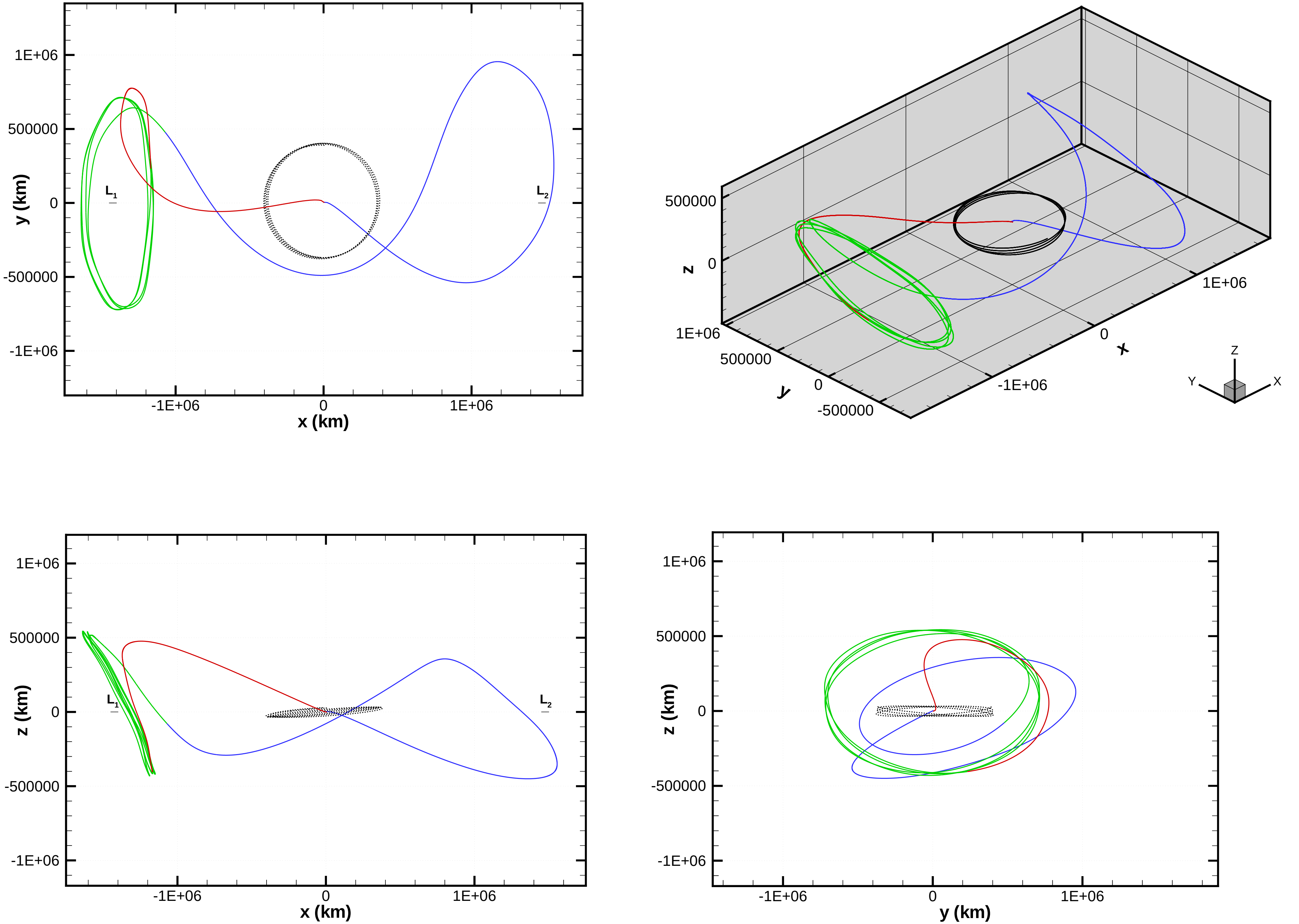


- UTTR 84x30 km Footprint
- Lifting Chute Slow Spiral Descent
- Mid-Air Retrieval By Helicopter At Entry + 14 Minutes
- Post-Recovery Process At NSAS JSC Curatorial Facility


GENESIS TRAJECTORY IN ROTATING COORDINATES
x 10^5
8
6
4
2
0
2
4
6
< SUN
L1
LUNAR ORBIT
HETEROCLINIC
RETURN ORBIT
EARTH
L2
RETURN PARKING ORBIT
0
0.5
1
1.5
2
2.5
3
x 10^6

Homoclinic-Heteroclinic Dynamical Chains Reveal Dynamics Underlying Genesis Mission Trajectory





1E+06
500000
0
-500000
-1E+06
y (km)
x (km)
L1
L2
z
x
y
Z
Y
X
z (km)
y (km)

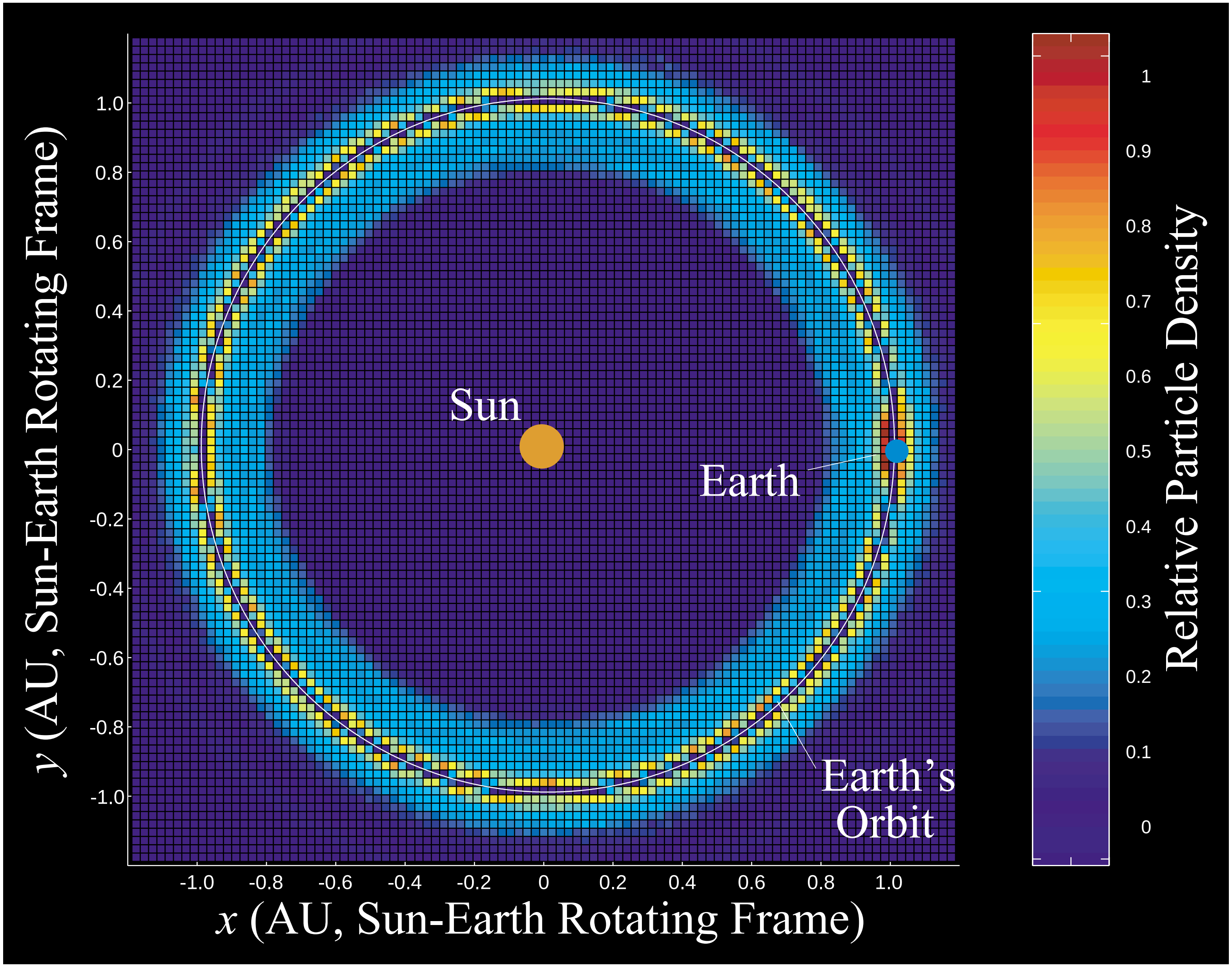
$\boxed{\texttt{Genesis movie}}$

- Potential planet-impacting asteroids may utilize dynamical channels as a pathway to Earth from nearby heliocentric orbits. This phenomena has been observed recently in the impact of comet ***Shoemaker-Levy 9*** with Jupiter.

- These ideas apply to ***any planet or moon system***. Mission flexibility is achieved post-launch making use of dynamical sensitivity — miniscule fuel expenditures can lead to dramatically different trajectories. One could turn a near-Earth mission into an asteroid rendezvous and return mission *in situ* with an appropriately placed small thrust. Rather than being a ***hindrance*** to orbital stability, sensitivity facilitates ***mission versatility***.

- Use homoclinic and heteroclinic structures to understand and compute ***transport*** rates of various objects around the solar system (eg, between Mars and Earth).

- ***Zodiacal Dust Cloud.*** Numerical simulations of the orbital evolution of asteroidal dust particles show that the Earth is embedded in a circumsolar ring of asteroidal dust known as the zodiacal dust cloud (Dermott *et al.* [1994]). Simulations and observations reveal that the zodiacal dust cloud has structure. Viewed in the Sun-Earth rotating frame, there are several high density clumps ($\sim$10% greater than the background) which are mostly evenly distributed throughout the Earth's orbit. The simulations considered the gravitational effects of the actual solar system and non-gravitational forces: radiation pressure, Poynting-Robertson light drag, and solar wind drag. The dust particles are believed to spiral in towards the Sun from the asteroid belt, becoming trapped temporarily in exterior mean motion resonances with the Earth. They are then scattered by close encounters with the Earth leading to further spiraling towards, and eventual collision with, the Sun.

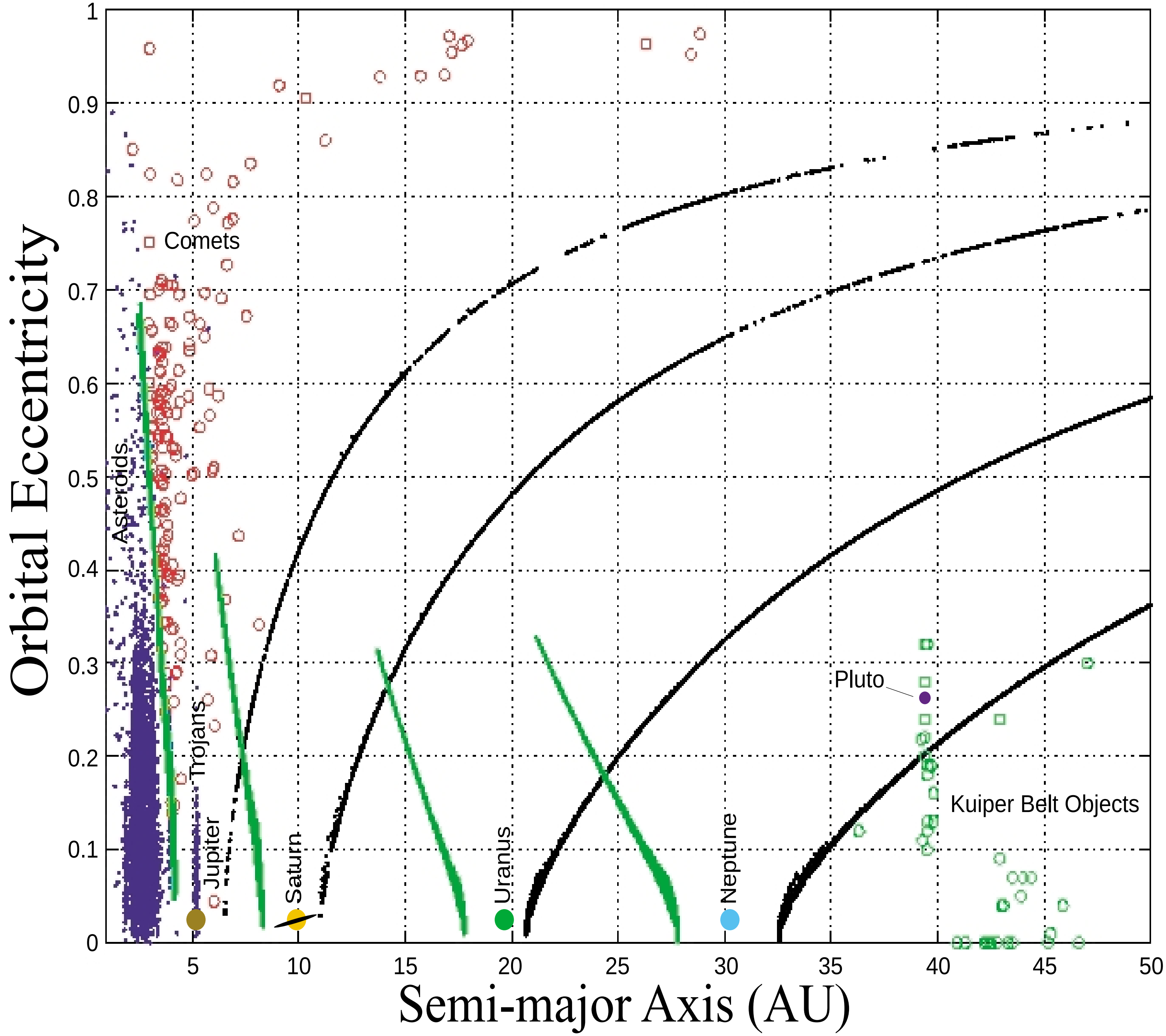


Sun
Earth
Earth's Orbit
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.0
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1.0
y (AU, Sun-Earth Rotating Frame)
x (AU, Sun-Earth Rotating Frame)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Relative Particle Density

- ***Variational and Symplectic Integrators.*** Symplectic integrators for the long time integrations of the solar system is well known through the work of Tremaine, Wisdom and others. In many problems in which the dynamics is delicate or where there are delicate controls, geometric integrators can be useful.
- ***Other Structures in the Solar System.***

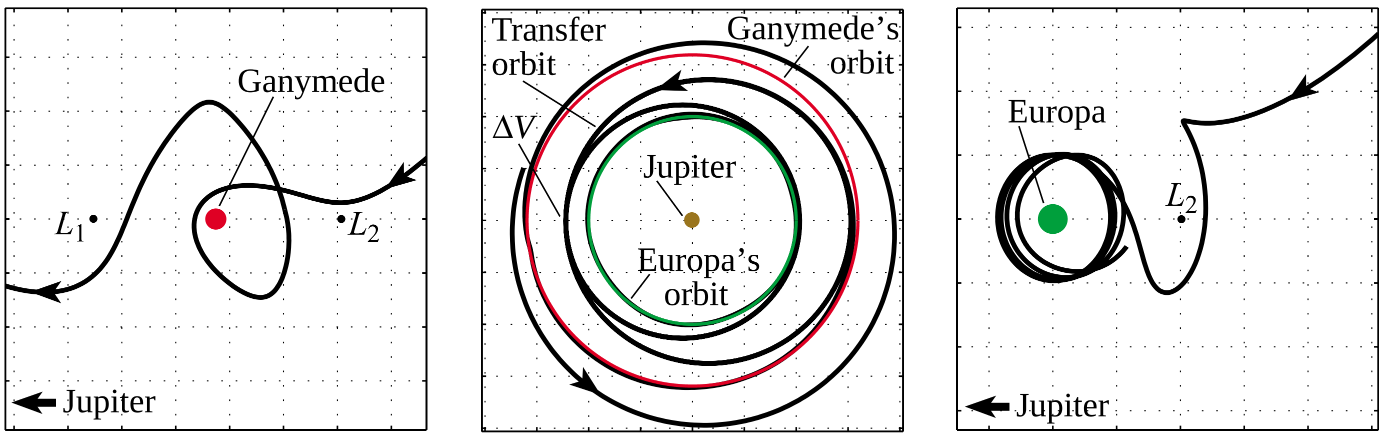
  As Lo and Ross [1997] suggested, further exploration of the phase space structure as revealed by the homoclinic-heteroclinic structures and their association with mean motion resonances may provide deeper conceptual insight into the evolution and structure of the asteroid belt (interior to Jupiter) and the Kuiper Belt (exterior to Neptune), plus the transport between these two belts and the terrestrial planet region.

  The figure plots the (local) semi-major axis versus the orbital eccentricity. We show the L_1 (green) and L_2 (black) manifolds for each of the giant outer planets. Notice the intersections between manifolds of adjacent planets, which leads to chaotic transport. Also shown are the asteroids (blue dots), comets (red circles), and Kuiper Belt objects (green circles).

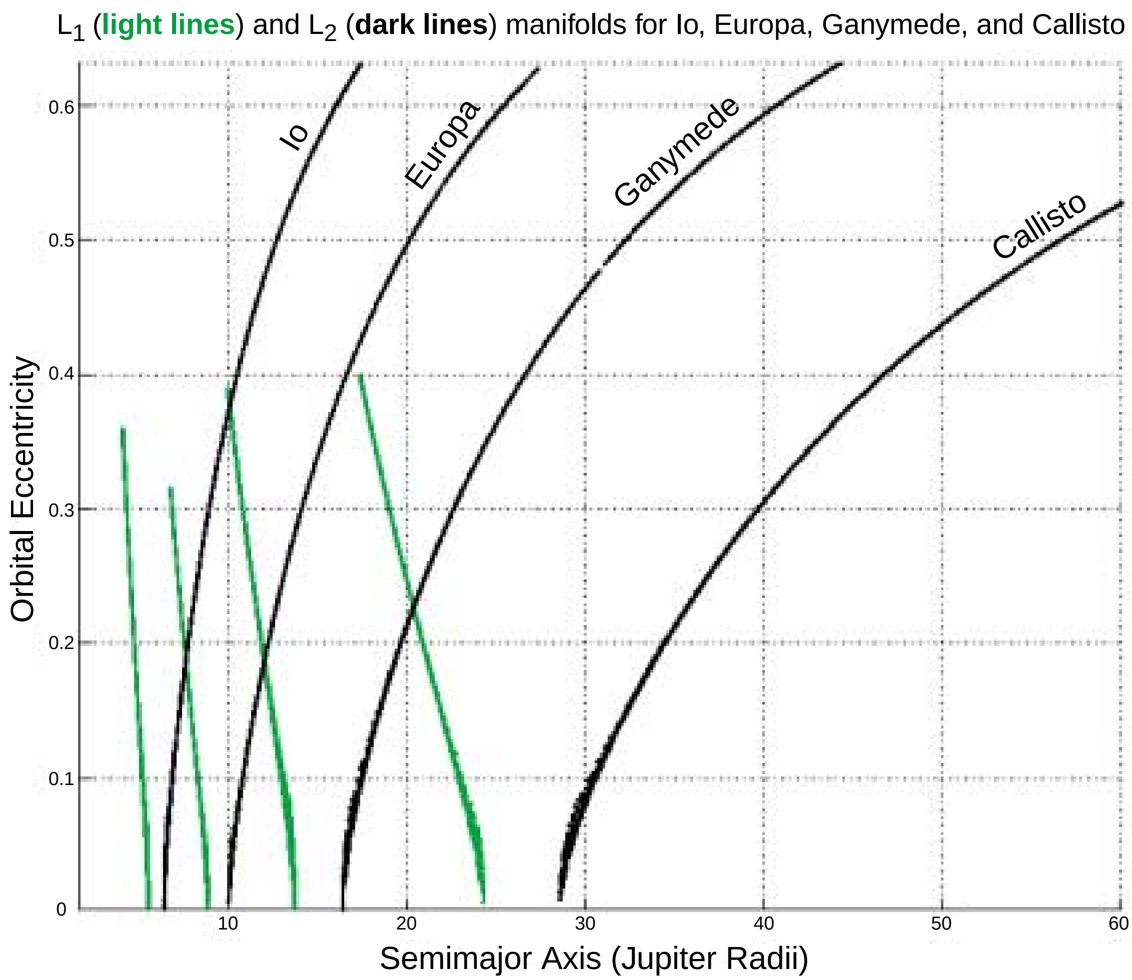

Orbital Eccentricity
Semi-major Axis (AU)
Comets
Asteroids
Trojans
Jupiter
Saturn
Uranus
Neptune
Pluto
Kuiper Belt Objects
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
5
10
15
20
25
30
35
40
45
50

"Petit Grand Tour" of Jupiter's moons

- Construction of some new trajectories that visit Europa and Ganymede.
- *Example*:
  - 1 orbit around **Ganymede**.
  - 4 orbits around **Europa**, etc.



- Idea is to use burns that enable a transfer from one three body system to another.


L1 (light lines) and L2 (dark lines) manifolds for Io, Europa, Ganymede, and Callisto
Io
Europa
Ganymede
Callisto
Orbital Eccentricity
0
0.1
0.2
0.3
0.4
0.5
0.6
10
20
30
40
50
60
Semimajor Axis (Jupiter Radii)

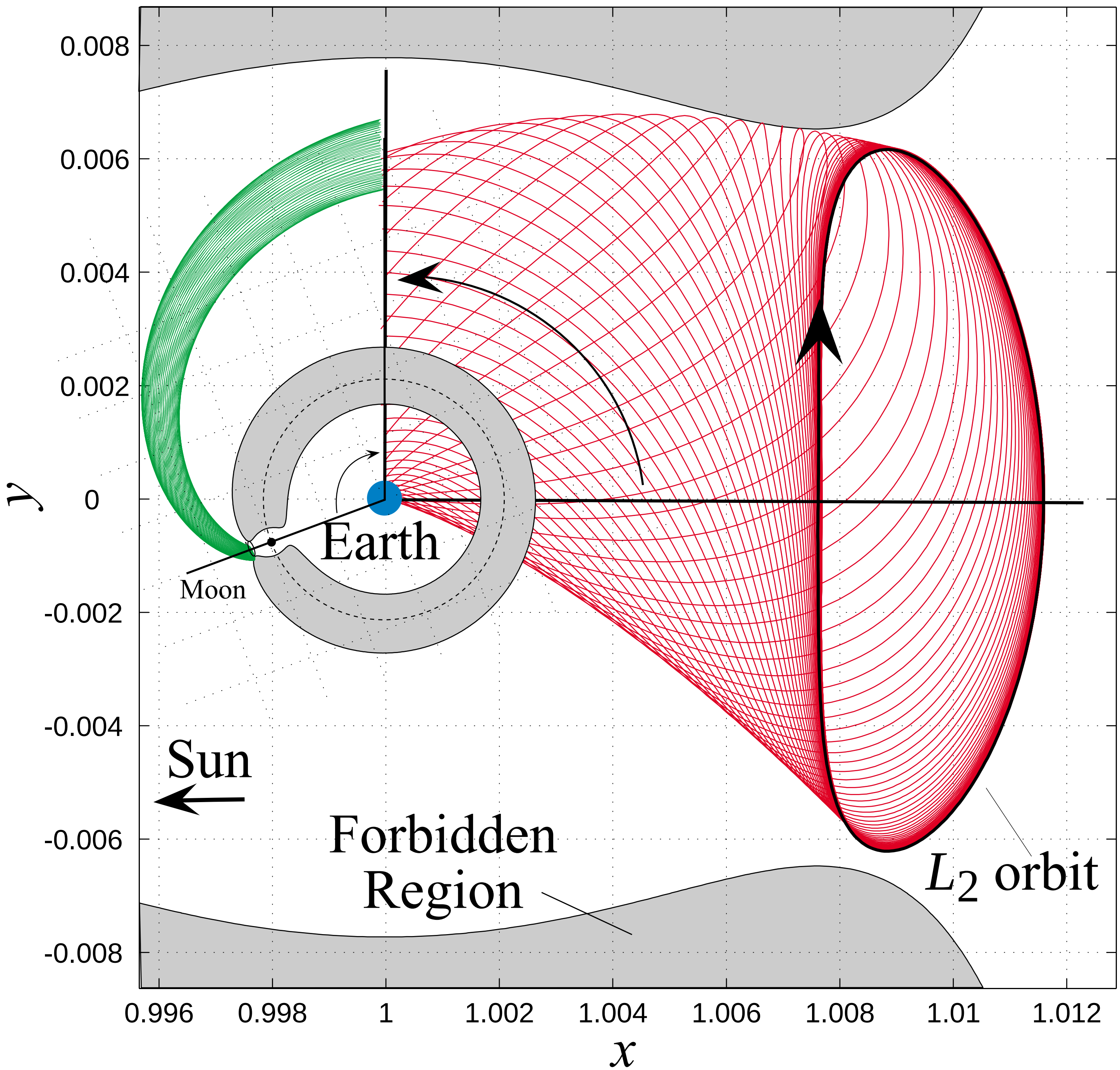
`Petit Grand Tour movie`

Lunar Capture: How to get to the Moon Cheaply

- In 1991, the failed Japanese mission, Muses-A (Uesugi [1986]), was given new life with a radical new mission concept and renamed as the ***Hiten Mission*** (Tanabe et al. [1982], Belbruno [1987], Belbruno and Miller [1993]).
- We present an approach to the problem of the orbital dynamics of this interesting trajectory by ***implementing in a systematic way*** the view (Barcelona group and Belbruno) that the Sun-Earth-Moon 4 body system can be modelled as ***two coupled 3 body systems***.
- Within each 3 body system, using our understanding of the invariant manifold structures associated with L_1 and L_2, we transfer from a 200 km Earth orbit into the region where the invariant manifold structure of Sun-Earth Lagrange points interacts with the

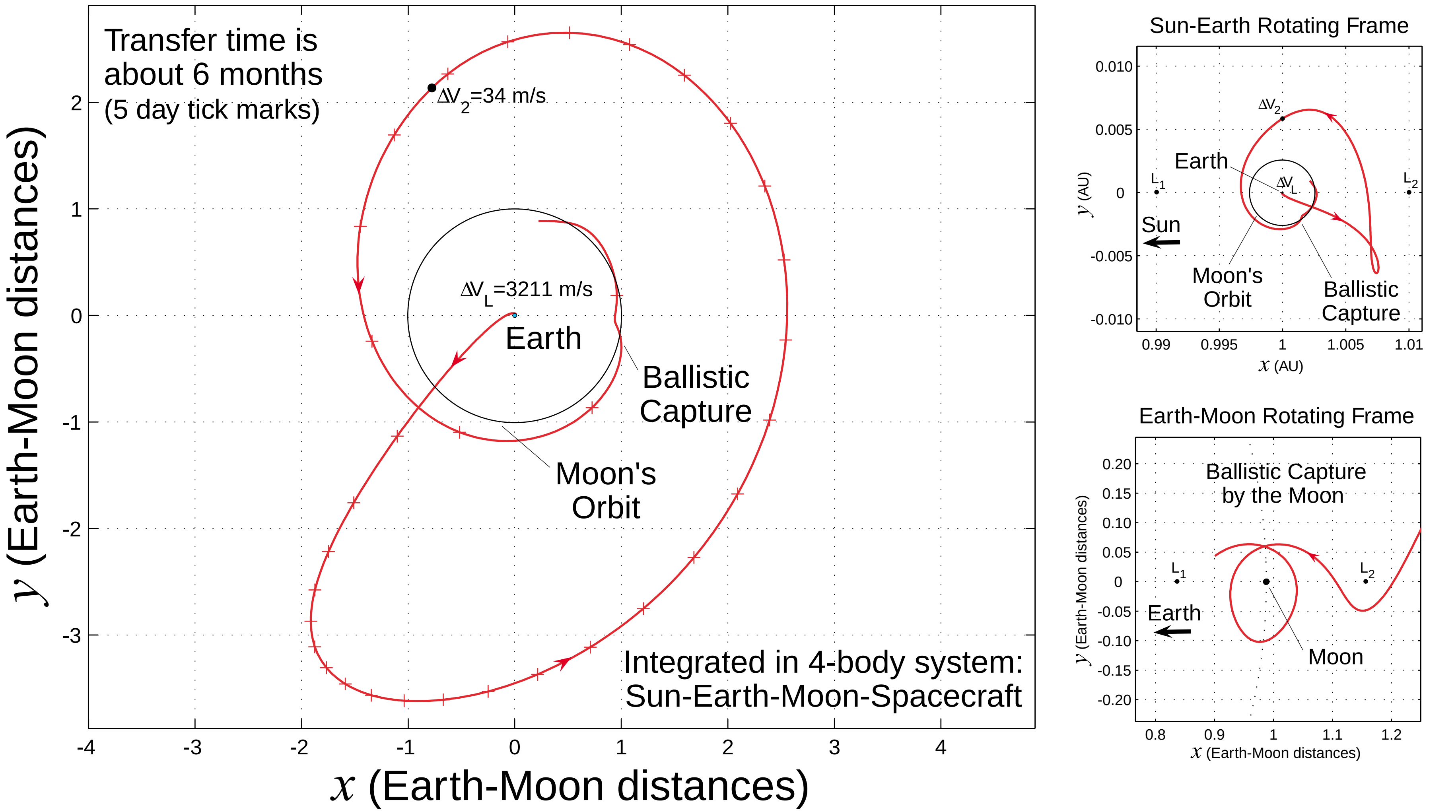
invariant manifold structure of the Earth-Moon Lagrange points.

- One utilizes the sensitivity of the "twisting" near the invariant manifold tubes to target back to a suitable Earth parking orbit.
- This interaction permits a low energy transfer from the Sun-Earth system to the Earth-Moon system. The invariant manifold tubes of the Earth-Moon system provide the dynamical channels in phase space that enable ballistic captures of the spacecraft by the Moon.
- The results are then checked by integration in the bicircular 4-body problem. It works!
- This technique is somewhat cheaper (18% less ΔV) than the usual Hohmann transfer (jumping onto an ellipse that reaches to the Moon, then accelerating to catch it, then circularizing). However, it also takes longer (6 months as opposed to 5 days).



0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
y
0.996
0.998
1
1.002
1.004
1.006
1.008
1.01
1.012
x
Earth
Moon
Sun
Forbidden Region
L_2 orbit

Ballistic Capture of a Spacecraft by the Moon from 200 km Altitude Earth Parking Orbit (Shown in an Inertial Frame)



Movie: Ballistic capture inertial

Movie: Ballistic capture rotating

Optimal Insertion into a Halo Orbit

- ***Halo Orbit Insertion*** goes back to the early days of L_1 halo-orbit missions (eg, Farquhar et al [1980] for the ISEE-3 mission launched in 1978).
- We study ***optimal control*** in the context of mission design with the aid of dynamical systems and invariant manifolds. In particular, we consider optimizing burns for ***halo orbit insertion*** of ***Genesis*** type missions, although the methods are rather general.
- ***Low thrust*** and ***impulsive burn*** contexts are both important and the techniques can handle either case.
- Optimization software COOPT (COntrol–OPTimization) is used to do an optimization of the ***cost function*** (minimizing ΔV) subject to the ***constraint of the equations of motion***. We vary the number of impulses and also consider the effect of delaying

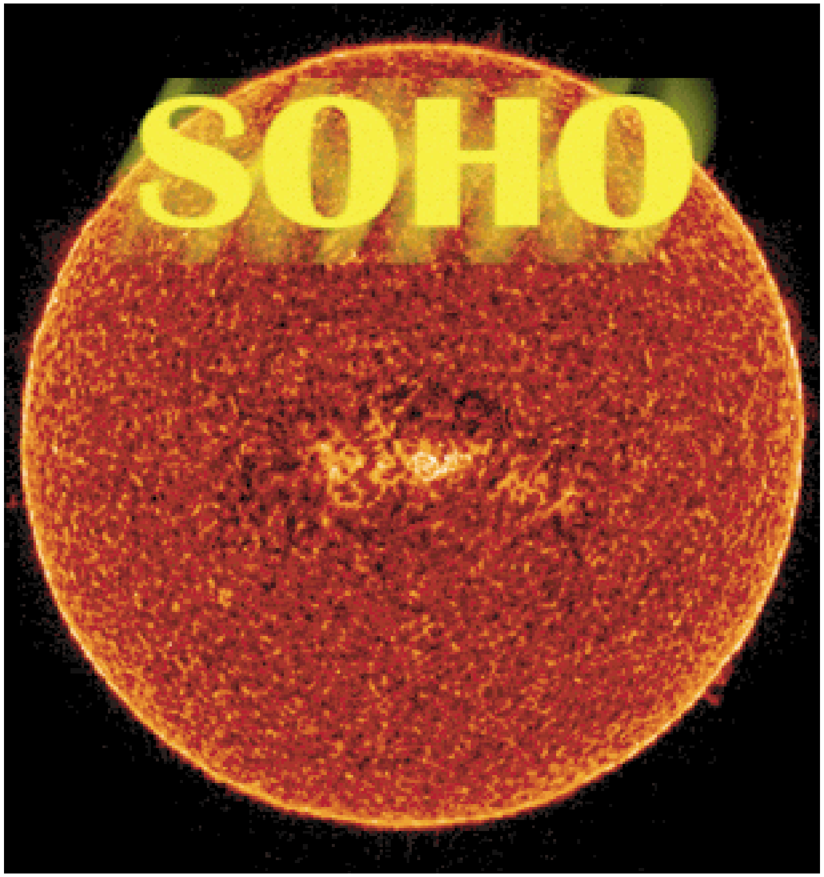
the first impulse.

- COOPT rather general software for ***optimal control and optimization of systems modeled by differential-algebraic equations*** (DAE), developed by the Computational Science and Engineering Group at University of California Santa Barbara. It has been designed to control and optimize a general class of DAE systems which may be quite large. It uses multiple shooting and SQP techniques to do the optimization.
- Due to the problem sensitivity and the ***instability*** of the halo orbit, an ***accurate first guess*** is essential—provided by a high order analytic expansion of minimum 3rd order using the Linstedt Poincaré method (Simo, Howell and Pernicka, Chua and Parker).
- Simo et al, in conjunction with the SOHO mission in the 1980's were the first to study invariant manifolds of the halo orbit.

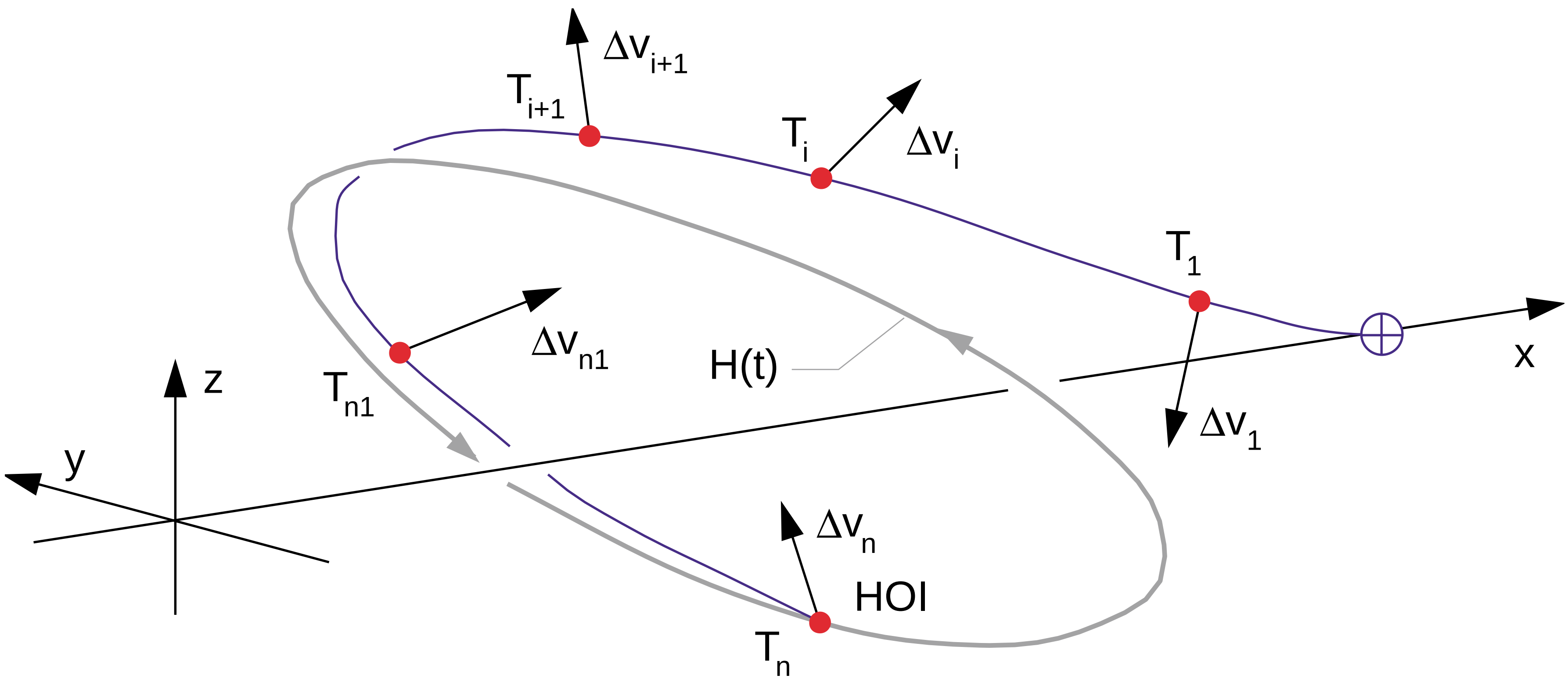
http://sohowww.nascom.nasa.gov/

THE SOLAR AND HELIOSPHERIC OBSERVATORY

March 09, 2000 21:47:27 UT - Mission Day: 1560 - DOY: 69

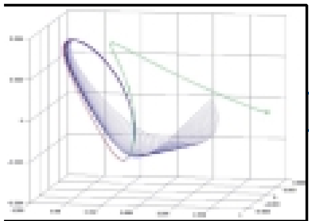


- ***Stable manifold of the halo orbit***–used to design the transfer trajectory which delivers the Genesis spacecraft from launch to insertion onto the halo orbit (HOI). ***Unstable manifold***–used to design the return trajectory which brings the spacecraft and its samples back to Earth via the ***heteroclinic connection***.
- Expected error due to launch is approximately 7 m/s for a boost of approximately 3200 m/s from a 200 km circular altitude Earth orbit. This error is then optimally corrected using impulsive thrusts. Halo orbit missions are ***very sensitive to launch errors***.
- ***Objective:*** *Find the maneuver times and sizes to minimize fuel consumption for a trajectory starting at Earth and ending on the specified halo orbit around the Lagrange point* L_1 *of the Sun-Earth system at a position and with a velocity consistent with the HOI time.*



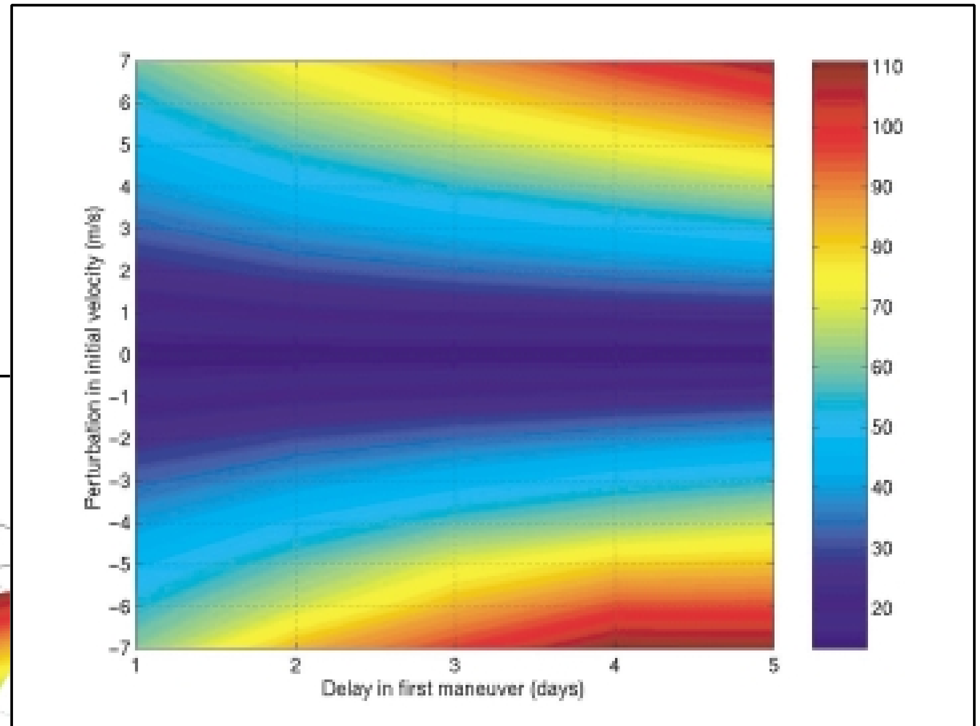
Δv_{i+1}
T_{i+1}
T_i
Δv_i
T_1
x
Δv_{n1}
H(t)
z
T_{n1}
Δv_1
y
Δv_n
HOI
T_n

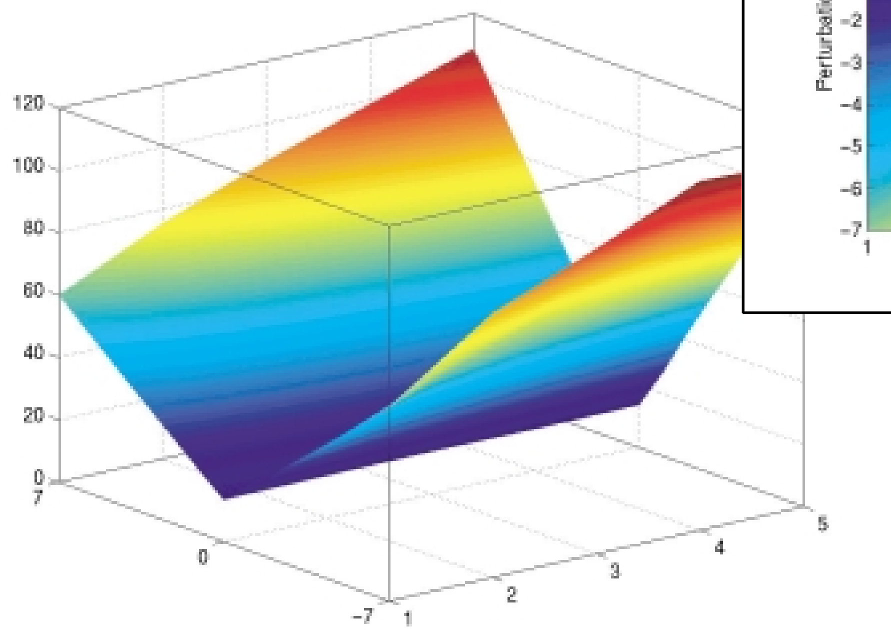
$\boxed{\texttt{Halo Insertion Movie}}$



Parametric Study of the Optimal Solution

Number of maneuvers:
- **Unperturbed injection velocity: 1**
- **Perturbed injection velocity: 2**





Influence of:
- **Delay in TCM1**
- **Perturbation in launching velocity**

Optimal solutions found for all cases

End

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